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Essays on Factor Reallocation and General Equilibrium Analysis

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by

Xintong Yang

Committee in Charge:

Professor Peter Rupert, Chair

Professor Javier Birchenall

Professor Marek Kapička

Professor Cheng-Zhong Qin

March 2016

The Dissertation of
Xintong Yang is approved:

Professor Javier Birchenall

Professor Marek Kapička

Professor Cheng-Zhong Qin

Professor Peter Rupert, Committee Chairperson

January 2016

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Acknowledgements

For me, graduate school is a journey to find who I truly want to be for life. Six years ago, I came to the US for the first time, knowing nothing about the connection between myself and the real world. During the six years, I have experienced lots of confusions and struggles. It was not until two years ago when I figured out what I truly want and who I truly want to be. It is a difficult transition for me. During this difficult process, first and foremost, I want to thank my family. My mom, Min Xu, gives me her self-less love and care from every aspect of life. It is her, who teaches me how to love and care for others. My dad, Luping Yang, provides me with unconditional supports and encourages me to accomplish my intellectual pursuit fearlessly. My husband, as well as a long-time friend, Fudong Zhang, keeps the company along this journey, and loves me in a way to challenge me and provoke me to reflect myself, and to grow from the reflection.

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Curriculum Vitæ

Xintong Yang

Education

Ph.D. Economics, University of California, Santa Barbara, 2016

M.A. Economics, University of California, Santa Barbara, 2010

B.S. Mathematics and Economics, Central University of Finance and Economics, 2009

Research Fields

Macroeconomics, Economic Growth, Housing and Real Estate

Committee Members

Peter Rupert (Chair), Javier Birchenall, Marek Kapička, Cheng-Zhong Qin

Honors and Fellowships

Excellent Teaching Assistant Award, UC Santa Barbara, 2013

Graduate Dean's Advancement Fellowship, UC Santa Barbrara, 2012

Jennifer Jo Williamson Fellowship, UC Santa Barbara, 2011

Top Scholarship of Academy, Central University of Finance and Economics, 2007-2009

Professional Experience

Referee Review for European Economic Reivew, 2013

Newsletter Contributor for Laboratory for Aggregate Economics and Finance, 2011 -present

Teaching Experience

Applied Probability Theory, 2015

Corporate Finance (Master level), 2013-2014

Macroeconomic Theory (Master level), Winter 2013

Corporate Finance, Fall 2011-2014

Microeconomic Theory, Fall 2010

Principles of Macroeconomics, Fall 2010

Abstract

Essays on Factor Reallocation and General Equilibrium Analysis

Xintong Yang

My dissertation studies the sectoral and regional reallocation of capital and labor for an economy in transition using general equilibrium model framework. The first chapter, "Capital in Transition: Housing and Sectoral Reallocation in the Long Run", studies the sectoral allocation of capital between housing and non-residential sectors using a two-sector general equilibrium model in a neoclassical growth environment. Calibrated to both the United States and China, the model can account for both the positive correlation between the share of housing capital and the consumption-output ratio in the United States and the negative correlation between the share of housing capital and the consumption-output ratio in China. The calibration to the Chinese economy implies that the rapid increase in the share of housing capital and the simultaneous decrease in the consumption-output ratio observed in China can be rationalized by a combination of three factors: a high elasticity of substitution between the two sectors, a high capital intensity of production of the housing sector, and a low initial share of housing capital before the Chinese housing market reform. This paper provides a tractable framework to understand the sectoral allocation of capital between housing and non-residential sectors across countries.

The second chapter, "Human Capital Spillover and Housing price", provides a model framework to study the relationship between the external effect of human capital and housing price growth in the SEZ economy in China. In a two-region model, high wage in the SEZ region reflects high level of human capital, and these jobs are not available to low human capital migrants from the non-SEZ economy. The migrants come to the SEZ economy for two reasons: on the one hand, the SEZ economy is a better place to accumulate human capital and earn a higher wage in the future; on the other hand, the SEZ economy has a better amenities for living. In the baseline model with migration, the share of population that choose to migrate to the SEZ economy is determined by the utility equalization between living in either economy. In the baseline model, the migration occurs all at once at the first period. Further, I extend the baseline model by incorporating the spillover effect of human capital: time invested in human capital accumulation has a higher return in high human capital environment. In this case, the migration to the SEZ economy becomes increasingly attractive as the gap between the human capital leaders and followers increase. By comparing the extended model with the baseline, I capture the significant positive impact of human capital spillover on the increase of housing prices.

The third chapter, ''Dynamic Arrow-Debreu Economy for General Equilibrium Analysis", coauthored with Cheng-Zhong Qin, develops a dynamic Arrow-Debreu abstract economy to more closely capture the timing of moves of Walrasian gen-

eral equilibrium model. Instead of inducing a pseudo game, the extensive form of the dynamic Arrow-Debreu abstract economy is well defined. As such, various game-theoretic solutions with and without symmetric information can be applied. We show that the set of subgame-perfect equilibrium allocations coincides with the set of Walrasian equilibrium allocations when information is symmetric. The set of perfect Bayesian equilibrium allocations coincides with the set of rational expectations equilibrium allocations when information is asymmetric. These results are useful for analyzing and refining Walrasian and rational expectations equilibrium allocations.

Contents

List of Figures

List of Tables

Chapter 1

Capital in Transition: Housing and Sectoral Reallocation in the Long Run

1.1 Introduction

Developed economies have experienced a sectoral reallocation of capital over the course of their long-run economic development. The share of housing capital has grown annually by approximately 0.2 percent for the United Kingdom, France, Germany, Canada, and the United States since 1700.[1] While the sectoral

[1]The decomposition of domestic capital follows [28]. Domestic capital is broken down into three categories: agricultural land, housing (including residential structure and land value), and nonresidential capital (including non-residential structure, equipment and machinery, and intellectual property products). From here on, the share of housing capital is defined as the share of value of housing capital out of the sum of value of non-residential and housing capital.

reallocation of capital happens gradually in these five developed economies, it has been rapid in China. Since the housing market reform in the 1980s, the share of housing capital in China has increased by 1.5 percent annually.[2]

The question I address in this paper is whether standard neoclassical growth theory can qualitatively and quantitatively explain the features of the sectoral reallocation of capital not only for developed economies but also for China. I begin by documenting the empirical regularities regarding the share of housing capital across developed economies and China. Alongside the different annual growth rate of the share of housing capital, another salient difference between the two types of economies is the correlation between the share of housing capital and the consumption-output ratio. On the one hand, the share of housing capital is positively correlated with the consumption-output ratio across developed economies, but on the other hand the correlation between the two variables has been negative in China since 1987. While there are many institutional differences between developed economies and China, I start with a simple framework to investigate the factors that determine the observed differences. I find that in a standard two-sector neoclassical growth framework, the distinction between two key parameters, the elasticity of substitution between the two sectors and the capital intensity of production of the housing sector, can explain the differences observed in the two types of economies.

[2]See Section 1.2 for a detailed discussion of the empirical regularities of sectoral allocation of capital.

I build a two-sector general equilibrium model with housing and non-residential sectors. The model features preferences with a constant elasticity of substitution between the two sectors, and Cobb-Douglas production technologies within each sector. The two types of capital are treated symmetrically and are endowed with dual functions: each unit of capital can be used as a factor input of production and a capital good that generates a rental return. In a frictionless environment, the equilibrium allocation features a balanced growth path with a constant sectoral allocation of capital between the two sectors. During the transitional dynamics, the correlation between the share of housing capital and the consumption-output ratio is determined by the interplay of the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors.

Calibrating the model to the United States and China, I examine whether the dynamics proposed by the model are consistent with empirical observations. With plausible parameters, the model generates reallocations that are consistent with the experiences of the United States and China. Moreover, the model can account for (on the one hand) the positive correlation between the share of housing capital and the consumption-output ratio in the United States, and (on the other hand) the negative correlation between the share of housing capital and the consumption-output ratio in China. In particular, the rapid increase in the share of housing capital and the decrease in the consumption-output ratio observed in China can be explained by a combination of three factors: a high elasticity of substitution between housing and non-residential sectors, a high capital intensity of production of the housing sector, and a low initial share of housing capital before the Chinese housing market reform.

To the extent that the model can reproduce the key features of the data in China, I apply the model to quantify the effect of the initial share of housing capital before the housing market reform on the Chinese economy. Given that there was no market for housing before the reform in the mid-1980s, the initial sectoral allocation when the market mechanism starts to work is a key factor to study the Chinese economy. When changing the Chinese sectoral capital allocation in 1987 to the US level of a comparable development stage, the comparative study suggests that the initial low share of housing capital before the housing market reform has led to an over-investment in housing, and an under-investment in non-residential capital since 1987 in China.

This paper makes four main contributions to the literature on structural change and housing. First, this paper highlights the role of the share of capital in the process of structural change, complementing the existing structural change literature that focuses on labor reallocation ([24], [27] and [6][3]). The model of [1] features both capital and labor reallocation between sectors with differentiated capital in-

[3] [24] study the structural change in production through labor reallocation using a nonhomothetic preference that features different income elasticities of demand among different sectors; [27] use a preference with constant elasticity of substitution and defines the structural change as a change in labor share; [6] investigate the role of agricultural productivity on the economic growth and sectoral allocation of labor and production for China's post-reform economy, using a two-sector model with nonhomothetic preferences and Cobb-Douglas production functions.

tensities of production, but does not consider the housing service consumption in the utility function. Building on the theory of [1], this paper stresses a novel mechanism to explain the different patterns of the correlation between the share of housing capital and the consumption-output ratio.

Second, this paper characterizes the transitional dynamics of sectoral allocation of capital. The model inherits the features of home production models where housing service consumption enters the preferences as an object of interest ([3], [20] and [11]). While most of the literature focuses on the business cycle properties of housing, few discuss long-term trends and the transitional dynamics of capital allocation along with the impact they have on economic development. A growing body of literature studies the effect of financial market liberalization on the acceleration of capital reallocation to the housing sector ([15], [23], and [32]). This literature features a life-cycle model with heterogeneous agents under an incomplete market environment. These models omit the supply side of housing capital. Also, the numerical solution procedure might obscure certain economic mechanisms. This paper complements the literature by studying the long-term trend of capital allocation with transitional dynamics and their macroeconomic implications.

Third, this paper provides a theoretical framework to study the correlation between the share of housing capital and the consumption-output ratio. The spillover effect of housing wealth on consumption growth is well studied by the empirical literature ([7], [17] and [22]). It is empirically shown that changes in housing wealth have a larger impact than changes in other financial assets in influencing household consumption. While data confirm the higher contemporary correlation between housing wealth and consumption, the causal relationship between housing wealth accumulation and consumption/saving motive is not clear. This paper proposes using the share of housing capital to study the interaction between the housing wealth and household consumption.

Last, this paper develops a unified theory to explain different empirical patterns across developed economies and China within the neoclassical growth framework. In the literature, the models applied to study developed economies and China are separated given the different empirical observations ([18] and [8]). This paper calibrates a standard two-sector neoclassical growth model to both the United States and China, and showcases the model's ability to reproduce the features that are consistent with both the United States and China. By providing a comparative perspective of the study of China with that of developed economies, this paper justifies the applicability of neoclassical growth theory.

The rest of the paper is organized as follows. Section 2 documents the empirical regularities regarding the sectoral allocation of capital across developed economies and China. Section 3 describes the model environment. Section 4 presents the model calibration to the United States. Section 5 presents the calibration to China. Section 6 draws conclusions.

1.2 Empirical Regularities of Sectoral Allocation of Capital

In this section, I document three major features regarding the share of housing capital across countries. First, I show that there is a consistent decline over time in the value of agricultural land, which is accompanied by a rise in the value of housing and non-residential capital. Second, I demonstrate a positive correlation between GDP per capita and the share of housing capital in the United States and China. Finally, I show that the correlation between the share of housing capital and the consumption-output ratio is positive across developed economies, but is negative in recent decades in China.

1.2.1 Structural Transformation of Capital and Share of Housing Capital

Figure 1.1 shows that in the United Kingdom, France, Germany, Canada, and the United States, the total value of capital, measured as a fraction of national income, has not changed much over time, but that the capital structure has been transformed: the value of land has gradually been replaced by the value of non-residential and housing capital.[4] In contrast, China experienced the same structural transformation of capital within a short period of 30 years.

[4] More precisely, the capital-output ratio presents a U-shaped pattern for the United Kingdom, France, Germany, and Canada due to WWII. For the United States, the U-Shaped pattern is less strong.

Table 1.1: Share of Housing Capital out of Total Value of Capital

	1700	1800	1900	2000	2010
UK	0.333	0.287	0.386	0.404	0.555
FRA	0.437	0.416	0.412	0.568	0.610
GER		0.227	0.345	0.581	0.621
USA		0.340	0.291	0.415	0.400
CAN		0.276	0.361	0.568	0.610
	1987	1994	2001	2008	2013
China	0.303	0.220	0.314	0.340	0.458

Data Sources:

Computed using non-residential capital and housing from [28] for the five developed economies; from Table 1.13 for China.

Table 1.1 summaries the evolution of the share of housing capital across the five developed economies since 1700, and in China between 1987 and 2013. As shown in the table, China has experienced a rapid increase in the share of housing capital since 1987. The percentage increase of the share of housing capital within 30 years in China is at the same level with that of the United States over a hundred years during the 20th century.

1.2.2 GDP Per Capita vs. Share of Housing Capital

Figures 1.2 and 1.3 show that the share of housing capital is positively correlated with GDP per capita. This is true for China during its 30-year transition and the United States throughout its longer-term transition. The correlation is insignificant for the US economy from 1950 to 2011, consistent with the notion that the postwar US economy is on a "balanced growth path".

1.2.3 Share of Housing Capital vs. Consumption-Output Ratio

Figure 1.4 documents an unconditional correlation between the average share of housing capital and the average consumption-output ratio between 1995 and 2013 for OECD countries. There is a significant positive correlation between the share of housing capital and the consumption-output ratio across OECD countries. When looking at the correlation between the two variables for the postwar US economy as shown in Figure 1.5 (b), the positive correlation remains.

However, during the 30-year transition in China, the correlation between the share of housing capital and the consumption-output ratio becomes negative. As shown in Figure 1.5 (a), the time-series plot of the unconditional correlation between the share of housing capital and the consumption-output ratio present a significant negative correlation.

The three empirical regularities regarding the share of housing capital point to three facts. First, the share of housing capital positively correlates with the income of an economy in transition. Second, the growth rate at which the share of housing capital increases has been dramatically different between developed economies and China. Lastly, the consumption-output ratio is negatively correlated with the share of housing capital in China, which contrasts with the positive relationship seen in the developed economies.

1.3 The Model

In this section, I present the neoclassical growth model environment, a two-sector model with exogenous technological progress. Capital and labor (re)allocation on both the balanced growth path and the transitional dynamics are characterized.

1.3.1 The Environment

The model economy is infinite horizon. Time is discrete. There is a representative household with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, s_t) \tag{1.1}$$

where c is the consumption of non-residential goods and services, s is the consumption of housing services, and β is the utility discount factor. Labor is supplied inelastically and normalized to one. The instantaneous utility function combines the two types of consumption with a constant elasticity of substitution $\epsilon \in [0, \infty)$:

$$u(c, s) = \frac{\left[(\eta c^{\frac{\epsilon-1}{\epsilon}} + (1-\eta) s^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}\right]^{1-\sigma} - 1}{1-\sigma}$$

where $\eta \in (0, 1)$ indicates the preference weight between the two types of consumption; and $\frac{1}{\sigma} \in [0, \infty)$ denotes the intertemporal elasticity of substitution.[5]

Final output in the two sectors are produced with the following production functions:

$$y_t = k_t^{\alpha_k}(z_t(1 - l_t))^{1-\alpha_k}, \quad s_t = h_t^{\alpha_h}(z_t l_t)^{1-\alpha_h} \tag{1.2}$$

where y and s denote the production of the non-residential sector and the housing sector respectively; k and h denote the factor inputs of capital in the two sectors respectively; 1 is the household's normalized time endowment, and l denotes the share of labor allocated to the housing sector. $\alpha_k \neq \alpha_h$ denote the capital intensities of production of the two sectors, and z represents the labor-augmenting technological progress, which evolves according to $z_t = z_0 \cdot A^t$, for $A > 1$ and $z_0 \geq 1$.

Non-residential capital and housing evolve as follows:

$$k_{t+1} = k_t(1 - \delta_k) + i_{kt}, \; 0 < \delta_k < 1 \tag{1.3}$$

and

$$h_{t+1} = h_t(1 - \delta_h) + i_{ht}, \; 0 < \delta_h < 1 \tag{1.4}$$

[5]Note that the CES instantaneous utility function is a homothetic preference, which implicitly assumes that the income elasticity of both types of consumption equals to one. In this paper, different income elasticities of demand between sectors are not considered.

where i_k and i_h are the non-residential and housing investment, δ_k and δ_h denote the depreciation rate for non-residential capital and housing respectively. Denote $a = k + h$ as the aggregate capital stock. Assume that $\delta_k = \delta_h$. The aggregate resource constraint is:

$$c_t + a_{t+1} \leq y_t + (1 - \delta)a_t \tag{1.5}$$

which requires consumption and investment to be less than output of the non-residential sector.

1.3.2 The Competitive Equilibrium and the Social Planner's Problem

Normalize the price of the consumption of non-residential goods and services to one. Denote the rental price of capital and the wage rate by R and w, and the interest rate by r. Let q denote the relative price of housing services. Define the share of housing capital as $\kappa = \frac{h}{a}$, and the share of labor allocated to the housing sector as l. A competitive equilibrium is defined as the paths of prices $(R_t, w_t, r_t, q_t)_{t\geq 0}$, the factor allocations $(l_t, \kappa_t)_{t\geq 0}$, and the consumption and stock holding decisions $(c_t, s_t, a_{t+1})_{t\geq 0}$ such that:

(a) Given the aggregate state $(a_t, z_t)_{t\geq 0}$ and the paths of prices $(R_t, w_t, q_t)_{t\geq 0}$, firms choose the factor allocations $(l_{it}, \kappa_{it})_{t\geq 0}$, for $i \in \{k, h\}$, to maximize profits at each period t:

$$\max_{l_{kt},\kappa_{kt}} \left\{ (\kappa_{kt}a_t)^{\alpha_k}(z_t l_{kt})^{1-\alpha_k} - R_t \cdot (\kappa_{kt}a_t) - w_t \cdot l_{kt} \right\}$$

and

$$\max_{l_{ht},\kappa_{ht}} \left\{ q_t \cdot (\kappa_{ht}a_t)^{\alpha_h}(z_t l_{ht})^{1-\alpha_h} - R_t \cdot (\kappa_{ht}a_t) - w_t \cdot l_{ht} \right\}$$

(b) Given the initial endowment of capital stock a_0 and the paths of prices $(r_t, w_t, q_t)_{t\geq 0}$, the household makes the consumption and saving decision $(c_t, s_t, a_{t+1})_{t\geq 0}$ to maximize the lifetime utility of (1.1):[6]

$$\max_{\{c_t,s_t,a_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$$

s.t.

$$c_t + q_t s_t + a_{t+1} \leq (1 + r_t)a_t + w_t$$

(c) All the markets clear s.t.

$$c_t + a_{t+1} = (\kappa_{kt}a_t)^{\alpha_k}(z_t l_{kt})^{1-\alpha_k} + (1-\delta)a_t$$

$$s_t = (\kappa_{ht}a_t)^{\alpha_h}(z_t l_{ht})^{1-\alpha_h}$$

$$l_{kt} + l_{ht} = 1$$

$$\kappa_{kt} + \kappa_{ht} = 1$$

Since markets are complete and competitive, the Second Welfare Theorem can be applied. The competitive equilibrium can be characterized by solving a

[6]Note that without aggregate uncertainty, the rate of return on both types of capital are the same. Hence, there is only one effective capital asset market that pools both types of capital together with a single rate of return.

social planner's problem: Given the state variables $\{z, k, h\}$, the factor allocations $\{l, k', h'\}$ are chosen to solve the following dynamic programming problem:

$$v(k, h, z) = \max_{l,k',h'} u(c, s) + \beta v(k', h', z')$$

s.t.

$$c + k' + h' = k^{\alpha_k}(z(1-l))^{1-\alpha_k} + (1-\delta)(k+h)$$

$$s = h^{\alpha_h}(zl)^{1-\alpha_h}$$

Once the solution is characterized, the competitive factor prices (R, w, r) and the factor allocations (l, κ) can be backed out. In particular, the relative price for housing services can be derived as:

$$q = \frac{u_s}{u_c} = \frac{1-\eta}{\eta} \cdot \left(\frac{c}{s}\right)^{\frac{1}{\epsilon}} \tag{1.6}$$

1.3.3 The Balanced Growth Path

Detrend the real variables by the growth rate of the economy, A. Denote $\hat{x}_t = \frac{x_t}{A^t}$, for $x_t = \{y_t, c_t, a_t, s_t\}$. In equilibrium, the equalization of the marginal product of capital and labor in both sectors within a period implies:

$$\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k(1-\alpha_h)}{\alpha_h(1-\alpha_k)} \cdot \frac{1-l}{l} \tag{1.7}$$

Denote the rate of return on non-residential capital and housing as $r_k = \alpha_k \cdot \frac{\hat{y}}{\hat{k}} - \delta$ and $r_h = \alpha_h \cdot \frac{q\hat{s}}{\hat{h}} - \delta$, respectively. The no-arbitrage condition $r_k = r_h = r$

implies $\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k}{\alpha_h} \cdot \frac{\hat{y}}{q\hat{s}}$, which indicates that the capital allocation depends on the relative value of final outputs in both sectors. Substituting $q = \frac{u_s}{u_c}$ in (1.6),

$$\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1-\eta} \cdot \left(\frac{\hat{y}}{\hat{s}}\right)^{1-1/\epsilon} \cdot \left(\frac{\hat{y}}{\hat{c}}\right)^{1/\epsilon} \tag{1.8}$$

Combining (1.7) and (1.8),

$$\kappa = \left\{1 + \frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1-\eta} \cdot \left(\frac{\hat{y}}{\hat{s}}\right)^{1-1/\epsilon} \cdot \left(\frac{\hat{y}}{\hat{c}}\right)^{1/\epsilon}\right\}^{-1} \tag{1.9}$$

and

$$l = \left\{1 + \frac{\alpha_h}{\alpha_k} \cdot \frac{1-\alpha_k}{1-\alpha_h} \cdot \frac{1-\kappa}{\kappa}\right\}^{-1} \tag{1.10}$$

Equations (1.9) and (1.10) imply the share of housing capital and the share of labor allocated to the housing sector in equilibrium. In particular, (1.10) shows that at each period, the share of labor allocated to the housing sector is monotonically increasing in the share of housing capital. In other words, labor and capital are always reallocated towards the same sector in equilibrium.

Dynamics of the economy are determined by the capital accumulation and the Euler equation. The capital accumulation implies:

$$\left(\frac{\hat{a}'}{\hat{y}'}\right) \cdot A = (1-\delta) \cdot \frac{\hat{a}}{\hat{y}} + 1 - \frac{\hat{c}}{\hat{y}} \tag{1.11}$$

The Euler equation implies:

$$\frac{u_c}{u_c'} = \beta A^{-\sigma} \left[\frac{\alpha_k}{1-\kappa'} \cdot \left(\frac{\hat{a}'}{\hat{y}'}\right)^{-1} + 1 - \delta\right] \tag{1.12}$$

where $u_c = \eta \cdot [\eta \hat{c}^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)\hat{s}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1-\sigma\epsilon}{\epsilon-1}} \cdot \hat{c}^{-\frac{1}{\epsilon}}$.[7]

A balanced growth path of the economy is defined as an equilibrium trajectory, along which the share of housing capital and the share of labor allocated to the housing sector stay constant, and all the real variables $\{y_t, c_t, s_t, a_t\}$ grow at the same rate.

Proposition 1: Assume that $A^\sigma > \beta\Big[\alpha_k A + (1-\alpha_k)(1-\delta)\Big]$. There exists a unique balanced growth path, along which the steady-state capital per capita is:

$$\hat{a}^* = z_0 \cdot \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{1}{1-\alpha_k}} \cdot \frac{\alpha_h(1-\alpha_k)}{\alpha_h(1-\alpha_k)(1-\kappa^*) + \alpha_k(1-\alpha_h)\kappa^*} \tag{1.13}$$

and the steady-state share of housing capital is:

$$\kappa^* = \frac{1-n(\Theta)}{1+m(\Theta)} \tag{1.14}$$

where $\Theta = \{\alpha_k, \alpha_h, A, \delta, \eta, \epsilon, \beta\}$ is a set of fundamental parameters, $n(\cdot)$ and $m(\cdot)$ are both functions of the fundamental parameters of the economy. Along the balanced growth path, all the real variables $\{y_t, c_t, s_t, a_t\}$ grow at the growth rate of technological progress, A.

Proposition 1 implies that the steady-state share of housing capital depends only on the fundamental parameters of the economy, whereas the steady-state cap-

[7]See Appendix 1.7.1 for an alternative derivation of the equilibrium conditions (1.7), (1.8) and (2.7) from FOCs of the dynamic programming problem. For all the propositions, see Appendix 1.7.2 for detailed proofs.

ital per capita is jointly determined by the labor-augmented technological progress and the capital allocation of the economy. The next two propositions demonstrate how the fundamental parameters and technological progress impact the steady-state share of housing capital and capital per capita.

Proposition 2: The steady-state share of housing capital, κ^*, is:

(1) increasing in the preference weight of the housing service consumption, i.e., $\frac{d\kappa^*}{d(1-\eta)} > 0$;

(2) increasing in the inverse of intertemporal elasticity of substitution, i.e., $\frac{d\kappa^*}{d\sigma} > 0$;

(3) increasing in the elasticity of substitution between the two types of consumption when the preference weight of the housing service consumption is sufficiently large, i.e., $\frac{d\kappa^*}{d\epsilon} > 0$, for $\eta \in \left(0, \frac{1}{1+\omega(\Theta)}\right)$ where $\omega(\cdot)$ is a function of fundamental parameters of the economy.

Proposition 3: The steady-state capital per capita, $\hat{a}^*$, is:

(1) increasing in the level of labor-augmented technological progress, i.e., $\frac{d\hat{a}^*}{dz_0} > 0$;

(2) decreasing in the growth rate of labor-augmented technological progress, i.e., $\frac{d\hat{a}^*}{dA} < 0$.

(3) decreasing in the steady-state share of housing capital if the production of the housing sector is labor intensive, i.e., $\frac{d\hat{a}^*}{d\kappa^*} < 0$ if $\alpha_k > \alpha_h$.

(4) increasing in the steady-state share of housing capital if the production of the

housing sector is capital intensive, i.e., $\frac{d\hat{a}^*}{d\kappa^*} > 0$ if $\alpha_k < \alpha_h$.

While the first two comparative statics in Proposition 3 are consistent with the one-sector neoclassical growth model, the last two statements have not been discussed by the one-sector model. It implies that the sectoral allocation of capital and the capital intensities of production of the two sectors determine the capital accumulation of an economy. In particular, in an economy where the production of the housing sector is labor intensive, a higher steady-state share of housing capital implies a lower investment-output ratio, i.e., a higher consumption-output ratio at steady state.[8] In an economy where the production of the housing sector is capital intensive, a higher steady-state share of housing capital implies a higher investment-output ratio, i.e., a lower consumption-output ratio at steady state.

1.3.4 The Transitional Dynamics

During the transitional dynamics, the correlation between the share of housing capital and the consumption-output ratio is determined by the interplay of the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors.

[8] This is because the steady-state capital per capita is determined when the actual investment per capita is equal to the break-even investment per capita, as discussed in a one-sector neoclassical growth model. A higher share of housing capital κ^* decreases the actual investment level without changing the break-even investment, leading to a higher capital per capita, a lower investment-output ratio and a higher consumption-output ratio at steady state.

Proposition 4: During the transitional dynamics, with capital deepening, if the production of the housing sector is labor (capital) intensive, i.e., $\alpha_k > \alpha_h$ ($\alpha_k < \alpha_h$), the share of housing capital increases, as the consumption-output ratio increases (decreases).

The mechanism behind the correlation between the share of housing capital and the consumption-output ratio lies in the return on housing capital. The no-arbitrage condition (1.8) implies that the rate of return on housing capital is always equal to that on non-residential capital. Consider the case when the production of the housing sector is labor intensive. As capital deepens, the relative output of the housing sector decreases. An increase in the return on housing capital induces a simultaneous increase in the share of housing capital and the consumption-output ratio. Otherwise, an opportunity to arbitrage can emerge.

1.4 Calibration: US (1948-2005)

In this section, I calibrate the model to the US postwar economy, and examine whether the dynamics generated by the model are consistent with the US data. Further, I investigate the effect of a lower initial share of housing capital on the economy. The benchmark calibration captures the key features of the US economy between 1948 and 2005, and the positive correlation between the share of housing capital and the consumption-output ratio is robust with respect to different values of parameters and the lower initial share of housing capital. But, the levels of the share of housing capital and the consumption-output ratio are sensitive to the elasticity of substitution between the two sectors, and the numerical exercise suggests that the initial share of housing capital has a large impact on the speed of sectoral reallocation and the investment structure of the economy.

1.4.1 Data

The measure of the flow variables of the model is from NIPA. In particular, the output of the non-residential sector includes the consumption of nondurable goods and services, the non-residential investment and the housing investment. The output of the housing sector is from the household expenditure on housing and utilities. Non-residential and housing capital are from the Fixed Assets Table.[9] Labor in the non-residential sector is computed as the total hours worked by the full-time and part-time workers in the private sector, divided by the total numbers of workers and hours in a year for normalization.[10] In addition, I refer to the current-price data as value, and the chain-type fixed-price quantity indices as quantity.

Table 1.2: Summary of Parameters

Parameter	Value	Description	Source/Target
Taken from the literature			
β	0.96	Utility discount factor	Standard value
σ	1.00*	Inverse of intertemporal elasticity of substitution	Standard value
Estimated from the data			
δ	0.06	Depreciation rate of capital	NIPA Table 1.5 and Fixed Assets Table 2.1
α_k	0.30	Capital intensity of production of the non-residential sector	NIPA Table 6.2
ϵ	1.81*	Elasticity of substitution between the two types of consumption	OLS regression in (1.15)
A	1.0149	Growth rate of the labor-augmented technological progress	Growth accounting
Calibrated in the model			
α_h	0.10	Capital intensity of production of the housing sector	Ratio of hours labor and leisure in 1948 $((1-l)/l)$
η	0.28	Preference weight of the consumption of nondurable goods and services	Ratio of non-residential and housing capital in 1948 (k/h)
z_0	10	Initial value of the labor-augmented technological progress	Relative output of non-residential sector in 1948 (y/s)

Note: parameters with $*$ will be varied for the robustness check.

1.4.2 Calibration

The frequency of the model is annual. The model economy is fully characterized by 8 parameters, β, δ, α_k, α_h, A, ϵ, η, σ, and three initial values, z_0, a_0

[9]Data from [11] are used for result comparison, in which the market value for housing includes both the value for land and residential structure, whereas the data from Fixed Assets Table only includes the value for residential structure. The results remain robust using data from [11].

[10]Labors are taken from the private sector because the output does not include the government expenditures.

and κ_0. Table 1.2 summarizes the model parameters. First, I adopt the standard parameter values for $\beta = 0.96$ and $\sigma = 1$. Then, I estimate δ, α_k, A and ϵ as follows:

$\delta_k = \delta_h = 0.06$. Using the capital accumulation equation (1.3) in the model and data for the real non-residential capital, I back out a series of the implied depreciation rates of non-residential capital $1 - (k' - i_k)/k$. The value reported is an average over the sample. The depreciation rate on housing capital is calculated in a similar way.

$\alpha_k = 0.30$. The labor income share of GNP net of housing services is about 70 percent during the sample period between 1948 and 2005. Hence, the capital intensity of production of the non-residential sector is chosen to be 0.30.

$A = 1.0149$. The labor-augmented technological progress is estimated through the growth accounting equation below for the sample period between 1948 and 2005.

$$\log z = \tfrac{1}{1-\alpha_k} \log y - \tfrac{\alpha_k}{1-\alpha_k} \log k - \log l$$

The average growth rate of the technological progress is estimated to be 1.0149.

$\epsilon = 1.81$. Equation (1.6) suggests a way to evaluate the elasticity of substitution between the two types of consumption:

$$\log \frac{c_{\text{value}}}{s_{\text{value}}} = \log \frac{\eta}{1-\eta} + \frac{\epsilon - 1}{\epsilon} \log \frac{c_{\text{quantity}}}{s_{\text{quantity}}} \tag{1.15}$$

Hence, the coefficient $\frac{\epsilon-1}{\epsilon}$ can be estimated by regressing the log ratio of the nominal expenditure value between the two sectors on the log ratio of the quantities between the two sectors. The regression yields an estimate of $\epsilon = 1.81$, with a two standard error coefficient interval of $[1.64, 2]$.

The remaining parameters that need to be assigned values are α_h and η. Equation (1.7) is referred to evaluate the capital intensity of production of the housing sector, using the ratio of hours for labor and leisure and the ratio of non-residential and housing capital in 1948 ($t = 0$ in the model). Further, (1.8) is referred to evaluate the preference weight between the two types of consumption to match the ratio of non-residential and housing capital, the consumption-output ratio, and the relative output ratio between the two sectors in 1948. For the initial values, I set $\kappa_0 = 0.4817$, which corresponds to the share of housing capital in 1948. $\hat{a}_0 = 10$ and $z_0 = 10$ are jointly chosen to match the range of the relative output ratio between the two sectors.[11]

Table 1.3 presents the comparison between the US data and the benchmark calibration. The first two rows show that the benchmark calibration is consistent with the allocation of capital and labor between the two sectors. In particular, the calibration matches the following feature of the data: capital is evenly allocated between the two sectors, and there is a slight reallocation of both capital and labor towards the housing sector between 1948 and 2005. The last two rows show that the benchmark calibration generates the increase in the consumption-output ratio and the relative output of the housing sector. Although the levels are slightly

[11] In fact, the choice of $\hat{a}_0$ and z_0 provides degree of freedom of the calibration. The dynamics is sensitive to the choice of the two initial conditions, and $\hat{a}_0 = 10$ and $z_0 = 10$ is the pair that better matches the US data between 1948 and 2005, among all the attempted trials.

Table 1.3: Data and Benchmark Calibration, 1948 - 2005

	US Data		Benchmark Calibration	
	1948	2005	1948	2005
κ	0.4817	0.4949	0.4817	0.5008
l	0.7740	0.8063	0.7819	0.7946
c/y	0.6710	0.7041	0.5092	0.6159
s/y	0.1506	0.2226	0.1579	0.1653

Note: US Data from NIPA. Calibration described in the text.

different, the increasing trend in both variables are captured by the benchmark calibration.

Table 1.4: Data and Model Calibration, 1948-2005 (Robustness)

	US Data		Model $\epsilon = 1.64$		Model $\epsilon = 2.52$		Model $\epsilon = 3.70$	
	1948	2005	1948	2005	1948	2005	1948	2005
κ	0.4817	0.4949	0.4817	0.4947	0.4817	0.5988	0.4817	0.7101
l	0.7740	0.8063	0.7819	0.7771	0.7819	0.8520	0.7819	0.9043
c/y	0.6710	0.7041	0.5333	0.6349	0.3555	0.5222	0.2044	0.3377
s/y	0.1506	0.2226	0.1684	0.1488	0.1684	0.1949	0.1684	0.2640

Note: US Data from NIPA. Calibration described in the text.

For the robustness check, I consider different values in the intertemporal elasticity of substitution, the growth rate of the economy, and the elasticity of substitution between the two sectors.[12] While results with different σ and A are similar to the benchmark calibration, the one with different ϵ in Table 1.4 shows that while the positive correlation between the share of housing capital and the consumption-output ratio is robust with respect to different elasticities of substitution, a higher elasticity of substitution between the two sectors leads to a higher share of housing capital and a lower consumption-output ratio.

To summarize, the calibration indicates that the model can generate dynamics that are consistent with the US data. It matches the allocation of capital and labor, as well as the positive correlation between the share of housing capital and the consumption-output ratio. The robustness check shows that the positive correlation between the share of housing capital and the consumption-output ratio is robust, but the elasticity of substitution between the two sectors impacts the levels of the two variables.

1.4.3 Initial Conditions

To the extent that the model can reproduce the US data, I investigate the effect of the initial sectoral capital allocation on the economy.[13] In particular, I

[12]See Appendix 1.7.3 for a detailed discussion on the robustness check.

[13]This is because among all the institutional differences between the United States and China, an important one is that when the market mechanism starts to work in China, the share of housing capital in China is low as shown in 1.13.

consider the counterfactual experiment of starting the calibrated US economy with half of the initial share of housing capital, and examine whether it can explain the negative correlation of the share of housing capital and the consumption-output ratio observed in China.

Table 1.5 shows that the positive correlation between the share of housing capital and the consumption-output ratio remains. But the lower initial share of housing capital decreases the relative output of the housing sector during the transition. Moreover, the lower initial share of housing capital has a large impact on the speed of the sectoral reallocation, the relative price of housing services and the investment structure as shown in Figure 1.6.

Table 1.5: Data, Benchmark Calibration and Counterfactual Experiment

	US Data		Benchmark Calibration		Counterfactual Experiment	
	1948	2005	1948	2005	1948	2005
κ	0.4817	0.4949	0.4817	0.5008	0.2409	0.5008
l	0.7740	0.8063	0.7819	0.7946	0.5503	0.7946
c/y	0.6710	0.7041	0.5092	0.6159	0.3699	0.6159
s/y	0.1506	0.2226	0.1579	0.1653	0.0650	0.1653

Note: US Data from NIPA. Calibration and counterfactual experiment described in the text.

When the economy starts at half of the initial share of housing capital of the benchmark calibration, Figure 1.6 shows that compared to the benchmark case, the share of housing capital experiences a rapid increase during the transitional dynamics. It increases to a level that is higher than the steady-state value first, and then gradually falls back to the steady-state value. Further, the relative price of housing services jumps to a high level, and gradually fall down during the transitional dynamics. Compared to the benchmark, the economy that starts at half of the initial share of housing capital present a different investment structure: non-residential investment-output ratio experiences a large drop, while the housing investment-output ratio experiences a large leap.

In summary, the result from this numerical exercise shows that the positive correlation between the share of housing capital and the consumption-output ratio is robust with respect to a lower initial share of housing capital. But, the initial share of housing capital has a large impact on the speed of sectoral reallocation of capital, the relative price of housing services and the investment structure of the economy. Compared to the benchmark case, the economy that starts at half of the initial share of housing capital experiences a rapid increase in the share of housing capital, a higher level of relative price of housing services and a different investment structure during the transitional dynamics.

1.5 Calibration: China (1987-2013)

In section 1.4, I showed that the elasticity of substitution between the two sectors and the initial share of housing capital have significant impacts on the dynamics of the economy, but neither different values in the elasticity of substitution nor the initial share of housing capital can explain the negative correlation between the share of housing capital and the consumption-output ratio observed in China. In this section, I calibrate the model to the Chinese economy between 1987 and 2013. I find that to match the negative correlation between the share of housing capital and the consumption-output ratio observed in the Chinese data, it requires a capital intensive production of the housing sector ($\alpha_k < \alpha_h$). Further, I quantify the effect of initial share of housing capital before the housing market reform on the Chinese economy during its 30-year transition.

1.5.1 Background Introduction and Data

In order to conduct a similar calibration exercise for China as the one for the US, I construct a dataset for capital stock in China. In particular, I choose the year 1987 as the initial period of the dataset. Based on the strand of literature that studies the Chinese housing market, I summarize the facts related to the housing market reform with respect to the following time frame, which justifies the reason for choosing 1987 as the initial period.

Pre-1978: Government established public ownership over all new housing stock. Housing was not a commodity, and urban households had little choices in housing consumption, which was provided by the government for a highly subsidized rental charge. In particular, housing investment was seriously neglected during the economic and political turmoil of the 1960s and 1970s. By 1978, per capita housing floor space was only 3.6 square meters.[14]

1979-1987: Reforms of the housing system started in 1979. The aim of the reform is to decentralize the housing investment. The decentralization led the housing investment as a proportion of GNP to rise from an average of 1.5 percent before reform to over 7 percent during the 1980s. As a result, per capita housing floor space rose to 5.2 square meters by 1985.[15]

Post-1987: In 1988, the Chinese government endorsed private property rights in urban land, and long-term land leases were granted for private real estate development. The housing construction has grown rapidly since then. In 1996, housing accounted for 86 percent of the building floor area sold. Since 1997, the housing investment has been high compared with that in other countries. Until 2012, per capita housing floor space has improved to 32.7 square meters and the housing investment accounts for about 15 percent of the total fixed asset investment in China.[16]

[14] See [34], [16] and [36].
[15] See [33], [13] and [36].
[16] See [21], [37] and [8].

Since the large scale of housing construction started after the Chinese government policy at the end of 1987, I choose the year 1987 as the initial period for the dataset. Data for the consumption of nondurable goods and services, the non-residential investment and the housing investment are from China Statistical Yearbook (CSY). A drawback of CSY is the lack of the categorized consumption data for the total population, with only the categorized consumption for per urban capita. To get a consistent measure, all the variables are in the unit of per urban capita, computed through dividing all the variables for urban population by the size of urban population.[17]

1.5.2 Calibration

Table 1.6 summarizes the parameters. For the calibration exercise for China, I take the parameter values $\beta = 0.96$, $\sigma = 1$ and $\delta = 0.08$.[18] Since there is no available hours worked for labor in China, the growth accounting for the growth rate of the labor-augmented technological progress cannot be conducted for China. For the benchmark calibration in China, I choose $A = 1.04$, leaving other possible values for A for the robustness check.

[17]CSY Data for the urban investment of total fixed assets starts from 1995. For the missing years between 1987 and 1994, data are estimated by 78% (1995 fraction) of the investment of total fixed assets; CSY Data for the urban investment for residential buildings starts from 1995. For the missing years between 1987 and 1994, data are estimated by 20% (1995 fraction) of the urban investment of total fixed assets; CSY Data for the urban consumption per capita miss the years 1991-1994, 1996-1999, and 2001-2009. For the missing years, data are estimated by interpolation.

[18]The depreciation rate is from [4], implying the useful lives of the capital stock is about 12 years.

Table 1.6: Summary of Parameters

Parameter	Value (China)	Value (US)	Source/Target
Taken from the literature or by assumption			
β	0.96	0.96	Standard value
σ	1.00	1.00	Standard value
δ	0.08	0.06	[4]
A	1.0400*	1.0149	By assumption
η	0.28	0.28	By assumption
α_k	0.30	0.30	By assumption
Estimated from the data			
ϵ	3.70*	1.81	OLS regression in (1.15)
Calibrated in the model			
α_h	0.50*	0.10	Consumption-output ratio
z_0	5	10	Relative output of housing sector

Note: The growth rate A is a parameter left for robustness check.

To estimate the elasticity of substitution between the two sectors, I follow the same approach for the US, by regressing the log ratio of nominal expenditure value between the two sectors on the log ratio of the quantities between the two sectors.[19]The estimation for the elasticity of substitution between the two sectors is 3.70, with a two standard error coefficient interval $[2.86, 5.26]$. Compared with $\epsilon = 1.81$ in the US (with a two standard error coefficient interval $[1.64, 2]$), the estimation indicates a more substitutable preference between the two sector for Chinese households.

[19]The nominal values are obtained from CSY. For quantities, I divide the nominal values by the price indices by categories in CSY, of which the available years are from 2001 to 2013.

Given that there is no labor-capital composition of national income in CSY as in NIPA, the calibration strategy is different for the capital intensities of production of both sectors and the preference weight between the two sectors, i.e., α_k, α_h and η. For the benchmark calibration, I assume that the preference weight between the two types of consumption for Chinese households is the same with that for US households, i.e. $\eta = 0.28$. To match the negative correlation between the share of housing capital and the consumption-output ratio, the capital intensity of production of the housing sector is set to be $\alpha_h = 0.50$ if assuming the capital intensity of production of the non-residential sector is the same with that of the US at $\alpha_k = 0.30$.

Table 1.7: Benchmark Calibration

	China Data		Benchmark Calibration	
	1987	2013	1987	2013
κ	0.3035	0.4580	0.3035	0.8703
c/y	0.3489	0.1985	0.5479	0.3230
s/y	0.0505	0.0390	0.0582	0.8411

Note: China Data from CSY(2014). Calibration described in the text.

Table 1.7 presents the benchmark calibration of China. It shows that the benchmark calibration generates the negative correlation between the share of housing capital and the consumption-output ratio in the data, although it does not match the level of both variables, given the potential issue of missing data. Meanwhile, the benchmark calibration captures the rapid increase in the share of housing capital as shown in Figure 1.7.

To summarize, I present a benchmark calibration for China between 1987 and 2013. The benchmark calibration assumes that the preference weight between the two types of consumption for Chinese households and the capital intensity of production of the non-residential sector are the same with the US economy. With a high elasticity of substitution between the two sectors, a high capital intensity of production of the housing sector and a low initial share of housing capital, the calibration accounts for the fast-growing share of housing capital, and the negative correlation between the share of housing capital and the consumption-output ratio between 1987 and 2013 in China.

1.5.3 Initial Sectoral Allocation

To the extent that the calibration can reproduce the key feature of the Chinese data, I apply the model to quantify the effect of the initial share of housing capital in 1987 on the Chinese economy. The approach is to compare two calibrated economies, one started with the share of housing capital at the China level in 1987, and the other started with the share of housing capital at the US level of a comparable development stage.[20]

[20] To be specific, both economies are calibrated to China.

In order to find an appropriate development stage in the US economy to refer to, I resample the US data between 1930 and 2013, using a moving block bootstrap approach proposed by [29]. I find that China in 1987 is most close to the United States in 1956, when using the joint correlation of the consumption-output ratio, the non-residential capital-output ratio, and the housing capital-output ratio as a measure of resemblance.[21]

Consider the two calibrated economies, one started with the share of housing capital at the US level in 1956 ($\kappa_0 = 0.47$), and the other started with the share of housing capital at the China level in 1987 ($\kappa_0 = 0.30$). Figure 1.8 shows the effect of raising the initial share of housing capital from the China level in 1987 to the US level in 1956 on the economy. The transitional dynamics for the non-residential investment-output ratio, the housing investment-output ratio, the relative output of the housing sector, and the relative price of housing services are compared. It is notable that the investment structure is strongly affected by the initial sectoral allocation of capital as shown in panel (a) and (b). This is because the low initial share of housing capital at the China level in 1987 raises the marginal return on the housing investment. An implication of Figure 1.8 is that the resources that should have been allocated to the non-residential sector are induced to housing during the 30-year transition in China.

Table 1.8 quantifies the impact of the initial share of housing capital at the China level in 1987 on the economy. It shows that when changing the initial share

[21] See the Appendix 1.9 for a statistical comparison between the US economy and China.

Table 1.8: China Benchmark vs. the US allocation

	$\overline{s/y}$	$\overline{i_k/y}$	$\overline{i_h/y}$	$\overline{q}$
China benchmark ($\kappa_0 = 0.30$)	0.1331	0.2493	0.3113	5.0176
US allocation ($\kappa_0 = 0.47$)	0.1482	0.3063	0.2611	4.6506

of housing capital from the China level in 1987 to the US level in 1956, the mean relative output of the housing sector and the mean non-residential investment-output ratio increase by 11.3 percent and 22.9 percent respectively. The mean housing investment-output ratio and the mean relative price of housing services decrease by 16.1 percent and 10 percent respectively. These numbers suggest that the initial low share of housing capital might lead to an over-investment in housing, an under-investment in non-residential capital, and a higher relative price of housing services during its 30-year transition in China.

1.6 Conclusion

I propose a two-sector general equilibrium model to study the sectoral allocation of capital between housing and nonresidential sector in a neoclassical growth environment. The model features preferences with a constant elasticity of substitution between the two sectors, and Cobb-Douglas production technologies within each sector. The equilibrium dynamics of the model imply that the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors hold the key to understanding the correlation between the sectoral capital allocation and the consumption-output ratio.

Calibrated to the United States and China, the model can account for the positive correlation between the share of housing capital and the consumption-output ratio in the United States on the one hand, and the negative correlation between the share of housing capital and the consumption-output ratio in China on the other hand. In particular, the calibration to China implies that the rapid increase in the share of housing capital and the simultaneous decrease in the consumption-output ratio observed in the Chinese data can be explained by a combination of three factors: a high elasticity of substitution between the two sectors, a high capital intensity of production of the housing sector, and a low initial share of housing capital before the Chinese housing market reform.

The initial low share of housing capital before the reform has a large impact on the Chinese economy. In a counterfactual experiment, when changing the initial share of housing capital at the China level in 1987 to the US level of a comparable development stage, the mean relative output of the housing sector and the mean non-residential investment-output ratio increase by 11.3 percent and 22.9 percent respectively. Meanwhile, the mean housing investment-output ratio and the mean relative prices of housing services decrease by 16.1 percent and 10 percent respectively. The comparative study suggests that the initial low share of housing capital in China has led to an over-investment in housing, an under-investment in non-residential capital and a higher relative price of housing services during its 30-year transition.

1.7 Appendix

1.7.1 Derivation of equilibrium conditions

Given the state variables $\{\hat{k}, \hat{h}\}$, i.e. the capital stock in the two sectors. $\{l, \hat{k}', \hat{h}'\}$ are chosen to maximize the following value function:

$$v\left(\hat{k}, \hat{h}\right) = \max_{l,\hat{k}',\hat{h}'} \left\{u\Big(\hat{c}, \hat{s}\Big) + \beta v\left(\hat{k}', \hat{h}'\right)\right\}$$

s.t.:

$$\hat{c} = \hat{k}^{\alpha_k}(1-l)^{1-\alpha_k} + (1-\delta)(\hat{k}+\hat{h}) - A\cdot(\hat{k}'+\hat{h}')$$

$$\hat{s} = \hat{h}^{\alpha_h} l^{1-\alpha_h}$$

Substituting $\hat{c}$ and $\hat{s}$ into the value function, one obtains the dynamic programming problem as follows:

$$v\left(\hat{k}, \hat{h}\right) = \max_{l,\hat{k}',\hat{h}'} u\Big(\hat{k}^{\alpha_k}(1-l)^{1-\alpha_k} + (1-\delta)(\hat{k}+\hat{h}) - A\cdot(\hat{k}'+\hat{h}'),\ \hat{h}^{\alpha_h} l^{1-\alpha_h}\Big) + \beta v\left(\hat{k}', \hat{h}'\right)$$

FOC w.r.t [l]:

$$\frac{u_c}{u_s} = \frac{(1-\alpha_h)\hat{h}^{\alpha_h} l^{-\alpha_h}}{(1-\alpha_k)\hat{k}^{\alpha_k}(1-l)^{-\alpha_k}} \tag{1.16}$$

FOC w.r.t $[\hat{k}']$:

$$A \cdot u_c = \beta v_k \left(\hat{k}', \hat{h}'\right) = \beta u_c' \cdot \left(\alpha_k (\hat{k}')^{\alpha_k - 1}(1 - l')^{1-\alpha_k} + 1 - \delta\right) \tag{1.17}$$

FOC w.r.t $[\hat{h}']$:

$$A \cdot u_c = \beta v_h \left(\hat{k}', \hat{h}'\right) = \beta \left[u_c' \cdot (1 - \delta) + u_s' \cdot \alpha_h (\hat{h}')^{\alpha_h - 1}(l')^{1-\alpha_h}\right] \tag{1.18}$$

Combining equation 1.17 and 1.18:

$$\frac{u_c}{u_s} = \frac{\alpha_h \hat{h}^{\alpha_h - 1} l^{1-\alpha_h}}{\alpha_k \hat{k}^{\alpha_k - 1}(1 - l)^{1-\alpha_k}} \tag{1.19}$$

Combining equation 1.16 and 1.19:

$$\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k(1 - \alpha_h)}{\alpha_h(1 - \alpha_k)} \cdot \frac{1 - l}{l} \tag{1.20}$$

Equation 1.20 is identical to equation 1.7 from firm profit optimization. Further, rearranging equation 1.19 gives:

$$\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k}{\alpha_h} \cdot \frac{u_c}{u_s} \cdot \frac{\hat{y}}{\hat{s}} \tag{1.21}$$

Equation 1.21 is identical to the non-arbitrage condition 1.8.

Further, equation 1.17 can be rewritten into Euler Equation as in 2.7:

$$\frac{u_c}{u_c'} = \beta \left[\alpha_k \cdot \frac{\hat{y}'}{\hat{k}'} + 1 - \delta\right] \tag{1.22}$$

Given a CES utility function, the marginal utility of c is given by:$u_c = \eta \cdot [\eta \hat{c}^{\frac{\epsilon-1}{\epsilon}} + (1 - \eta)\hat{s}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1-\sigma\epsilon}{\epsilon-1}} \cdot \hat{c}^{-\frac{1}{\epsilon}}$. Therefore, one obtains the same Euler equation with 2.7.

1.7.2 Proof

1. *Proof for Proposition 1:*

First, show that all economic variables grow at the same rate A. The aggregate resource constraint 1.5 implies that y, c, a all have to grow at the same rate, denoted by G along a balanced growth path. $a = k + h$ implies that k and h also grows at G. Further, production function in the nonresidential sector implies $G = G^{\alpha_k} A^{1-\alpha_k}$. Also, production function in the housing sector implies that growth rate in the housing sector is $G = A^{\alpha_h} \cdot A^{1-\alpha_h} = A$. Thus, along the balanced growth path, variable y, c, s, a all grow at $G = A$.

Next, compute the steady-state share of capital in the housing sector κ^* and the steady-state capital per capita $\hat{a}^*$. Euler equation 2.7 implies that along the BGP,

$$\frac{\hat{a}^*}{z_0 \cdot (1 - l^*)} = \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{1}{1-\alpha_k}} (1 - \kappa^*)^{-1} \tag{1.23}$$

Dynamics of capital accumulation 1.11 implies that along the BGP,

$$\begin{aligned}\left(\frac{\hat{c}}{\hat{y}}\right)^* &= 1 - (A + \delta - 1)\left(\frac{\hat{a}}{\hat{y}}\right)^* \\ &= 1 - (A + \delta - 1)\left[\frac{\hat{a}^*}{z_0 \cdot (1 - l^*)}\right]^{1-\alpha_k} (1 - \kappa^*)^{-\alpha_k}\end{aligned}$$

Combined with 1.23 implies that along the BGP,

$$\left(\frac{\hat{c}}{\hat{y}}\right)^* = 1 - (A + \delta - 1) \cdot \frac{\alpha_k}{A^\sigma/\beta - (1-\delta)} \cdot (1 - \kappa^*)^{-1} \tag{1.24}$$

Production functions of both sectors 1.2 imply that along the BGP,

$$\hat{y}^* = (1-\kappa^*)^{\alpha_k} \cdot \left[\frac{\hat{a}^*}{z_0(1-l^*)}\right]^{\alpha_k} \cdot [z_0(1-l^*)]$$

$$\hat{s}^* = (\kappa^*)^{\alpha_h} \cdot \left[\frac{\hat{a}^*}{z_0(1-l^*)}\right]^{\alpha_h} \cdot \left(\frac{1-l^*}{l^*}\right)^{\alpha_h} \cdot (z_0 l^*)$$

Substituting equation 1.10 and 1.23 into the expressions above implies:

$$\hat{y}^* = \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{\alpha_k}{1-\alpha_k}} \cdot [z_0(1-l^*)]$$

$$\hat{s}^* = \left[\frac{\alpha_h(1-\alpha_k)}{\alpha_k(1-\alpha_h)}\right]^{\alpha_h} \cdot \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{\alpha_h}{1-\alpha_k}} \cdot (z_0 l^*)$$

Hence,

$$\left(\frac{\hat{y}}{\hat{s}}\right)^* = \left[\frac{\alpha_h(1-\alpha_k)}{\alpha_k(1-\alpha_h)}\right]^{1-\alpha_h} \cdot \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{\alpha_k-\alpha_h}{1-\alpha_k}} \cdot \frac{1-\kappa^*}{\kappa^*} \tag{1.25}$$

Substituting equations 1.24 and 1.25 into equation 1.9, one obtains:

$$\kappa^* = \frac{1-n}{1+m}$$

where $m = \left[\frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1-\eta}\right]^{\epsilon} \left[\frac{\alpha_k(1-\alpha_h)}{\alpha_h(1-\alpha_k)}\right]^{(\alpha_h-1)(\epsilon-1)} \left[\frac{\alpha_k}{A^\sigma/\beta-(1-\delta)}\right]^{\frac{(\alpha_k-\alpha_h)(\epsilon-1)}{1-\alpha_k}}$, $n = \frac{\alpha_k(A+\delta-1)}{A^\sigma/\beta-(1-\delta)}$.

Substituting κ^* into 1.23, one obtains:

$$\hat{a}^* = z_0 \cdot \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{1}{1-\alpha_k}} \cdot \frac{\alpha_h(1-\alpha_k)}{\alpha_h(1-\alpha_k)(1-\kappa^*) + \alpha_k(1-\alpha_h)\kappa^*}$$

For capital per effective labor in equation 1.23 to be nonnegative, it needs:

$$\frac{A^\sigma}{\beta} > 1-\delta \tag{1.26}$$

Further, for κ^* to take a plausible value within $(0,1)$, n needs to be within $(0,1)$, which implies:

$$\frac{A^\sigma}{\beta} > \alpha_k A + (1-\alpha_k)(1-\delta) \quad \& \quad A > 1-\delta \tag{1.27}$$

The two inequalities 1.26 and 1.27 implies the following parameter ranges:[22]

$$\frac{A^\sigma}{\beta} > \alpha_k A + (1-\alpha_k)(1-\delta) \quad \& \quad A > 1-\delta \tag{1.28}$$

Q.E.D.

2. *Proof for Proposition 2:*

$$\frac{d\kappa^*}{d(1-\eta)} = -\frac{d\kappa^*}{\eta} = \frac{(1-n)m}{(1+m)^2} \cdot \frac{d\log m}{d\eta}$$

Since $n < 1$ and $\frac{d\log m}{d\eta} = \frac{\epsilon}{(1-\eta)\eta}$, $\frac{d\kappa^*}{d(1-\eta)} > 0$.

$$\begin{aligned}\frac{d\ln\kappa^*}{d\sigma} &= -\frac{n}{1-n} \cdot \frac{d\ln n}{d\sigma} - \frac{m}{1+m} \cdot \frac{d\ln m}{d\sigma} \\ &= \frac{A^\sigma \cdot \ln A}{A^\sigma - \beta(1-\delta)} \left[\frac{n}{1-n} + \frac{m}{1+m} \cdot \frac{(\alpha_k-\alpha_h)(\epsilon-1)}{1-\alpha_k}\right]\end{aligned}$$

Therefore, $\frac{d\kappa^*}{d\sigma} > 0$, if $\epsilon > 1 - \frac{n}{1-n} \cdot \frac{1+m}{m} \cdot \frac{1-\alpha_k}{\alpha_k-\alpha_h}$. Given the parameter ranges of the model, $\frac{n}{1-n} \cdot \frac{1+m}{m} \cdot \frac{1-\alpha_k}{\alpha_k-\alpha_h} > 1$. Therefore, $\frac{d\kappa^*}{d\sigma} > 0$ for $\forall\epsilon > 0$.

$$\begin{aligned}\frac{d\ln\kappa^*}{d\epsilon} &= -\frac{m}{1+m} \cdot \frac{\partial \ln m}{\partial\epsilon} \\ &= -\frac{m}{1+m} \cdot \ln\left\{\frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1-\eta} \cdot \left[\frac{\alpha_k(1-\alpha_h)}{\alpha_h(1-\alpha_k)}\right]^{(\alpha_h-1)} \cdot \left[\frac{\alpha_k}{A^\sigma/\beta-(1-\delta)}\right]^{\frac{\alpha_k-\alpha_h}{1-\alpha_k}}\right\}\end{aligned}$$

[22]Note that along BGP, the intuition for $\frac{A^\sigma}{\beta} = 1 + r^*$, i.e. another interpretation of the parameter range is $r^* \in \Big(\alpha_k(A+\delta-1)-\delta, \infty\Big)$.

Therefore, $\frac{d\kappa^*}{d\epsilon} > 0$, if $\eta \in \left(0, \frac{1}{1+\omega}\right)$, where $\omega = \left(\frac{\alpha_k}{\alpha_h}\right)^{\alpha_h} \cdot \left(\frac{1-\alpha_h}{1-\alpha_k}\right)^{\alpha_h - 1} \cdot \left(\frac{\alpha_k}{A^\sigma/\beta-(1-\delta)}\right)^{\frac{\alpha_k-\alpha_h}{1-\alpha_k}}$.

Q.E.D.

3. *Proof for Proposition 3:*

$$\frac{d\hat{a}^*}{dz_0} = \left[\frac{\alpha_k}{A^\sigma/\beta - (1-\delta)}\right]^{\frac{1}{1-\alpha_k}} \cdot \frac{\alpha_h(1-\alpha_k)}{\alpha_h(1-\alpha_k)(1-\kappa^*) + \alpha_k(1-\alpha_h)\kappa^*} > 0$$
$$\frac{d\hat{a}^*}{dA} = \frac{d\ln a^*}{dA} \cdot a^* = -\frac{1}{1-\alpha_k} \cdot \frac{A^\sigma \cdot \ln A}{A^\sigma - \beta(1-\delta)} < 0$$
$$\frac{d\hat{a}^*}{d\kappa^*} = \frac{d\ln a^*}{d\kappa^*} \cdot a^* = -\frac{(\alpha_k - \alpha_h)a^*}{\alpha_h(1-\alpha_k)(1-\kappa^*) + \alpha_k(1-\alpha_h)\kappa^*}$$

In the last expression, if $\alpha_k > \alpha_h$, $\frac{d\hat{a}^*}{d\kappa^*} < 0$; if $\alpha_k < \alpha_h$, $\frac{d\hat{a}^*}{d\kappa^*} > 0$ Q.E.D.

4. *Proof for Proposition 4:*

Equation 1.9 implies:

$$\ln\kappa = -\ln\left(1 + \frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1-\eta} \cdot \left(\frac{\hat{y}}{\hat{s}}\right)^{1-1/\epsilon} \cdot \left(\frac{\hat{c}}{\hat{y}}\right)^{-1/\epsilon}\right)$$

It follows:

$$
\begin{aligned}
\frac{d\ln\kappa}{d\hat{a}} &= -\frac{1}{\kappa}\cdot\frac{\alpha_k}{\alpha_h}\cdot\frac{\eta}{1-\eta}\cdot\frac{d\left(\frac{\hat{y}}{\hat{s}}\right)^{1-1/\epsilon}\cdot\left(\frac{\hat{c}}{\hat{y}}\right)^{-1/\epsilon}}{d\hat{a}} \\
&\propto -\left[\left(\frac{\hat{y}}{\hat{s}}\right)^{1-1/\epsilon}\cdot\frac{d\,(\hat{c}/\hat{y})^{-1/\epsilon}}{d\hat{a}}+\left(\frac{\hat{c}}{\hat{y}}\right)^{-1/\epsilon}\cdot\frac{d\,(\hat{y}/\hat{s})^{1-1/\epsilon}}{d\hat{a}}\right] \\
&= -\left[-\frac{1}{\epsilon}\left(\frac{\hat{c}}{\hat{s}}\right)^{1-1/\epsilon}\cdot\left(\frac{\hat{c}}{\hat{y}}\right)^{-1}\cdot\frac{d\ln(\hat{c}/\hat{y})}{d\hat{a}}+\left(\frac{\hat{c}}{\hat{s}}\right)^{-1/\epsilon}\cdot\frac{(\epsilon-1)(\alpha_k-\alpha_h)}{\epsilon}\cdot\frac{\hat{y}}{\hat{s}}\cdot\frac{1}{\hat{a}}\right] \\
&= \frac{1}{\epsilon}\cdot\frac{1}{\hat{a}}\cdot\left(\frac{\hat{c}}{\hat{s}}\right)^{1-1/\epsilon}\cdot\left(\frac{\hat{c}}{\hat{y}}\right)^{-1}\left[\frac{d\ln(\hat{c}/\hat{y})}{d\ln\hat{a}}-(\epsilon-1)(\alpha_k-\alpha_h)\right] \\
&\propto \frac{d\ln(\hat{c}/\hat{y})}{d\ln\hat{a}}-(\epsilon-1)(\alpha_k-\alpha_h)
\end{aligned}
$$

This is because $\frac{\hat{y}}{\hat{s}}=\frac{(1-\kappa)^{\alpha_k}(1-l)^{1-\alpha_k}}{\kappa^{\alpha_h}l^{1-\alpha_h}}\cdot\hat{a}^{\alpha_k-\alpha_h}$ and therefore:

$$
\frac{d\ln\kappa}{d\ln\hat{a}}\propto\left\{\frac{d\ln(\hat{c}/\hat{y})}{d\ln\hat{a}}-(\epsilon-1)(\alpha_k-\alpha_h)\right\}
$$

Hence, $\frac{d\ln\kappa}{d\ln\hat{a}}>0\iff\frac{d\ln(\hat{c}/\hat{y})}{d\ln\hat{a}}>(\epsilon-1)(\alpha_k-\alpha_h)$.

The expression above shows that the correlation between the share of housing capital and the consumption-output ratio is determined by the interplay of the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors. To demonstrate the implications and the mechanism of this proposition, I use the following numerical example for illustration. Suppose that a model economy has already reached its balanced growth path.[23] Suppose that there is a permanent unexpected shock to the growth rate of the economy A,

[23]Three key parameters of the model ϵ, α_k and α_h are varied for illustration. The other parameters of the model economy are set as: $\beta=0.96$, $\eta=0.28$, $\sigma=1$, $\delta=0.06$, $A=1.04$, $z_0=10$.

such that the economy enters the transitional dynamics with capital deepening. I consider two cases: (1) the production of the housing sector is labor intensive, i.e. $\alpha_k > \alpha_h$; (2) the production of the housing sector is capital intensive, i.e. $\alpha_k < \alpha_k$.[24] In each case, I compare two scenarios: (a) when the two sectors are more substitutable ($\epsilon > 1$); (b) when the two sectors are more complementary ($\epsilon < 1$).[25]

Case 1: When the production of housing sector is labor intensive ($\alpha_k > \alpha_h$)

Suppose that there is a permanent shock to the growth rate A such that the economy enters transitional dynamics with capital deepening. Figure 1.9 shows the comovement of the share of housing capital and consumption-output ratio for both scenarios when $\epsilon > 1$ and $\epsilon < 1$.

Case 2: When the production of housing sector is capital intensive ($\alpha_k < \alpha_h$)

Suppose that there is a permanent shock to the growth rate A such that the economy enters transitional dynamics with capital deepening. Figure 1.10 shows

[24] In particular, for case 1, $\alpha_k = 0.3$ and $\alpha_h = 0.1$; for case 2, $\alpha_k = 0.3$ and $\alpha_h = 0.5$.

[25] For scenario (a), $\epsilon = 1.81$; for scenario (b), $\epsilon = 0.76$.

the countermovement of the share of housing capital and consumption-output ratio for both scenarios when $\epsilon > 1$ and $\epsilon < 1$.

Mechanism

Consider the case when the production of housing sector is labor intensive, and the two sectors are more substitutable ($\alpha_k > \alpha_h$ and $\epsilon > 1$). By Proposition 4, the share of housing capital comoves with the consumption-output ratio. Figure 1.11 (a) present the arbitrage opportunities if Proposition 4 is violated, and Figure 1.11 (b) shows the case otherwise.

Q.E.D.

1.7.3 Robustness Check

Tables 1.9 and 1.10 show alternative calibrations of the model economy, in which I consider different values for the intertemporal elasticity of substitution and the growth rate of the economy. The results in Table 1.9 are similar to those of the benchmark calibration. The implication of capital and labor reallocation are basically identical to the benchmark calibration. Also, the relative output between the non-residential and the housing sector increases in the cases for all three σ, implying the increase of the relative prices of housing services. Meanwhile, the general patterns implied by the different values of A in Table 1.10 are also similar to the results of the benchmark calibration.

Table 1.9:

Data and Model Calibration, 1948-2005 (Robustness I)

	US Data		Model $\sigma = 0.5$ $\epsilon = 1.81,\ A = 1.0149$		Model $\sigma = 2$ $\epsilon = 1.81,\ A = 1.0149$		Model $\sigma = 3$ $\epsilon = 1.81,\ A = 1.0149$	
	1948	2005	1948	2005	1948	2005	1948	2005
κ	0.4817	0.4949	0.4817	0.5008	0.4817	0.5009	0.4817	0.5008
l	0.7740	0.8063	0.7819	0.7947	0.7819	0.7947	0.7819	0.7947
c/y	0.6710	0.7041	0.4766	0.6159	0.5168	0.6159	0.5396	0.6157
s/y	0.1506	0.2226	0.1523	0.1653	0.1684	0.1653	0.1684	0.1653

Table 1.10:

Data and Model Calibration, 1948 - 2005 (Robustness II)

	US Data		Model $A = 1.0049$ $\epsilon = 1.81, \sigma = 1$		Model $A = 1.0200$ $\epsilon = 1.81, \sigma = 1$		Model $A = 1.0400$ $\epsilon = 1.81, \sigma = 1$	
	1948	2005	1948	2005	1948	2005	1948	2005
κ	0.4817	0.4949	0.4817	0.5023	0.4817	0.5002	0.4817	0.4986
l	0.7740	0.8063	0.7819	0.7956	0.7819	0.7942	0.7819	0.7932
c/y	0.6710	0.7041	0.5378	0.6335	0.5590	0.6080	0.5266	0.5826
s/y	0.1506	0.2226	0.1370	0.1451	0.1438	0.1496	0.1519	0.1555

1.8 Data Constructions for China: 1987-2013

In [28], data for the developed economies are from their national capital accounts, including consistent annual balance sheets[26]. In contrast, China does not have such a national account for capital stock, giving rise to challenges in measuring capital allocations in China. In this paper, I construct a dataset of capital stock in China that is similar to the measure of capital stock for the five developed economies as in [28]. In the following, I describe the approaches to construct this dataset.

1.8.1 Agricultural Land Value in China

In order to measure the value of agricultural land in China, data for both the land price and the land area are needed. The data for the land price index in China is not available until 2000.[27] To obtain a consistent time series data for the land price in China, I take the 30 percent of the average sales price for commercial residential housing in China as a proxy for the missing data between 1987 and 2000.[28]. It is not the most accurate measure, but is still reasonable given a high correlation between the land price index and the average sales price for commercial residential housing after 2000 in China.

[26]Following new international guidelines, the balance sheets report on the market value of all the non-financial and financial assets and liabilities held by each sector of the economy (households, government and corporations) and by the rest of the world

[27]Land price index data after 2000 can be found from Ministry of Land and Resource of China.

[28]This is because the availability of the average sales price for commercial residential housing from [9]. In [12], the land value is approximately 30% of the housing sales value.

Table 1.11: Data Sources for Areas of Agricultural Land in China

Year	Mha	Source
1987	137.3	Institute of Applied Remote Sensing CAS
1994	136.4	[10]
1995	131.1	State Land Administration Bureau
1996	133.3	State Land Administration Bureau
2008	96.0	China Statistical Yearbook (2014)
2012	65.0	Land and Resource Ministry of China

To obtain the time series data for the area of agricultural land between 1987 and 2012, I interpolate the time series from the data points obtained from various sources as shown in Table 1.11[29]. Figure 1.12 shows that the total area of China's agricultural land has been steadily decreasing. Table 1.12 shows the dataset constructed for the agricultural land value to output ratio in China between 1987-2013, which shows that while the unit land price (yuan per sq.m) has been increasing, the area of agricultural land keeps decreasing, and the agricultural land value to output ratio is also decreasing.

[29] [31] made the effort in collecting data for the agricultural land use from various sources, and I found two more recent data points from China Statistical Yearbook (2014) and the Land and Resource Ministry of China

1.8.2 Non-residential and Housing Capital in China

To measure the value of non-residential and housing capital in China, I take the standard perpetual inventory approach. I initialize the housing capital in 1994 as the ratio of the housing investment in 1995[30] to the sum of the average growth rate of the housing investment between 1995 and 2000 and the depreciation rate of capital. Further, I estimate the housing investment for the missing years between 1987 and 1994 by 40 percent of the investment in construction and installment[31], and then back out the housing capital between 1987 and 1993 following the same perpetual inventory approach. The values of nonresidential capital are estimated using the same approach. In particular, the depreciation rate is assumed to be 8 % for housing, and 24% for machinery and equipment.[32] To account for the price effect on the housing capital in China, I construct the time series of housing price indices, using the price indices of the investment in fixed assets between 1987 and 2003, and the real residential land price indices from [35] between 2004-2013. Table 1.13 shows the capital allocation in China between 1987 and 2013.

[30]the first year when the investment data in housing are available

[31]In China Statistical Yearbook, data for the investment in construction and installment started earlier from 1981. The available data show that the housing investment is about 40 percent of the investment in construction and installment.

[32]In [4], estimates of the useful lives of structures and buildings is 38 years, and of machinery and equipment is 12 years

1.9 A Statistical Comparison between the United States and China

The aim is to find an appropriate development stage in the US economy for comparison in the counterfactual experiment. CSY started to categorize the investment of total fixed assets into non-residential and housing from 1995 in Table 10.4, which allows a 19-year time series for analyzing the investment behavior in China. In order to find an appropriate episode in the US economic history to compare with, I resample the US data between 1930 and 2013, using a moving block bootstrap approach proposed by [29]. In particular, I fix the length of the block to be 19 years, and obtain 66 blocks of 19 years between 1930 and 2013 in the US economy. Then, I calculate the joint correlations of the consumption-output ratio, the non-residential capital-output ratio and the housing capital-output ratio for the 66 blocks.

Figure 1.13 shows a comparison between the dynamics of the sectoral allocation of capital in the US and China. Table 1.14 shows the joint correlation of the three targeted variables between the US economy and China. The highlighted row in Table 1.14 shows the highest joint correlation among the 66 sampled blocks. Hence, I choose the episode of the US economy between 1956 and 1982 as a reference for the study of China between 1987 and 2013.

1.10 Numerical Solution Algorithm

The social planner's problem in this paper is an extension of the standard dynamic programming problem, with a two-dimensional state vector of nonresidential and housing capital stock, (k, h). In particular, the detrended SP problem of interest can be summarized by the following Bellman equation:

$$v(\hat{k}, \hat{h}) = \max_{\hat{k}', \hat{h}'} \frac{\left[(\eta \hat{c}^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)\hat{s}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \right]^{1-\sigma} - 1}{1-\sigma} + \beta v(\hat{k}', \hat{h}')$$

subject to

$$\hat{c} = \hat{k}^{\alpha_k}(1-l)^{1-\alpha_k} + (1-\delta)(\hat{k}+\hat{h}) - A \cdot (\hat{k}' + \hat{h}')$$

$$\hat{s} = \hat{h}^{\alpha_h} l^{1-\alpha_h}$$

$$l = \left\{ 1 + \frac{\alpha_h(1-\alpha_k)}{\alpha_k(1-\alpha_h)} \left(\frac{\hat{k}}{\hat{h}} \right) \right\}^{-1}$$

The solution algorithm follows:

1. Compute the steady-state value of κ^* and $\hat{a}^*$ according to the assigned parameters.

2. Discretize the state space $(\hat{k}, \hat{h})$ on the gridded domain, where $\hat{k} \in [\hat{k}_{\min}, \hat{k}_{\max}]$ and $\hat{h} \in [\hat{h}_{\min}, \hat{h}_{\max}]$. $[\hat{k}_{\min}, \hat{k}_{\max}]$ and $[\hat{h}_{\min}, \hat{h}_{\max}]$ are chosen according to the steady-state value of κ^* and $\hat{a}^*$.

3. Taking $\hat{h}'$ as given, find the optimal $\hat{k}'(\hat{k}, \hat{h}, \hat{h}')$ by iterating on the following value function until convergence:

$$v^{j+1}(\hat{k}, \hat{h}) = \max_{\hat{k}' \in [\hat{k}_{\text{lb}}, \hat{k}_{\text{ub}}]} u(\hat{c}(\hat{k}, \hat{h}, \hat{k}', \hat{h}')) + \beta v^j(\hat{k}', \hat{h}')$$

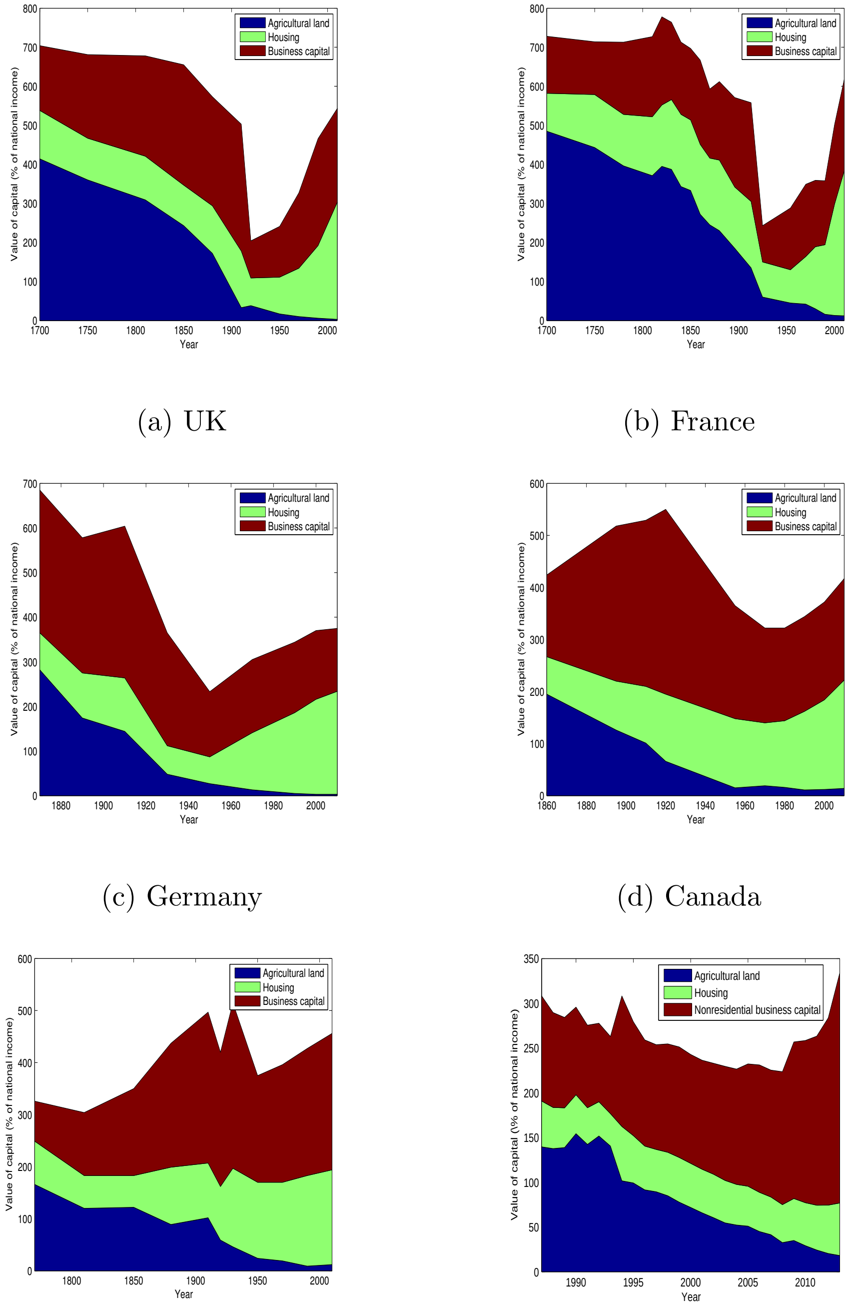
where the lower and upper bound for $\hat{k}'$ are $\hat{k}_{\text{lb}} = \max\left\{\frac{(1-\delta)\hat{k}}{A}, \hat{k}_{\min}\right\}$ and $\hat{k}_{\text{ub}} = \min\left\{\frac{\hat{k}^{\alpha_k}(1-l)^{1-\alpha_k}+(1-\delta)\hat{k}-(A\hat{h}'-(1-\delta)\hat{h})}{A}, \hat{k}_{\max}\right\}$. Note that the upper bound is chosen as in the extreme case when the consumption of the nondurable goods and services is zero.

4. Taking the function $\hat{k}'(\hat{k}, \hat{h}, \hat{h}')$ as given, find the optimal $\hat{h}'(\hat{k}, \hat{h})$ by interating on the following value function until convergence:

$$v^{j+1}(\hat{k}, \hat{h}) = \max_{\hat{h}' \in [\hat{h}_{\text{lb}}, \hat{h}_{\text{ub}}]} u(\hat{c}(\hat{k}, \hat{h}, \hat{k}'(\hat{k}, \hat{h}, \hat{h}'), \hat{h}')) + \beta v^j(\hat{k}'(\hat{k}, \hat{h}, \hat{h}'), \hat{h}')$$

where the lower and upper bound for $\hat{h}'$ are $\hat{h}_{\text{lb}} = \max\left\{\frac{(1-\delta)\hat{h}}{A}, \hat{h}_{\min}\right\}$ and $\hat{h}_{\text{ub}} = \min\left\{\frac{\hat{k}^{\alpha_k}(1-l)^{1-\alpha_k}+(1-\delta)\hat{h})}{A}, \hat{h}_{\max}\right\}$. Note that the upper bound is chosen as in the extreme case when both the consumption of the nondurable goods and services and the nonresidential investment is zero, as if all the resources were used for the housing investment.

With the policy functions $\hat{k}'(\hat{k}, \hat{h})$ and $\hat{h}'(\hat{k}, \hat{h})$, one can obtian a time-serise simulation for the policy functions $\hat{k}(t)$ and $\hat{h}(t)$ by feeding in the intial values $(\hat{k}_0, \hat{h}_0)$.



(a) UK (b) France

(c) Germany (d) Canada

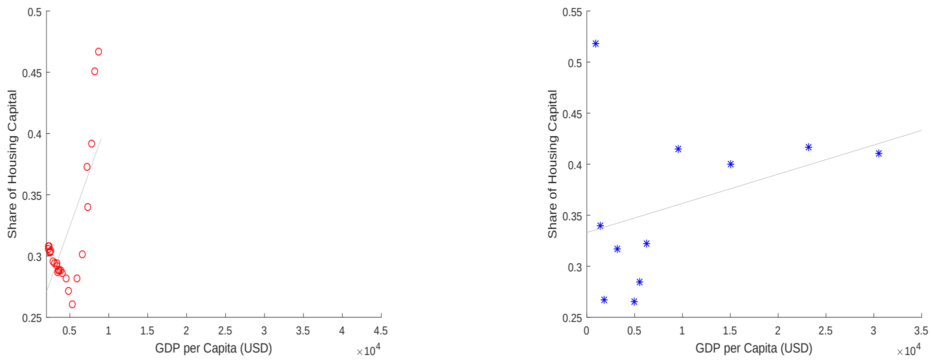
(e) US (f) China

Figure 1.1: Allocation of Domestic National Capital

Data Sources:

1. [28] for five developed economies.

2. Constructed for China. (See Appendix 1.8 for detailed construction approaches)



(a) China: 1987-2013

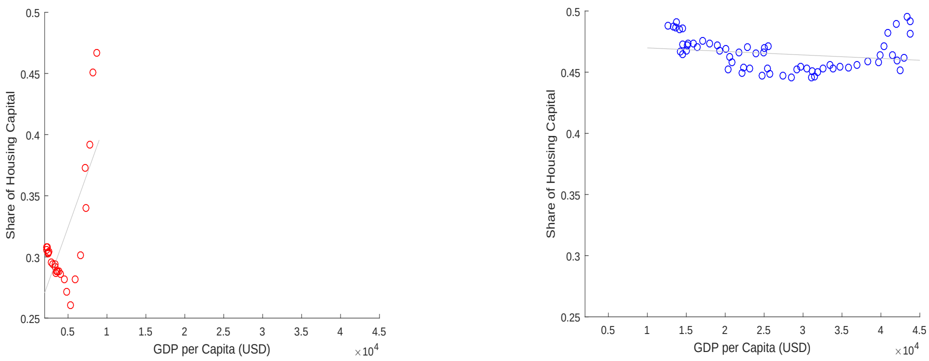
(b) US: 1720 -2011

Figure 1.2: GDP Per Capita vs. Share of Housing Capital

Data Sources:

1. GDP per capita: [5] for the United States; Penn World Table 8.1 for China. Both are in 2005 PPP-adjusted USD.

2. Share of housing capital: Computed using non-residential capital and housing (real) from [28] for the United States; from Table 1.13 for China.



(a) China: 1987-2011

(b) US: 1950 -2011

Figure 1.3: GDP Per Capita vs. Share of Housing Capital

Data Sources:

1. GDP per capita: Penn World Table 8.1 for both the United States and China. Both are in 2005 PPP-adjusted USD.

2. Share of housing capital: Computed using non-residential capital and housing (real) from Fixed Assets Table for the United States; from Table 1.13 for China.

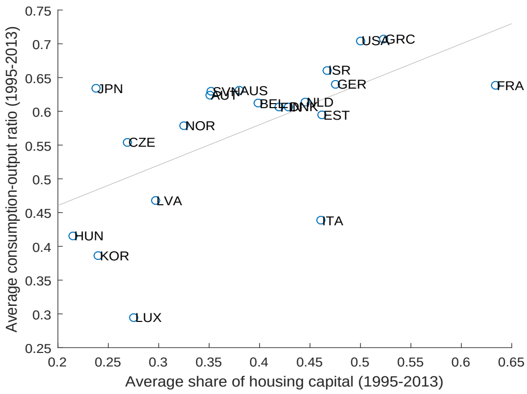
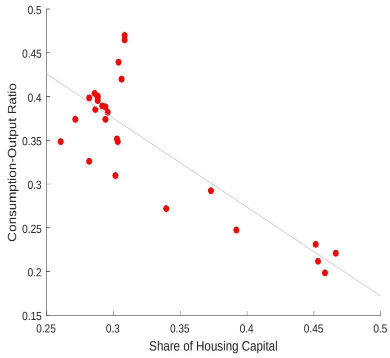


Figure 1.4: Share of Housing Capital vs. Consumption-Output Ratio

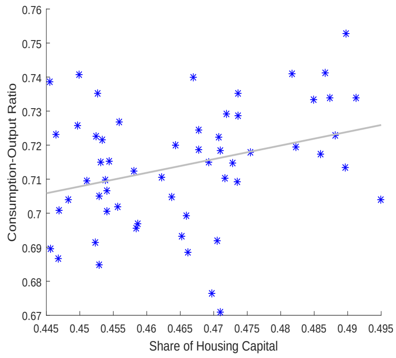
Data Sources: OECD Statistics

1. Average share of housing capital: Computed using non-residential capital and housing (real) from the Balance Sheets for Non-financial Assets.

2. Average consumption-output ratio: Computed using consumption and output (real) from the Final Consumption Expenditure of Households Table.



(a) China: 1987-2011



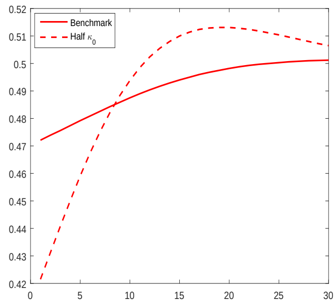
(b) US: 1950 -2011

Figure 1.5: Share of Housing Capital vs. Consumption-Output Ratio

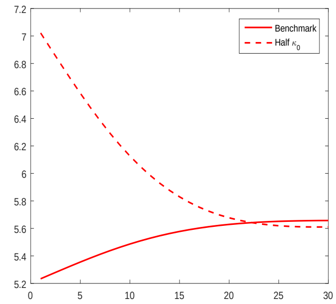
Data Sources:

1. Share of housing capital: Computed using non-residential capital and housing (real) from Fixed Assets Table for the United States; from Table 1.13 for China.

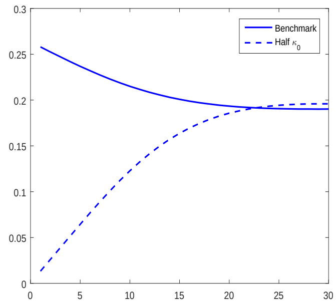
2. Consumption-output ratio: Computed using consumption and GDP (real) from NIPA for the United States; from China Statistical Yearbook (2014) for China.



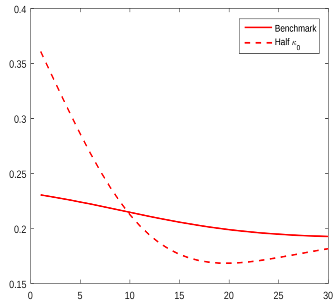
(a) κ



(b) q

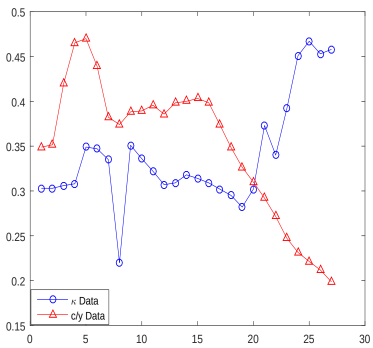


(c) i_k/y

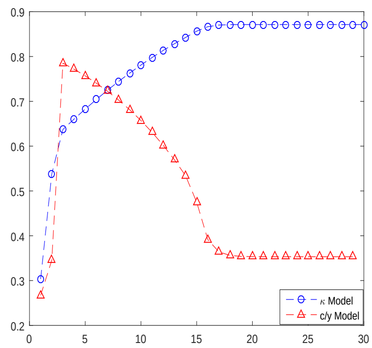


(d) i_h/y

Figure 1.6: Benchmark vs. Half initial κ_0

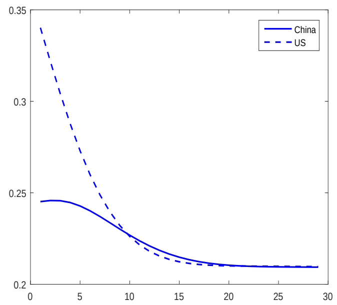


(a) Data

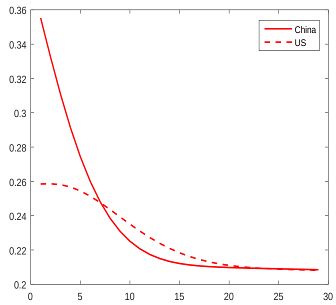


(b) Model

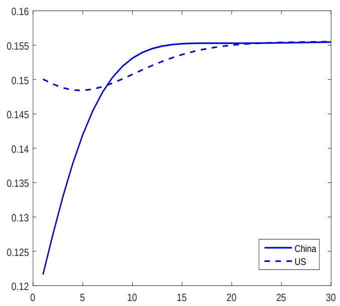
Figure 1.7: Negative Correlation between κ and c/y in China



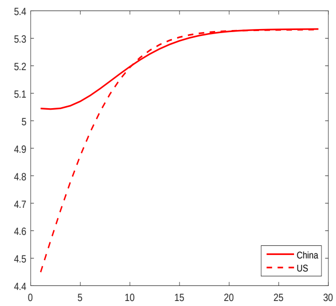
(a) i_k/y



(b) i_h/y

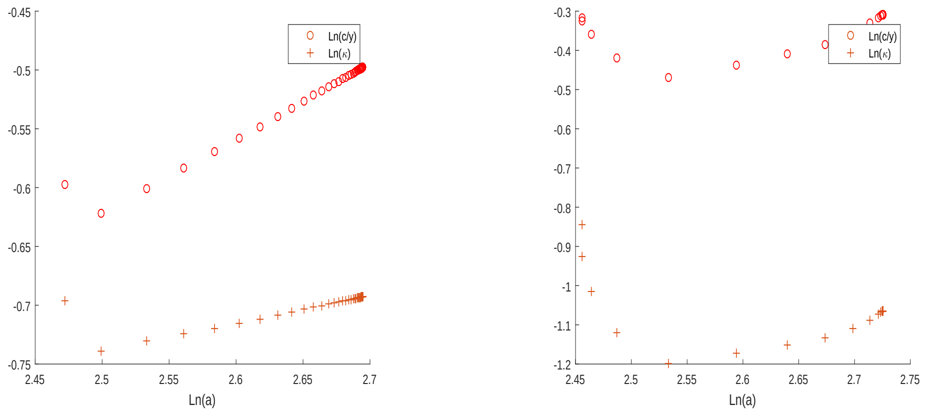


(c) s/y



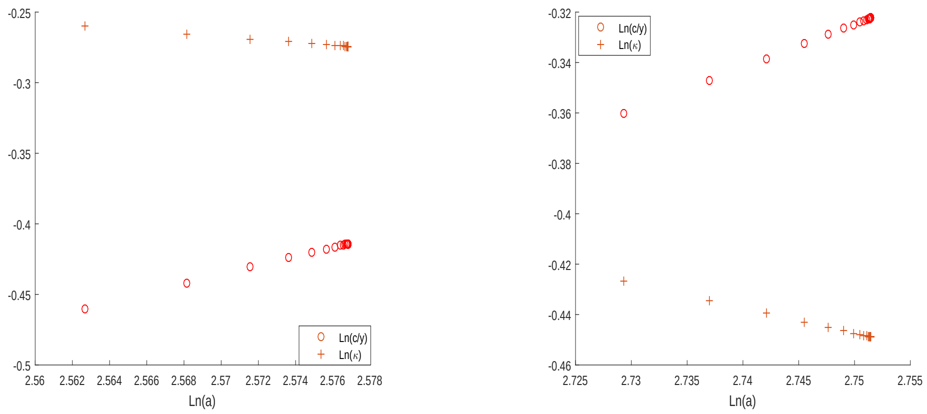
(d) q

Figure 1.8: China Benchmark vs. US allocation



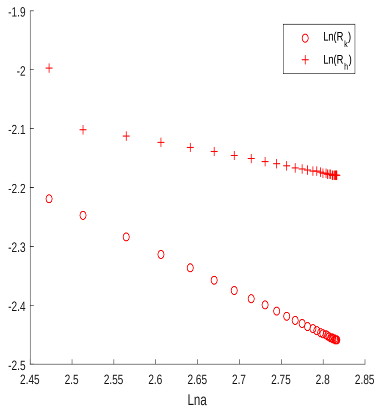
(a) $\epsilon = 1.81$ (b) $\epsilon = 0.76$

Figure 1.9: Comovement of κ and c/y

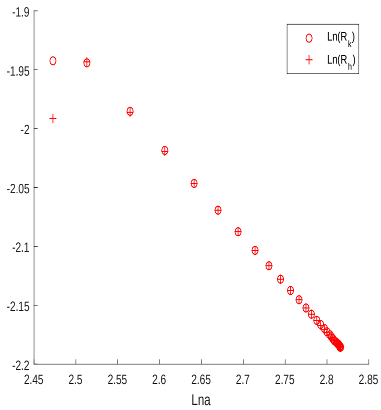


(a) $\epsilon = 1.81$ (b) $\epsilon = 0.76$

Figure 1.10: Countermovement of κ and c/y



(a) Without Comovement



(b) With Comovement

Figure 1.11: Return to the housing captial R_h

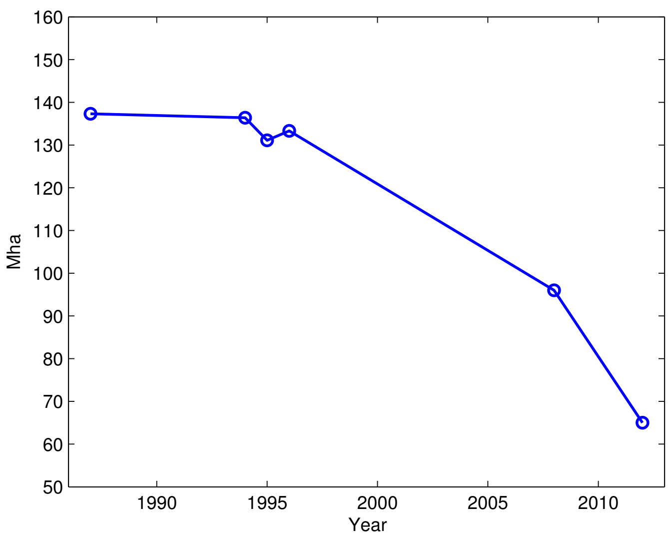


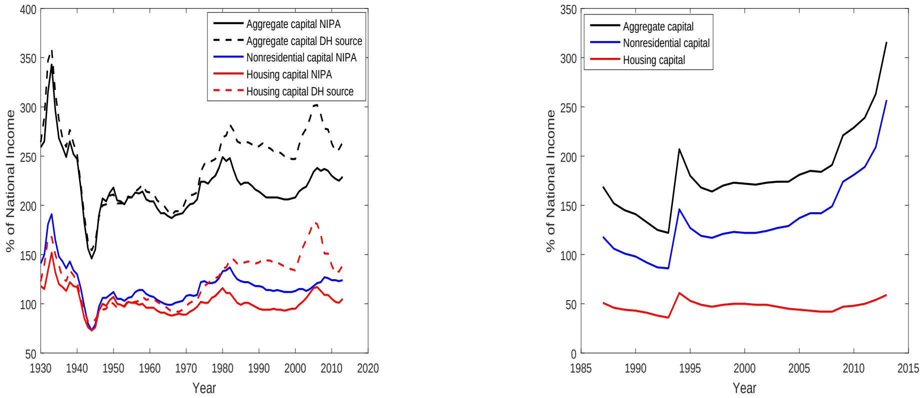
Figure 1.12: Areas of Agricultural Land in China: 1987- 2012

Table 1.12: Agricultural Land Value to Output Ratio

Year	**Mha** (10^{10} sq.m)	**Land price** (yuan/sq.m)	**Land value to output Ratio** (%)
1987	137.3*	122.5	139.52
1988	137.2	150.9	137.65
1989	137.1	172.1	138.72
1990	137.0	210.9	154.28
1991	136.8	226.9	142.25
1992	136.7	298.9	151.74
1993	136.6	362.5	140.45
1994	136.4*	358.2	101.56
1995	131.1*	452.7	99.22
1996	133.3*	481.4	91.48
1997	130.0	536.9	89.40
1998	126.7	556.1	84.88
1999	123.6	557.1	77.81
2000	120.5	584.5	71.87
2001	117.5	605.0	65.78
2002	114.6	627.5	60.36
2003	111.7	659.2	54.55
2004	108.9	764.6	52.23
2005	106.2	881.1	50.96
2006	103.6	935.8	44.88
2007	101.0	1093.6	41.45
2008	96.0*	1072.7	32.58
2009	88.8	1337.8	34.91
2010	82.1	1417.5	29.12
2011	76.0	1498.0	24.29
2012	65.0*	1629.0	20.43

Table 1.13: Allocation of National Capital

Year	Capital-output ratio			
	Non-residential (%)	Housing (%)	Agricultural Land (%)	Aggregate (%)
1987	117.55	51.22	139.52	308.29
1988	105.95	46.01	137.65	289.60
1989	100.92	44.48	138.72	284.12
1990	97.84	43.60	154.28	295.72
1991	92.23	41.08	142.25	275.56
1992	87.64	38.29	151.74	277.66
1993	86.30	36.22	140.45	262.96
1994	145.84	60.83	101.56	308.24
1995	127.25	52.94	99.22	279.40
1996	118.63	48.94	91.48	259.05
1997	116.96	47.34	89.40	253.70
1998	121.08	48.65	84.88	254.61
1999	123.46	49.97	77.81	251.24
2000	121.67	49.26	71.87	242.80
2001	121.78	48.81	65.78	236.37
2002	123.94	48.65	60.36	232.95
2003	127.48	47.49	54.55	229.52
2004	128.80	45.42	52.23	226.45
2005	136.58	53.63	50.96	241.18
2006	142.34	61.49	44.88	248.72
2007	141.75	84.27	41.45	267.46
2008	148.51	76.41	32.58	257.50
2009	174.94	112.73	34.91	322.58
2010	181.23	148.96	29.12	359.31
2011	188.90	165.19	24.29	378.37
2012	209.10	173.02	20.43	402.56
2013	257.01	217.41	18.08	492.50



(a) US: 1930-2013 (b) China: 1987-2013

Figure 1.13: Sectoral Allocation of Captial: US v.s. China

Table 1.14: Joint Correlation of c/y, k/y and h/y

	c/y	h/y	k/y
1963-81	0.9422	0.5407	0.7401
1964-82	0.9421	0.6663	0.9003
1965-83	0.8958	0.5695	0.9479
1966-84	0.8371	0.4348	0.9376
1967-85	0.7376	0.2732	0.9099
1968-86	0.6073	0.0702	0.8879

Chapter 2

Human Capital Spillover and Housing Price

2.1 Introduction

Human capital concentration and housing price growth usually go hand in hand. Compared with other regions in the United States, the agglomeration in San Francisco, California attracts workers with high level of human capital, giving rise to both higher local wages and higher local housing prices. Similar empirical patterns are also observed in China. Since the Special Economic Zone (SEZ) policy, SEZ regions have drawn a large number of migrants from the non-SEZ regions in China.[1] At the same time, housing price of the SEZ regions stay higher than that of the non-SEZ regions.

Some aspects of the migration process are puzzling. During this process, many of the new migrants seem to worse off than they were where they come from. They are faced with higher housing prices, live in more congested space, and have no regular jobs. What drives them to keep coming? [30] shows that 60 percent of human capital concentration is due to enhanced wage, with the rest caused by growth in living quality, all of which in turn drive up housing price. As shown in Table 2.1, compared with the non-SEZ cities and the national average, Beijing and Shanghai, as representatives of the SEZ cities, experience a significantly higher growth in population, housing price indices, and level of GDP per capita.

Literature has studies the external effect of human capital that leads to human capital concentration.[2] A natural question that arises is that what is the external effect of human capital on the housing price. In this paper, I propose a two-region model to address this question. In the model, high wage in the SEZ region reflect high level of human capital, and these jobs are not available to low human capital migrants from the non-SEZ economy. The migrants come to the SEZ economy for two reasons: on the one hand, the SEZ economy is a better place to accumulate human capital and earn a higher wage in the future; on the other hand, the SEZ

[1]Special Economic Zone policy is one of the series of economic reform undertaken in China since 1979. The Chinese government selected a few regions along the coastal areas, where special treatment policies are provided, including special tax incentives for investments, and independence on trade activities.

[2]See [14] and [26].

Table 2.1

	Growth in Share of Population (%)	Growth in Housing Price Index (%)	Average GDP Per Capita in 2005 USD
	2006-2013	2006-2013	2006-2013
National	4.3	169.3	5238
Representative SEZ Cities			
Beijing	27.1	353.4	11652
Shanghai	18.8	286.0	11262
Representative Non-SEZ Cities			
Wuhan	3.6	60.1	5327
Xi'an	6.2	73.0	4352

Data Sources:

[1] Share of population: China Statistical Yearbook 2014

[2] Housing price index: [35]

[3] GDP per capita: China Statistical Yearbook 2014

[4] SEZ: Special Economic Zone, regions that have received subsidiary policy of Chinese government

[5] Non-SEZ: regions that have not received subsidiary policy of Chinese government

economy has a better amenities for living. The theory shows that during the migration process, the spillover effect of human capital has a significant positive impact on the increase of housing prices.

The model is based on the two-region economy of [26]. The non-SEZ economy has a Cobb-Douglas production technology with a single labor input, where as the SEZ economy has a human-capital based production technology. In particular, each migrant to the SEZ economy allocate a fixed time endowment between laboring for wage that is indicated by one's current level of human capital and accumulating human capital so as to increase future wage. I start with considering the equilibrium allocations in the non-SEZ economy and in the SEZ, human-capital based economy, both in isolation first. In the baseline model with migration, the share of population that choose to migrate to the SEZ economy is determined by the utility equalization between living in either economy. In the baseline model, the migration occurs all at once at the first period. Further, I extend the baseline model by incorporating the spillover effect of human capital: time invested in human capital accumulation has a higher return in high human capital environment. In this case, the migration to the SEZ economy becomes increasingly attractive as the gap between the human capital leaders and followers increase. By comparing the extended model with the baseline, the effect of human capital spillover can be captured.

2.2 The Model

Consider a two-sector (non-SEZ and SEZ) economy, with a fixed total population of identical households, viewed as infinitely-lived dynasties. Every household has preferences

$$\sum_{t=0}^{\infty} \frac{u(c_t, m_t, a_t)}{(1+\rho)^t} \tag{2.1}$$

over paths c_t of nondurable consumption goods, m_t of housing services, and a_t of amenities within living environment, which is positively correlated with GDP per capita. ρ is a measure of time preference.

Assume that the functional form of the households' preference is

$$u(c, m, a) = \mu_c \log(c) + \mu_m \log(m) + \mu_a \log(a)$$

where μ_c, μ_m and μ_a denote the household's preference weights of consumption in nondurable good, housing services, and living amenity, respectively. Each household is endowed with one unit of time, supplied inelastically to income-directed activities: working for wages and accumulating human capital. Each household is endowed with one unit of production land, which yields a rental return.[3] In both economies, total population and housing services are normalized to unity, and housing services are assumed to be supplied inelastically. The main focus of the model will be on the labor reallocation from non-SEZ to SEZ, but I begin by setting notation and describing resource allocation for two polar economies.

2.2.1 Non-SEZ Polar Economy

In the non-SEZ polar economy, each household is endowed with one unit of production land, which is combined with labor to produce nondurable goods using a Cobb-Douglas technology:

$$F(x_t) = Ax_t^{\alpha}$$

[3]For simplicity of analysis, I assume that the only physical capital in the model is land.

where x denote the employment, A denotes the constant labor productivity, and α denotes the labor intensity of production.

Human capital is assumed to have no effect on the labor productivity in the non-SEZ polar case, so no time is spent to human capital accumulation in the non-SEZ economy. In an economy where the entire workforce is employed in production, the competitive wage is $w_t = F'(1)$, and the equilibrium nondurable good consumption is $c_t = F(1) = A$. Capital return on production land is $F(1) - F'(1)$, and interest rate is constant at $r_t = \rho$. Meanwhile, the equilibrium housing service consumption $m_t = 1$ due to the inelastic supply and the market-clearing of housing services. Normalize the price of nondurable consumption goods to one. The competitive equilibrium relative price of housing services is

$$p_t = \frac{u_m(c_t, m_t, a_t)}{u_c(c_t, m_t, a_t)} = \frac{A\mu_m}{\mu_c} \tag{2.2}$$

which indicates that the equilibrium relative price of housing services in the non-SEZ polar economy stays constant, and is determined by the labor productivity of production of the economy. Note that the preference weight of living amenities does not affect the relative price of housing services in the non-SEZ polar economy.

2.2.2 SEZ Polar Economy

In the SEZ polar economy, the production technology for nondurable goods is linear in the labor inputs.

$$G(\nu_t, h_t) = \nu_t h_t \tag{2.3}$$

which means that a worker with human capital h_t who spends ν_t units of time labor will produce $\nu_t h_t$ units of nondurable consumption goods.

Human capital accumulation follows:

$$h_{t+1} - h_t = \delta h_t (1 - \nu_t) \tag{2.4}$$

where $1 - \nu(t)$ represents the time spent for the knowledge-improving activities,[4] and δ denotes the rate of return for the time spent to accumulate human capital.

Denote the rental price of production land and wage rate by R and w, and the interest rate by r. Let p denote the relative price of housing services. The budget constraint for the households in the SEZ polar economy for each period t is

$$c_t + p_t m_t \leq w_t \nu_t + R_t \tag{2.5}$$

which means that the expenditure on consumption of nondurable goods and housing services is no more than the sum of labor and non-labor earnings.

Without loss of generality, assume that the living amenity a_t is linear in the total production of the economy, i.e., $a_t = \gamma G(\nu_t, h_t)$, $\gamma > 0$. Assume that the measure of households are continuous such that each household is infinitesimal, which means that each single household cannot decide the level of living amenities. Taking the path of living amenities $(a_t)_{t \geq 0}$ and the paths of prices $(r_t, R_t, w_t, p_t)_{t \geq 0}$ as given, each household makes consumption decision $(c_t, m_t)_{t \geq 0}$, the time allocation between labor and accumulating human capital $(\nu_t)_{t \geq 0}$, and the stock of

[4] [26] gives examples of knowledge-improving activities, including useful experience on and off the job, as well as schooling.

human capital $(h_t)_{t\geq 0}$ to maximize the lifetime utility of (2.1), subject to dynamics of human capital accumulation (2.4) and the budget constraint (2.5).

The equilibrium interest rate must equal to the rate of return of human capital accumulation whenever households engage in both labor and human capital accumulation.

$$r_t = \delta \tag{2.6}$$

This is from the linearity in time for both production technology (2.3) and human capital accumulation (2.4).

Euler equation implies

$$\frac{c_{t+1}}{c_t} = \frac{1+r_t}{1+\rho} = \frac{1+\delta}{1+\rho} \tag{2.7}$$

which implies that the consumption of nondurable goods grows at a constant rate.

The equilibrium relative price of housing services is determined by the marginal rate of substitution between the nondurable goods and housing services, $p_t = \frac{\mu_m}{\mu_c} \cdot \frac{c_t}{m_t}$. In equilibrium, due to market clearing, $m = 1$ for all t, we have:

$$\frac{p_{t+1}}{p_t} = \frac{c_{t+1}}{c_t} = \frac{1+\delta}{1+\rho} \tag{2.8}$$

which means that the relative price of housing services in the SEZ polar economy is growing at the same constant rate as consumption over time.

A balanced growth path of the SEZ economy is defined as an equilibrium, along which the time allocation between labor and human capital accumulation stays constant, and all the variables of the economy $\{w_t, h_t, p_t, c_t, a_t\}$ grow at the same rate.

Proposition 1: Assume that $\rho \leq \delta$. There exists a unique balanced growth path, along which the steady-state time allocation between labor and human capital accumulation is:

$$\nu^* = \frac{\rho + \delta\rho}{\delta + \delta\rho} \tag{2.9}$$

Further, the variables of the economy $\{w_t, h_t, p_t, c_t, a_t\}$ grow at rate of $\frac{\delta-\rho}{1+\rho}$.

The result from Proposition 1 aligns with AK model of endogenous growth, where all economic variables grow at the same rate, and the growth rate depends on the accumulation of human capital.

2.2.3 A Baseline Model of Migration

With two polar economies specified, I turn to study the economy with migration. The household's optimization problem remain the same, except that each household is free to move, and is able to choose to work in either economy. Assume that all households start from the non-SEZ economy, and are endowed with a common human capital level h_0. With identical households, the migration occurs only once at period 0, leaving a constant fraction of households permanently staying in the non-SEZ economy.

Denote x_0 as the share of population migrating to the SEZ economy, and $1-x_0$ stands for the share of population staying in the non-SEZ economy. For the non-SEZ economy, market clearing conditions in both nondurable good consumption and housing service consumption, $(1-x_0)c_t^n = F(1-x_0) = A(1-x_0)^\alpha$ and $(1-x_0)m_t^n = 1$, imply that the equilibrium consumption of nondurable goods and housing services in the non-SEZ economy are functions of x_0:

$$c_t^n(x_0) = A(1-x_0)^{\alpha-1} \tag{2.10}$$

$$m_t^n(x_0) = \frac{1}{1-x_0} \tag{2.11}$$

The equilibrium living amenity can also be written as a function of x_0:

$$a_t^n(x_0) = A\gamma(1-x_0)^\alpha \tag{2.12}$$

Equation (2.2) implies that the equilibrium relative price of housing services in the non-SEZ economy is:

$$p_t^n(x_0) = \frac{\mu_m}{\mu_c} A(1-x_0)^\alpha \tag{2.13}$$

Meanwhile, with migration, the emerging SEZ economy is populated with x_0 measure of households. Market clearing conditions in nondurable good and housing service consumption imply:

$$c_t^s \cdot x_0 = G(u_t^s, h_t^s) \tag{2.14}$$

$$m_t^s(x_0) = \frac{1}{x_0} \tag{2.15}$$

Equations (2.6), (2.9), (2.7) and (2.19) imply:

$$r_t^s = \delta \tag{2.16}$$

$$\nu_t = \nu^* = \frac{\rho(1+\delta)}{\delta(1+\rho)} \tag{2.17}$$

$$c_t^s = c_0^s \left(\frac{1+\delta}{1+\rho}\right)^t \tag{2.18}$$

$$p_t^s(x_0) = x_0 c_0^s \cdot \frac{\mu_m}{\mu_c}\left(\frac{1+\delta}{1+\rho}\right)^t \tag{2.19}$$

where $c_0^s = \nu^* h_0 = \frac{\rho(1+\delta)}{\delta(1+\rho)} h_0$ represents the initial nondurable good consumption. Further, the living amenity each period can be written as:

$$a_t^s(x_0) = \gamma G(u_t^s, h_t^s) \tag{2.20}$$

Denote the maximized lifetime utility for the households who stay in the non-SEZ economy by the value function $V^n(x_0) = \sum_{t=0}^{\infty} \frac{u(c_t^n(x_0), m_t^n(x_0), a_t^n(x_0))}{(1+\rho)^t}$. Denote the maximized lifetime utility for the households who migrate to the SEZ economy by the value function $V^s(x_0) = \sum_{t=0}^{\infty} \frac{u(c_t^s(x_0), m_t^s(x_0), a_t^s(x_0))}{(1+\rho)^t}$.

Lemma 1: In the baseline model of migration, there exists a unique equilibrium in which both V^n and V^s are functions of x_0. In particular,

$$V^n(x_0) = \frac{1+\rho}{\rho}\Big\{f(\Theta) - \Big[\mu_c + \mu_m - \alpha(\mu_c + \mu_a)\Big]\log(1-x_0)\Big\} \tag{2.21}$$

where Θ is a set of parameters, and $f(\cdot) = (\mu_c + \mu_a)\log A + \mu_a \log \gamma$, and

$$V^s(x_0) = \frac{1+\rho}{\rho}\Big\{g(\Theta) + (\mu_a - \mu_m)\log x_0\Big\} \tag{2.22}$$

where $g(\cdot) = (\mu_c + \mu_a)\Big[\log\left(\frac{\rho}{\delta}\right) + \frac{\rho+1}{\rho}\log\left(\frac{1+\delta}{1+\rho}\right) + \log(h_0)\Big] + \mu_a \log\gamma$. The unique equilibrium share of population staying in the non-SEZ economy x_0^* is determined by

$$V^n(x_0^*) = V^s(x_0^*).$$

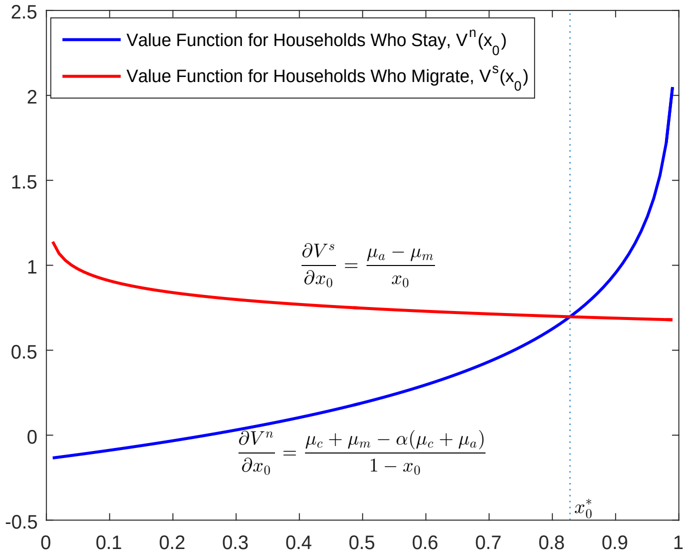


Figure 2.1: x_0^* in the Baseline Model of Migration

Figure 2.1 shows that with more households migrating to the SEZ economy, i.e., as x_0 increases, the households who stay in the non-SEZ economy gain more consumption per capita of both nondurable goods and housing services, which is represented by the first expression of the slope of the value function, $\frac{\partial V^n}{\partial x_0}$. And, with more labor engaging in production, the total production and thus the living amenity level decrease. This causes utility loss, which is represented by the second expression of the slope of the value function, $\frac{\partial V^n}{\partial x_0}$.[5] Hence, the value function of households staying in the non-SEZ economy is increasing in the share of population migrating, if the gain from more consumption of nondurable good and housing service consumption is higher than the loss from deteriorating living amenities, i.e., $\mu_c + \mu_m > \alpha(\mu_c + \mu_a)$, and vice versa.

Meanwhile, with more households populated in the SEZ economy, more labor inputs in production lead to a higher level of total production. Hence, the migrating households gain from the improving living amenities, which is represented by the first expression of the slope of the value function, $\frac{\partial V^s}{\partial x_0}$. At the same time, more population cause congestion, i.e. less consumption per capita of housing services given the fixed supply of housing services. This causes utility loss, which is represented by the second expression of the slope of the value function, $\frac{\partial V^s}{\partial x_0}$. Hence, the value function of households migrating to the SEZ economy is increasing in the share of population migrating if the preference weight of living amenities is higher than that of housing service consumption, i.e., $\mu_a > \mu_m$, and vice versa.

Denote the share of production in the SEZ economy as ω_t, and the ratio of relative price of housing services between the two economies as λ_t.

[5] This is because of the positive correlation between living amenities and GDP per capita by assumption.

Proposition 2: In the equilibrium of migration in the baseline economy, the share of production in the SEZ economy, ω_t, is:

$$\omega_t = \frac{x_0^* h_0 \nu^* \left(\frac{1+\delta}{1+\rho}\right)^t}{x_0^* \nu^* h_0 \left(\frac{1+\delta}{1+\rho}\right)^t + A(1-x_0^*)^\alpha} \tag{2.23}$$

and the ratio of relative price of housing services between the two economies, λ_t, is:

$$\lambda_t = \frac{\rho}{\delta} \cdot \frac{x_0^* h_0}{A(1-x_0^*)^\alpha} \left(\frac{1+\delta}{1+\rho}\right)^{t+1} \tag{2.24}$$

Figure 2.2 shows the dynamics of the share of production in the SEZ economy and the ratio of relative price of housing services between the two economies over time. Over the 50-period transition, both the share of production in the SEZ economy and the ratio of relative price of housing services increases.

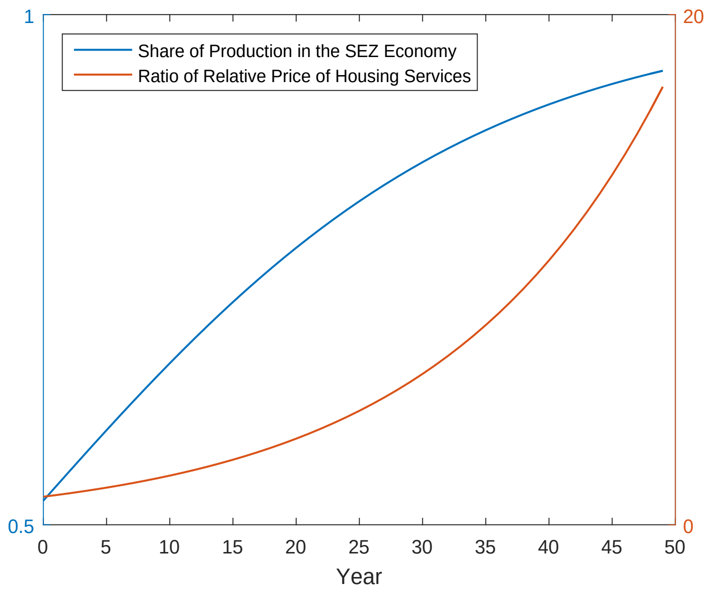


Figure 2.2: Share of Production and Ratio of Relative Price of Housing Services

The baseline model captures the increase in the share of production and the increase in the relative of housing prices in the SEZ economy. However, the migration only occurs once and for all, leaving a constant fraction of population permanently staying in the non-SEZ economy.

2.2.4 Migration with Human Capital Spillover Effect

In order to incorporate the multi-period migration process into the model, I extend the baseline model such that an externality affects the human capital accumulation.[6] In this case, the model suggests a multi-period migration and a population in which different cohorts of migrants have different levels of human capital in the SEZ economy.

Let $h_{s,t}$ denote the human capital at period t of a household who migrate to the SEZ economy at period $s \leq t$. Note that households in the migration cohort s behave identically. Assume that the entry-level of human capital is $h_{t,t} = h_0$. Further let H_t denote the highest level of human capital that any worker in the SEZ economy has attained at date t, and assume that the human capital accumulation technology is given by:

$$h_{s,t+1} = h_{s,t} + \delta \left(\frac{H_t}{h_{s,t}} \right)^{\theta} h_{s,t}(1 - \nu_{s,t}) \qquad (2.25)$$

[6] [19] suggested that production can be affected by an externality, in the sense that more productive people nearby can make any individual more productive. [14] and [26] propose a formulation in which an externality affects the technology for accumulating human capital rather than the technology for goods production. In this paper, I adopt the latter formulation.

where $\theta > 0$ is a parameter that captures the spillover effect of the human capital of the leaders on those of the followers.[7] Compared with the human capital accumulation technology of the baseline model implied by (2.4), (2.25) magnifies the return of δ by an increasing function of the gap between the followers' human capital $h_{s,t}$ and the leaders' human capital H_t. Hence, migration to the SEZ economy will become increasingly attractive over time, as those who have migrated earlier accumulate higher level of human capital.

As before, the equilibrium interest rate is also equal to δ. In this case, the human capital leaders will be the only labor engaging in production in equilibrium, and all the new migrants will specialize in accumulating human capital until they catch up and become leaders themselves. This is because of the linearity in labor for both production technology and human capital accumulation technology.[8]The external effects create two classes of cohorts of migrants in the SEZ economy: labor and full-time learners.

Let z_t be the number of leaders at time t, i.e., the number of migrants who have attained the skill level H_t of the leaders and are now producing goods as well as accumulating human capital in the SEZ economy. Their human capital evolves according to:

$$H_{t+1} = H_t + \delta(1 - \nu_t)H_t \tag{2.26}$$

[7]One way to understand the spillover effect is that leaders can lead followers to catch up faster.

[8]This point is also proved in [26] and [14].

The human capital for someone who migrate at period s evolves for $t \geq s$ according to:

$$h_{s,t+1} = h_{s,t} + \delta \left(\frac{H_t}{h_{s,t}} \right)^{\theta} h_{s,t} \tag{2.27}$$

which is the case of (2.25) but with $\nu_{s,t} = 0$.

For every migration cohort s, there will be a catch-up date T_s, at which $h_{s,t} = H_t$ first holds. Figure 2.3 shows the levels of human capital for different cohorts of migrants. For the first cohort of migrants, $s = 1$, the catch-up date is $T_1 = 4$. That being said, the first cohort of migrants specialize in accumulating human capital until period $t = 4$. For the second cohort of migrants, $s = 2$, the catch-up date is $T_s = 6$, i.e., the second cohort migrants specialize in accumulating human capital until period $t = 6$. Denote x_t as the share of population who migrates to the SEZ economy at period t. We have the number of leaders, who are also labors engaging in production, satisfy the following equation:

$$z_{T_s} = x_s \tag{2.28}$$

In Figure 2.3, we have $z_4 = x_1$, i.e., the number of leaders at the catch-up date for the first cohort of migrants, $T_1 = 4$, is equal to the share of population who have migrated to the SEZ economy in period 1, which consist of the leaders and the first cohort of migrants. Further, $z_6 = x_2$, i.e., the number of leaders at the catch-up date for the second cohort of migrants, $T_2 = 6$, is equal to the share of population who have migrated to the SEZ economy in period 2, which consist of the leaders, the first and the second cohorts of migrants.

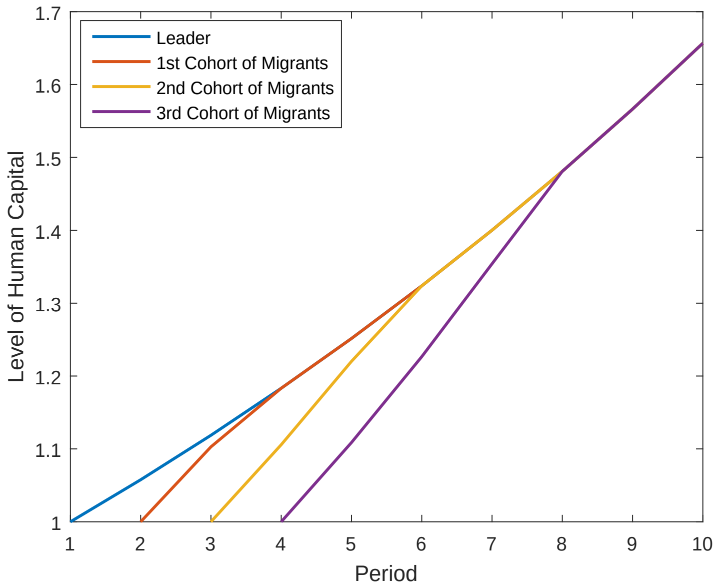


Figure 2.3: Levels of Human Capital for Different Cohorts of Migrants

Households choose to migrate whenever the expected utility from migrating is greater or equal to that from staying. An equilibrium with a multi-period migration is defined as a collection of allocations in the non-SEZ economy $\{c_{n,t}, m_{n,t}, a_{n,t}\}$, and a collection of allocations in the SEZ economy, $\{c_{s,t}, m_{s,t}, \nu_{s,t}, a_t, H_t, z_t, T_s\}$,[9] the relative price of housing services in both non-SEZ and SEZ economies $\{p_t^n, p_t^s\}$, and the share of population migrating to the non-SEZ economy $\{x_t\}$, that satisfy:

1. Given the relative prices of housing services and the share of population migrating to the SEZ economy, households maximize utilities in either economy, such that the allocations in both economies are implied by the equations (2.10), (2.11), (2.12), (2.18), (2.19), (2.20), and (2.28)

[9]In particular, $c_{s,t}, m_{s,t}, u_{s,t}$ denote the consumption of nondurable goods and housing services, and the time spent in labor for migration cohort s at time t.

(1a) $c_{n,t} = A(1-x_t)^{\alpha-1}$ and $c_{s,t} = c_{n,t}$, for $t \leq T_s$, and $c_{s,t} = c_{s,T_s} \cdot \left(\frac{1+\delta}{1+\rho}\right)^{t-T_s}$, for $t \geq T_s$, where $c_{s,T_s} = \nu^* h_0$.

(1b) $m_{n,t} = \frac{1}{1-x_t}$ and $m_{s,t} = \frac{\mu_m}{\mu_c} \cdot \frac{c_{s,t}}{p_t^s}$.

(1c) $\nu_{s,t} = 0$ for $t \leq T_s$ and $\nu_{s,t} = \nu^*$ for $t > T_s$.

(1d) $z_{T_s} = x_s$ for $\forall s \geq 0$.

(1e) $a_{n,t} = \gamma A(1-x_t)^{\alpha}$ and $a_t = \gamma \cdot z_t \cdot (\nu^* H_t)$.

2. Both markets clear in the SEZ economy, i.e., for $\forall t \geq 0$,

(2a) $x_0 m_{0,t} + \sum_{s=1}^{t}(x_s - x_{s-1})m_{s,t} = 1$.

(2b) $x_0 c_{0,t} + \sum_{s=1}^{t}(x_s - x_{s-1})c_{s,t} = z_t \cdot (\nu^* H_t)$.

I focus on the equilibrium in which households correctly anticipate the share of population who migrate to the SEZ economy at each period without uncertainty. An alternative interpretation is that a social planner with perfect information is solving for an optimal labor allocation problem. Suppose that there are N cohorts of migration.[10] Note that I assign the consumption of nondurable goods for the followers before they turn to leaders equal to the consumption in the non-SEZ economy, as shown in (1a).

[10]Note that N is also determined in equilibrium.

The flow utility if staying in the non-SEZ economy at period t is a function of x_t:

$$u_t^n(x_t) = \mu_c \log(c_{n,t}) + \mu_m \log(m_{n,t}) + \mu_a \log(a_{n,t})$$

$$= f(\Theta) - [\mu_c + \mu_m - \alpha(\mu_c + \mu_a)] \log(1 - x_t)$$

where $f(\Theta) = (\mu_c + \mu_a) \log A + \mu_a \log \gamma$ is defined as in (2.21). Market clearing conditions (2a) and (2b) imply the relative price of housing services in the SEZ economy is:

$$p_t^s = \frac{\mu_m}{\mu_c} \cdot z_t \cdot (\nu^* H_t) \tag{2.29}$$

Hence, the consumption of housing services for households who choose to migrate at cohort s at period t is:

$$m_{s,t} = \begin{cases} \frac{A(1-x_t)^{\alpha-1}}{z_t \nu^* H_t}, \text{ if } t < T_s \\ z_t \cdot \left(\frac{1+\delta}{1+\rho}\right)^{-T_s}, \text{ if } t > T_s \end{cases}$$

And, the flow utility of a household who migrate to the SEZ economy at cohort s at period t is a function of both x_t and z_t:

$$u_t^s(x_t, z_t) = \begin{cases} (\mu_c + \mu_m)\Big[\log A + (\alpha - 1)\log(1 - x_t)\Big] + \mu_a \log \gamma + (\mu_a - \mu_m)\Big[\log z_t + ... \\ \log\left(\frac{\rho}{\delta}\right) + \log h_0 + (t+1)\log\left(\frac{1+\delta}{1+\rho}\right)\Big], \text{ if } t < T_s \\ (\mu_c + \mu_m)\Big[\log\left(\frac{\rho}{\delta}\right) + \log h_0 + (t+1)\log\left(\frac{1+\delta}{1+\rho}\right)\Big] + \mu_a \log \gamma + (\mu_m + \mu_a)\log z_t - \\ (\mu_c + \mu_m) T_s \log\left(\frac{1+\delta}{1+\rho}\right), \text{ if } t > T_s \end{cases}$$

For the last cohort of migrants, i.e., the N^{th} cohort, the share of population in the non-SEZ economy who are willing to migrate to the SEZ economy is determined by the indifference between living in either economy, i.e., x_N is determined by

$$V^n(x_N) = V^N(x_N, z_N)$$

where $V^n(\cdot) = \sum_{t=N}^{\infty} \frac{u_t^n(x_t)}{(1+\rho)^t}$ and $V^N(\cdot) = \sum_{t=N}^{\infty} \frac{u_t^N(x_t,z_t)}{(1+\rho)^t}$, with u_t^n and u_t^s defined above.

For the second last cohort of migrants, i.e., the $N-1^{\text{th}}$ cohort, by the same reasoning, the share of population in the non-SEZ economy who are willing to migrate to the SEZ economy is determined by the indifference between living in either economy, i.e., x_{N-1} is determined by

$$V^n(x_{N-1}) = V^{N-1}(x_{N-1}, z_{N-1})$$

where $V^n(\cdot) = \sum_{t=N-1}^{\infty} \frac{u_t^n(x_t)}{(1+\rho)^t}$ and $V^N(\cdot) = \sum_{t=N-1}^{\infty} \frac{u_t^{N-1}(x_t,z_t)}{(1+\rho)^t}$, with u_t^n and u_t^s defined above. By induction, I construct an equation system with $N+1$ equations as follows:

$$\begin{cases} V^n(x_N) = V^s(x_N, z_N) \\ V^n(x_{N-1}) = V^{N-1}(x_{N-1}, z_{N-1}) \\ \dots \\ V^n(x_1) = V^1(x_1, z_1) \\ V^n(x_0) = V^0(x_0, z_0) \end{cases} \tag{2.30}$$

Lemma 2: In the baseline model of migration, there exists an equilibrium in which $\{x_0^*, ..., x_N^*, z_0^*, ..., z_N^*\}$ solves the following equation system (2.30), (2.26), (2.27) and (2.28).

Once solved the equilibrium, I compute the share of production in the SEZ economy, ω_t, and the ratio of relative price of housing services between the two economies, λ_t.

Proposition 3: In the equilibrium of migration with human capital spillover effect, the share of production in the SEZ economy, ω_t, is:

$$\omega_t = \frac{z_t \cdot \nu^* h_0 \left(\frac{1+\delta}{1+\rho}\right)^t}{z_t \cdot \nu^* h_0 \left(\frac{1+\delta}{1+\rho}\right)^t + A(1-x_t)^\alpha} \tag{2.31}$$

and the relative price ratio of housing services between the two economies, λ_t, is:

$$\lambda_t = \frac{\rho}{\delta} \cdot \frac{z_t h_0}{A(1-x_t)^\alpha} \left(\frac{1+\delta}{1+\rho}\right)^{t+1} \tag{2.32}$$

Comparing (2.36) and (2.32), the effect of human capital spillover on the housing price can be computed as:

$$\lambda_t^{\text{ratio}} = \frac{(1-x_0^*)^\alpha}{x_0^*} \cdot \frac{z_t}{1-x_t} \tag{2.33}$$

Equation (2.33) shows that as there are more leaders in the SEZ economy, i.e., larger z_t, there will be more households who migrate to the SEZ economy, i.e., smaller x_t, both of which contribute to higher housing prices, compared to the baseline economy.

2.3 Conclusion

In this paper, I propose a two-region model to address this question. In the model, high wage in the SEZ region reflect high level of human capital, and these jobs are not available to low human capital migrants from the non-SEZ economy. The migrants come to the SEZ economy for two reasons: on the one hand, the SEZ economy is a better place to accumulate human capital and earn a higher wage in the future; on the other hand, the SEZ economy has a better amenities for living. The theory shows that during the migration process, the spillover effect of human capital has a significant positive effect on the increase of housing prices.

In the model, the non-SEZ economy has a Cobb-Douglas production technology with a single labor input, where as the SEZ economy has a human-capital based production technology. In particular, each migrant to the SEZ economy allocate a fixed time endowment between laboring for wage that is indicated by one's current level of human capital and accumulating human capital so as to increase future wage. I start with considering the equilibrium allocations in the non-SEZ economy and in the SEZ, human-capital based economy, both in isolation first. In the baseline model with migration, the share of population that choose to migrate to the SEZ economy is determined by the utility equalization between living in either economy. In the baseline model, the migration occurs all at once at the first period. Further, I extend the baseline model by incorporating the spillover effect of human capital: time invested in human capital accumulation has a higher return in high human capital environment. In this case, the migration to the SEZ economy becomes increasingly attractive as the gap between the human capital leaders and followers increase. By comparing the extended model with the baseline, the effect of human capital spillover can be captured.

2.4 Appendix

Proof for Proposition 1 Suppose that there exists a defined balanced growth path. Equation (2.5) implies

$$\frac{h_{t+1}}{h_t} = \frac{c_{t+1}}{c_t} = 1 + \delta(1 - u_t) \tag{2.34}$$

Combined with (2.7), we have a unique constant equilibrium time allocation between labor and human capital accumulation, denoted by ν.

$$\nu = \frac{\rho + \delta\rho}{\delta + \delta\rho}$$

Since $\nu \in [0, 1]$, it requires that $\rho \leq \delta$. Further, (2.5) implies that:

$$\frac{w_{t+1}}{w_t} = \frac{h_{t+1}}{h_t} = \frac{p_{t+1}}{p_t} = \frac{c_{t+1}}{c_t} = \frac{a_{t+1}}{a_t} = 1 + \delta(1 - \nu) = \frac{1 + \delta}{1 + \rho}$$

Q.E.D.

Proof for Lemma: Equations (2.10), (2.11), (2.12) and (2.13) imply that the value function of households staying in the non-SEZ economy is the maximized lifetime utility, a function of x_0, as follows:

$$\begin{aligned} V^n(x_0) &= \sum_{t=0}^{\infty} \frac{\mu_c \log\left(c_t^n(x_0)\right) + \mu_m \log\left(m_t^n(x_0)\right) + \mu_a \log(a_t^n(x_0))}{(1+\rho)^t} \\ &= \frac{1+\rho}{\rho}\Big\{(\mu_c + \mu_a)\log A + \mu_a \log\gamma - \Big[\mu_c + \mu_m - \alpha(\mu_c + \mu_a)\Big]\log x_0\Big\} \end{aligned}$$

Equations (2.15), (2.16), (2.17), (2.18), (2.19) and (2.20) imply that the value function of households migrating to the SEZ economy is the maximized lifetime utility, a function of x_0, as follows:

$$
\begin{aligned}
V^s(x_0) &= \sum_{t=0}^{\infty} \frac{\mu_c \log(c_t^s) + \mu_m \log(m_t^s(x_0)) + \mu_a \log(a_t^s(x_0))}{(1+\rho)^t} \\
&= \Big\{(\mu_c + \mu_a)\Big[\log\left(\tfrac{\rho}{\delta}\right) + \log\left(\tfrac{1+\delta}{1+\rho}\right) + \log(h_0)\Big] + \mu_a \log\gamma + (\mu_a - \mu_m)\log(1-x_0)\Big\} \ldots \\
&\sum_{t=0}^{\infty} \tfrac{1}{(1+\rho)^t} + (\mu_c + \mu_a)\log\left(\tfrac{1+\delta}{1+\rho}\right)\sum_{t=1}^{\infty} \tfrac{t}{(1+\rho)^t} \\
&= \tfrac{1+\rho}{\rho}\Big\{(\mu_c + \mu_a)\Big[\log\left(\tfrac{\rho}{\delta}\right) + \log\left(\tfrac{1+\delta}{1+\rho}\right) + \log(h_0)\Big] + \mu_a \log\gamma + (\mu_a - \mu_m)\log(1-x_0)\Big\} \\
&+ \tfrac{1+\rho}{\rho^2}(\mu_c + \mu_a)\log\left(\tfrac{1+\delta}{1+\rho}\right) \\
&= \tfrac{1+\rho}{\rho}\Big\{g(\Theta) + (\mu_a - \mu_m)\log(1-x_0)\Big\}
\end{aligned}
$$

where $g(\Theta) = (\mu_c + \mu_a)\Big[\log\left(\frac{\rho}{\delta}\right) + \frac{1+\rho}{\rho}\log\left(\frac{1+\delta}{1+\rho}\right) + \log(h_0)\Big] + \mu_a \log\gamma$. Both value functions of $V^n(x_0)$ and $V^s(x_0)$ are common knowledge. x_0^* is determined when $V^n(x_0^*) = V^s(x_0^*)$. Q.E.D.

Proof for Proposition 2: Once solving for the equilibrium share of population between the non-SEZ and SEZ economies, we obtain the solution for all the variables of the economy. In particular, the share of production in the SEZ economy, ω_t, can be computed as:

$$\omega_t = \frac{x_0 G(u_t^s, h_t^s)}{x_0 G(u_t^s, h_t^s) + F(x_0^*)} = \frac{x_0 \nu_t h_t}{x_0 \nu_t h_t + A(1-x_0^*)^\alpha} \tag{2.35}$$

where $\nu_t = \nu^*$, and $h_t = h_0 \left(\frac{1+\delta}{1+\rho}\right)$. The ratio of relative price of housing services between the two economies, λ_t, can be computed as:

$$\lambda_t = \frac{p_t^s(x_0^*)}{p_t^n(x_0^*)} = \frac{c_0^s \cdot x_0^*}{A(1 - x_0^*)^\alpha} \left(\frac{1+\delta}{1+\rho}\right)^t \tag{2.36}$$

where $c_0^s = \nu^* h_0 = \frac{\rho(1+\delta)}{\delta(1+\rho)} h_0$. Q.E.D.

Chapter 3

Dynamic Arrow-Debreu Abstract Economy for General Equilibrium Analysis

3.1 Introduction

In their classic 1954 paper, Arrow and Debreu introduce a market participant to Walrasian general equilibrium model, who plays the price-setting role and maximizes the value of the market excess demand.[1] They assume that the market participant and other economic agents move simultaneously and independently. The simultaneity of the moves implies that the market participant does not maximize the value of the instantaneous market excess demand, but it takes market

[1]See [2].

excess demand as given when revising the prices. The simultaneity also makes the resulting economy with the market participant, known as an abstract economy, not well behaved in that the feasibility of an economic agent's choice depends on the market participant's choice which the agent cannot directly observe.

The process underlying the determination of equilibrium price by the Walrasian auctioneer in a competitive market works as follows. The Walrasian auctioneer sets a price first and then the economic agents observe the price and decide what quantities they want to supply or demand at that price. To capture this dynamic nature, we consider a dynamic variation of the Arrow-Debreu abstract economy with two stages. The market auctioneer selects a price vector in Stage 1 and subsequently each consumer observes the auctioneer's selection and simultaneously and independently chooses an affordable bundle in Stage 2. The dynamic variation makes it natural for the consumers' feasible choices to depend on the market auctioneer's choice. Thus, instead of inducing a pseudo game as with the simultaneous move Arrow-Debreu abstract economy, the extensive form game of our dynamic variation is well defined. As such, various game-theoretic solution concepts with or without symmetric information can be applied. We show that the set of subgame-perfect equilibrium allocations coincides with the set of Walrasian equilibrium allocations when information is symmetric. The set of perfect Bayesian equilibrium allocations coincides with the set of rational expectations equilibrium with asymmetric information.

The rest of the paper is organized as follows. Section 2 considers the dynamic abstract economy with symmetric information. Section 3 presents the dynamic abstract economy with asymmetric information. Section 4 concludes.

3.2 Dynamic Abstract Economy with Complete Information

Let $\mathcal{E} = (X_i, \omega_i, u_i)_{i=1}^n$ be an n-person exchange economy with consumer i's consumption set X_i, endowment ω_i and his utility function u_i for $i = 1, 2, \cdots, n$. We assume u_i is strictly monotonic for all i.

3.2.1 Abstract Economy

The associated abstract economy with $\mathcal{E}$ introduced in the seminal paper of Arrow and Debreu (1954) has $n+1$ agents, with agent 0 being the fictitious agent called the market auctioneer and the other n agents being the consumers in $\mathcal{E}$. The choice set of agent 0 consists of non-negative price vectors and his objective is to minimize the extent to which the economy is out of equilibrium. More precisely, agent 0 has choice set

$$X_0 = \{(p, 1) \in \Re_+^l \mid p \in \Re_+^{l-1}\}, \tag{3.1}$$

where l is the number of commodities and the l-th good is chosen as the numeraire. Agent 0 has the following objective function:

$$u_0(x,p) = \sum_{h=1}^{L} p_h \min\{\sum_{i=1}^{n}(x_{ih} - \omega_{ih}), 0\}, \quad (x,p) \in \prod_{i=1}^{n} X_i \times X_0. \tag{3.2}$$

A justification of the preceding objective function and its consistency with the objective of the auctioneer are provided in the following remark.

Remark 1. *The objective function of the market auctioneer formulated in (2) focuses on commodities with positive excess supplies only. A justification for this formulation is as follows. First, the formulation is equivalent to the minimization of the total value of excess supplies over commodities whose excess supplies are positive. Due to the* Walras Law*, no commodity has positive excess supply if and only if no commodity has positive excess demand. Thus, to minimize the value of total excess supply is consistent with the auctioneer's objective to minimize the extent to which the economy is out of equilibrium.*

Each consumer $1 \leq i \leq n$ chooses an affordable bundle $x_i \in X_i$ so as to maximize his utility function. The affordability of a consumption bundle for consumer i is determined by his initial endowment and the price vector chosen by the market auctioneer. Thus, given the auctioneer's choice $p \in X_0$, agent i's affordable bundles consist of $X_i(p) = \{x \in X_i \mid p \cdot x_i \leq p \cdot \omega_i\}$. His objective is to solve:

$$\max u_i(x_i) \text{ subject to } x_i \in X_i(p). \tag{3.3}$$

In summary, the Arrow-Debreu abstract economy associated with economy $\mathcal{E}$ is the collection $(X_i, A_i, u_i)_{i=0}^n$ as specified in (1)-(3). The abstract economy is, however, a pseudo-game, in that the feasibility of agent i's choice depends on the simultaneous choice by agent 0. Nevertheless, Nash equilibria of the abstract economy are competitive equilibria of economy $\mathcal{E}$ and vice versa.

3.2.2 Dynamic Variation

We consider a dynamic variation of the abstract economy that has the following two stages. In stage 1, the auctioneer moves by choosing a price vector $p \in X_0$. In Stage 2, consumers observe the auctioneer's choice of a price vector p and then simultaneously and independently make choices, with $X_i(p)$ as consumer i's feasible choice set for all $i = 1, 2, \cdots, n$. While a strategy of the auctioneer is a price vector in X_0, a strategy of consumer i is a mapping $x_i : X_0 \longrightarrow X_i$ such that $x_i(p) \in X_i(p)$ for all $p \in X_0$.

It turns out that Nash equilibrium allocations of the dynamic abstract economy need not be the same as Walrasian equilibrium allocations. The following example provides an illustration.

Example 1: Consider an economy $\mathcal{E} = (X_i, \omega_i, u_i)_{i=1}^n$, where $n = 2$, $\omega_1 = (1, 0)$, $\omega_2 = (0, 1)$, $X_i = \Re_+^2$ and $u_i(x_i) = x_{i1}x_{i2}$ for $x_i \in X_i$ and $i = 1, 2$. Assume that good 2 is the numeraire. This economy has a unique competitive equilibrium consisting of $p^* = (1, 1)$ and $x_i^* = (\frac{1}{2}, \frac{1}{2})$ for $i = 1, 2$. Now consider strategies $x_1(\cdot)$

for consumer 1 and $x_2(\cdot)$ for consumer 2, where

$$x_1(p) = \begin{cases} (\frac{1}{2}, 1), & \text{if } p = (2,1) \\ \omega_1, & \text{otherwise} \end{cases} \quad \text{and} \quad x_2(p) = (\frac{1}{2p_1}, \frac{1}{2})$$

Neither consumer would deviate as their consumption bundles are utility maximizing subject to budget constraint at price vector p. Nor would the auctioneer, because a payoff of $-\frac{1}{2}$ when choosing price vector $p = (2, 1)$ is the highest payoff he can possibly obtain given the consumers' strategies $(x_1(\cdot), x_2(\cdot))$. Therefore, consumers' strategies $(x_1(\cdot), x_2(\cdot))$ together with the auctioneer's price vector $p = (2, 1)$ constitute a Nash equilibrium. However, neither market clears at this Nash equilibrium, implying that this Nash equilibrium allocation fails to be the unique competitive equilibrium allocation.

Consumer 1's Nash equilibrium strategy $x_1(\cdot)$ in Example 1 is sequentially irrational, since choosing $\omega_1 = (1, 0)$ at price vectors $p \neq (2, 1)$ always leads to a 0 payoff which is strictly dominated. Subgame perfection eliminates this type of sequential irrationality. Indeed, it turns out that the set of subgame-perfect equilibria coincides with the set of Walrasian equilibria. This is shown in the following theorem.

Theorem 1. *Let $\mathcal{E} = (X_i, \omega_i, u_i)_{i=1}^n$ be an exchange economy. Then, each subgame-perfect equilibrium allocation of the dynamic abstract economy is a Walrasian equilibrium allocation and vice versa.*

Proof. First, we show that a WE, $(p^*, x^*) \in \Re^l_+ \times \Re^{nl}_+$, for $\mathcal{E}$ is a SPE allocation. To this end, consider strategy profile $(p^*, \{x_i^*(\cdot)\}_{i=1}^n)$, where $\{x_i^*(\cdot)\}_{i=1}^n$ is the profile of consumers' demand functions that are derived by the following utility maximization problems: for $i = 1, 2, \cdots, n$ and $p \in X_0$, $x_i^*(p)$ solves

$$\max_{x_i \in \Re^L_+} u_i(x_i)$$
$$\text{subject to} \quad x_i \in \{x \in X_i \mid \sum_{h=1}^{L} p_h(x_{ih} - \omega_{ih}) \leq 0\}.$$

The auctioneer's strategy p^* is optimal given the consumers' strategies because

$$u_0(x^*(p^*), p^*) = \sum_{h=1}^{L} p_h^* \min\{\sum_{i=1}^{n}(x_{ih}^*(p^*) - \omega_{ih}), 0\} = 0$$

and $u_0(x^*(p), p) \leq 0$ for $p \in X_0$. Therefore, $u_0(x^*(p^*), p^*) \geq u_0(x^*(p), p)$ for $p \in X_0$. Since $x_i^*(p)$ is utility maximizing for all i and for all $p \in X_0$, it follows that $(x_i^*(p))_{i=1}^n$ form a Nash equilibrium for the subgame led to by the the auctioneer's strategy p. This shows that $(p^*, \{x_i^*(\cdot)\}_{i=1}^n)$ is a SPE. Furthermore, by construction, $x_i^* = x_i^*(p^*)$ for all i. Consequently, the WE allocation $(x_i^*)_{i=1}^n$ is a SPE allocation.

Next, let $(\bar{p}, \bar{x}(\cdot)) \in \Re^l_+ \times \Re^{nl}_+$ be a SPE. Notice that the payoff of the auctioneer in SPE must be zero, since he can always choose a WE market-clearing price vector that generates him a zero payoff. It follows from the auctioneer's payoff functional form that it must be

$$\sum_{h:\ \sum_{i=1}^n(\bar{x}_{ih}(\bar{p}) - \omega_{ih}) < 0} \bar{p}_h \min\{\sum_{i=1}^{n}(\bar{x}_{ih}(\bar{p}) - \omega_{ih}), 0\} = 0.$$

Consequently, for commodity h with $\sum_{i=1}^{n}(\bar{x}_{ih}(\bar{p})-\omega_{ih}) < 0$, we must have $\bar{p}_h = 0$. However, the strict monotonicity of consumers utility functions implies that there cannot be any free good in SPE. Therefore, we have $\sum_{i=1}^{n}(\bar{x}_{ih}(\bar{p}) - \omega_{ih}) = 0$, for all $h = 1, ..., L$. This concludes that $(\bar{p}, \bar{x}(\cdot))$ is a Walrasian equilibrium. □

Our dynamic abstract economy differs from the abstract economy only in terms of the timing between the move by the auctioneer and those by the consumers. Theorem 1 shows that the dynamic abstract economy matches Walrasian general equilibrium very well both in terms of the timings of the moves and the solutions. We show in the next section that results of this paper can be extended to allow for information asymmetry.

3.3 Dynamic Abstract Economy with Asymmetric Information

Our analysis in the previous section was based on complete information. In this section, we consider the case with incomplete information. When information is symmetric (i.e., all agents have the same information), the case can be analyzed in a similar way as with complete information using expected utility functions. Our analysis in this section focuses on asymmetric information.

3.3.1 Rational Expectations Equilibrium

Following Mas-Colell et al. (1995), we consider an economy with asymmetric information and rational expectations as follows. There is a single period in which one of the states $s \in \mathcal{S} = \{1, 2, \cdots, S\}$ will occur. Denote by p_s the price vector and x_{is} agent i commodity bundle in state s. There is a common prior $\pi = (\pi(1), \cdots, \pi(S))$ over the states. Agent i's von Neumann-Morgenstern expected utility function over random vectors $x_i = (x_{i1}, \cdots, x_{iS})$ of state-dependent commodity bundles is given by

$$U_i(x_i) = \sum_{s=1}^{S} \pi_{is} u_{is}(x_{is}),$$

where $\pi_i = (\pi_i(1), \cdots, \pi_i(S))$ is consumer i's *updated* belief about the states and u_{is} is consumer i's Bernoulli utility function in state s. Consumer i's initial endowment is state-dependent $\omega_i = (\omega_{i1}, \cdots, \omega_{iS})$, where ω_{is} is the endowment consumer i receives conditional on the occurrence of state s. In addition, consumer i's *private information* is described by a signal function $\sigma_i : \mathcal{S} \longrightarrow \Re$. Consumer i can distinguish states $s, s' \in \mathcal{S}$ if and only if $\sigma_i(s) \neq \sigma_i(s')$. It is assumed that for all i and $s, s' \in \mathcal{S}$, $\omega_{is} = \omega_{is'}$ whenever $\sigma_i(s) = \sigma_i(s')$. This means that consumer i can extract no information on the state from his endowment not already revealed to him by his signal.

The *timing* of the model is as follows: (i) a state $s \in \mathcal{S}$ occurs in the beginning of the period; (ii) consumer i receives endowment ω_{is} and signal $\sigma_i(s)$;[2] (iii) the

[2]That is, the auctioneer observes all the signals of consumers as discussed in papers on rational expectations equilibrium models (e.g., [25]).

spot commodity markets operate after all consumers receive their endowments and signals; (iv) the state gets revealed and consumption takes place at the end of the period.

For $s \in \mathcal{S}$, let $p(s)$ be the price vector that are expected to prevail in state s by consumers. Viewing $p(s)$ as a public signal, when state s occurs, consumer i knows that event $\mathcal{S}_i(\sigma_i(s), p(s))$ occurs where

$$\mathcal{S}_i(\sigma_i(s), p(s)) = \{s' \in \mathcal{S} : \sigma_i(s') = \sigma_i(s),\ p(s') = p(s)\}. \tag{3.4}$$

Thus, consumer i can use (4) to update the common prior π:

$$\pi_i(s'|\sigma_i(s), p(s)) = \begin{cases} \frac{\pi(s')}{\sum_{s'' \in \mathcal{S}_i(\sigma_i(s), p(s))} \pi(s'')}, & s' \in \mathcal{S}_i(\sigma_i(s), p(s)), \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

It follows that when sate s occurs, consumer i's *interim* utility at random vector $x_i = (x_i(1), \cdots, x_i(S))$ is given by

$$U_i(x_i|\sigma_i(s), p(s)) = \sum_{s' \in \mathcal{S}} \pi_i(s'|\sigma_i(s), p(s)) u_{is'}(x_i(s')). \tag{3.6}$$

With (4), (5), and (6) in place, we are now ready to define rational expectations equilibrium:

Definition 1. *A REE is composed of a vector of state-dependent prices* $p^* = (p^*(s))_{s \in \mathcal{S}}$ *and a vector of state-dependent commodity bundles* $x_i^* = (x_i^*(s))_{s \in \mathcal{S}}$ *for* $i = 1, 2, \cdots, n$*, such that* *i) for all i and $s \in S$, $x_i^*(s)$ solves*

$$\max_{x_i \in \Re_+^{lS}} \sum_{s' \in S_i(\sigma_i(s), p^*(s))} \pi_i(s' \mid \sigma_i(s), p^*(s)) u_{is'}(x_i(s'))$$

$$\text{subject to} \quad p^*(s) \cdot x_i(s) \leq p^*(s) \cdot \omega_i(s) \qquad \text{(affordability)}$$

$$x_i(s') = x_i(s), \;\; s \in S_i(p^*(s), \sigma_i(s)) \qquad \text{(measurability)}$$

ii) $\sum_{i=1}^n x_i^*(s) = \sum_{i=1}^n \omega_i(s)$, $s \in \mathcal{S}$.

The notion of the rational expectations equilibrium recognizes the role of the market price as an information aggregator? Furthermore, it takes into consideration the fact that the market participants infer information from the market price.

3.3.2 Perfect Bayesian Equilibrium

Consider the dynamic abstract economy with asymmetric information. As with the previous section, there are $n + 1$ players with agent 0 as the market auctioneer and the consumers as the other n agents. The *timing* of the game is as follows: (i) nature draws a state $s \in S$ according to the common prior π, and then reveals the private signal $\sigma_i(s)$ to consumer $i = 1, 2, \cdots, n$ and the collection $\sigma(s) = (\sigma_1(s), \cdots, \sigma_n(s))$ to the market auctioneer; (ii) the auctioneer chooses a price vector p; (iii) consumers observe the market auctioneer's choice and simultaneously and independently choose affordable bundles; (iv) the state gets revealed and consumption takes place.

An information set of the auctioneer is a subset of states which he cannot distinguish using his signal $\sigma(\cdot)$. Formally, the auctioneer's information set containing state s is

$$\mathcal{S}_0(s) = \{s' \in S \mid \sigma(s') = \sigma(s)\}, \forall s \in S. \tag{3.7}$$

In comparison, as a later mover, each consumer i gets to observe the auctioneer's choice and receives his private signal. Correspondingly, an information set of consumer i containing state s and spot market price vector p is identified as

$$\mathcal{S}_i(s,p) = \{(s',p) | \sigma_i(s') = \sigma_i(s)\}.$$

A *strategy* is a complete plan of action that specifies in advance what moves a player would make in every contingency that might arise. Here, a contingency that may arise from player i's point of view is that one of his information set has been reached. Thus, a strategy for the auctioneer is a mapping that maps information sets $\mathcal{S}_0(s)$ into price vectors in $\Re^l_+$. For convenience we write a strategy for the auctioneer as $p(\cdot) = (p(s))_{s\in\mathcal{S}}$, where it is required that

$$p(s) = p(s') \text{ for } s, s' \in \mathcal{S} : \mathcal{S}_0(s) = \mathcal{S}_0(s') \qquad \text{(measurability)}$$

On the other hand, a strategy for agent $i \geq 1$ is a mapping that maps information set $\mathcal{S}_i(s,p)$ into a bundle $x_i(s,p) \in \Re^l_+$ such that

$$p \cdot x_i(s,p) \leq p \cdot \omega_{is} \qquad \text{(affordability)}$$

For convenience, we write the strategy as $x_i(\cdot) = (x_i(s,p))_{s\in\mathcal{S},p\in\Re^l_+}$, where

$$x_i(s,p) = x_i(s',p) \text{ for } s,s' \in \mathcal{S},\ p \in \Re^l_+ : \mathcal{S}_i(s,p) = \mathcal{S}_i(s',p) \qquad \text{(measurability)}$$

Given that consumer i anticipates that the auctioneer will play strategy $p(\cdot) = (p(s))_{s\in\mathcal{S}}$ and observes spot market price vector p, he knows that the event $\mathcal{S}_i(s,\sigma_i(\cdot),p,p(\cdot))$ occurs after the occurrence of $s \in \mathcal{S}$, where

$$\mathcal{S}_i(s,\sigma_i(\cdot),p,p(\cdot)) = \begin{cases} \{s'|p(s') = p(s),\ \sigma_i(s') = \sigma_i(s)\}, & p = p(s), \\ \{s'|p(s') = p,\ \sigma_i(s') = \sigma_i(s)\}, & p \neq p(s),\ p \in p(S), \\ \{s'|\sigma_i(s') = \sigma_i(s)\}, & p \notin p(S). \end{cases} \tag{3.8}$$

The middle information set of consumer i on the right hand side follows the event that signal $\sigma_i(s)$ is received and spot market price vector p is observed, where p is the price vector the auctioneer will choose for some state the auctioneer is able to distinguish from state s.

When state s occurs, the auctioneer can update his belief about the states as follows

$$\pi_0(s') = \begin{cases} \frac{\pi(s')}{\sum_{s''\in S_0(s)} \pi(s'')}, & s' \in S_0(s), \\ 0, & \text{otherwise.} \end{cases} \tag{3.9}$$

In contrast, conditional on state s occurring and observing spot price vector p, agent i can update his belief about the states as follows.

$$\pi_i(s'|\sigma_i(s),p,p(\cdot)) = \begin{cases} \frac{\pi(s)}{\sum_{s''\in\mathcal{S}(s,\sigma_i(s),p,p(\cdot))} \pi(s'')}, & s' \in \mathcal{S}_i(s,\sigma_i(s),p,p(\cdot)), \\ 0, & \text{otherwise.} \end{cases} \tag{3.10}$$

With (7)-(10) in place, we are ready to define perfect Bayesian equilibrium for the dynamic abstract economy with asymmetric information.

Definition 2. *A PBE of the dynamic abstract economy is a strategy profile* $(p^*(\cdot), x^*(\cdot))$ *and a belief system* $\pi^*(\cdot) = (\pi_0^*(\cdot), \pi_1^*(\cdot), \cdots, \pi_n^*(\cdot))$ *such that (i)* $(p^*(\cdot), x^*(\cdot))$ *is sequentially rational given the belief system* $\pi^*(\cdot)$*: for all* s *and price vector* $p \in \Re_+^l$*,* $p^*(s)$ *solves*

$$\max \sum_{s' \in S_0(s)} u_0(x^*(s', p^*(s')), p^*(s'))\pi_0^*(s' \mid \sigma(s)) \text{ subject to measurability}$$

and $x_i^*(s, p)$ *solves*

$$\max \sum_{s' \in S_i(s,p)} u_{si}(x_i^*(s', p))\pi_i^*(s' \mid \sigma(s), p) \text{ subject to afforability } \textit{and} \text{ measurability}$$

for $i = 1, 2, \cdots, n$*; (ii)* $\pi_i^*(\cdot)$ *is obtained by updating the prior using strategy profile* $(p^*(\cdot), x^*(\cdot))$ *for* $i = 0, 1, \cdots, n$*.*

In PBE, players' strategies are sequentially rational given their belief systems represented by $\pi^*(\cdot)$. In turn, players' belief systems are compatible with their strategies according to Bayes rule.

3.3.3 Equivalence between REE and PBE

With complete information, we provide an example in which Nash equilibrium allocations of the dynamic abstract economy are not identical to the Walrasian equilibrium allocations. Likewise, with asymmetric information, the Bayesian

Nash equilibrium allocations of the dynamic abstract economy are not necessarily identical to rational expectation equilibrium allocations. It is illustrated by the following example:

Example 2: Consider a model economy with 4 states of nature: $S = \{(q, j) : (1, 0), (1, 1), (3, 0), (3, 1)\}$, each occurring with a probability of 1/4. There are two agents, i.e., $n = 2$. There are two types of goods $(x, m) \in \mathbb{R}^2$ in the economy: one is the non-numeraire goods, the supply of which is state-dependent; the other is the numeraire goods with a state-independent constant supply $\bar{m} \to \infty$. The endowments for both agents are in the form of $(\omega_{is}, \frac{\bar{m}}{2}) \in \mathbb{R}^2$, where the non-numeraire endowment $\omega_{is} = j$ if a state of nature $s = (q_s, j)$ occurs, i.e.,

$$\omega_{is} = \begin{cases} 0, & \text{if} \quad s \in \{(1, 0), (3, 0)\} \\ 1, & \text{otherwise} \end{cases}$$

for $i = 1, 2$. Agent 1 is risk averse and receives information signal. In particular, $u_1(x_1, m_1) = z_1 - z_1^2$, where $z_1 = q_s \cdot x_1 + m_1$. Further, the information signal for agent 1 is:

$$\sigma_1(s) = \begin{cases} c_1, & \text{if} \quad s \in \{(1, 0), (3, 1)\} \\ c_2, & \text{otherwise} \end{cases}$$

where $c_1 \neq c_2$.

Agent 2 is risk neutral and receives no information. In particular, $u_2(x_2, m_2) = z_2$, where $z_2 = q_s \cdot s_2 + m_2$. Further, $\sigma_2(s) = c$ for $\forall s \in S$. Each agent chooses consumption bundle $(x_i, m_i) \in \mathbb{R}^2$ to maximize their utilities as follows:

$$\max_{x_i, m_i} u_i(x_i, m_i)$$

$$\text{s.t.} \quad p \cdot x_i + m_i \leq p \cdot \omega_i + \frac{\bar{m}}{2}$$

Since $\bar{m}$ is a constant, the utility maximization problem above is equivalent to solving the following one-dimensional optimization with respect to the non-numeraire goods only: $\max_{x_i} u_i(z_i)$, where $z_i = q_s \cdot x_i + p \cdot (\omega_i - x_i)$.

Then there is a unique R.E.E of the economy is: $p(s)^* = 2$, $x_1^*(\sigma_1(s)) = \begin{cases} -\frac{1}{2} & \text{for } \sigma_1 = c_1 \\ \frac{1}{2} & \text{for } \sigma_1 = c_2 \end{cases}$, and $x_2^*(s) = \begin{cases} \frac{1}{2} & \text{for } s = (1,0) \text{ and } (1,1) \\ \frac{3}{2} & \text{for } s = (3,1) \\ -\frac{1}{2} & \text{for } s = (3,0) \end{cases}$.[3]

Consider a strategy $x_1(\cdot)$ for agent 1 and $x_2(\cdot)$ for agent 2 , where:

$$x_1(\sigma_1(s), p) = \begin{cases} -\frac{1}{(1-p)^2+(3-p)^2} & \text{if} \quad \sigma_1(s) = c_1 \\ \frac{1}{(1-p)^2+(3-p)^2}, & \text{if} \quad \sigma_1(s) = c_2 \end{cases}, \qquad x_2(p) = \begin{cases} \frac{1}{2}, & \text{if } p = 1 \\ -1, & \text{otherwise} \end{cases}$$

Neither consumers would deviate as their consumption bundles are utility maximizing at price $p = 1$. Nor would the auctioneer, since an expected payoff of 0 is the highest payoff he can possibly get given the two agents strategies. Therefore, agentrs' strategies and $p = 1$ constitute a BNE. However, the market is not clear in every state.

Agent 2's Bayesian Nash equilibrium strategy $x_2(\cdot)$ is sequential irrational, because choosing -1 at price $p = 1$ leads to negative payoff which are strictly

[3]See Appendix for proof.

dominated by any other positive demand. Perfect Bayesian game eliminates this type of sequential irrationality. Parallel to the equivalence between SPE and WE allocations with complete information, we now show that PBE and REE allocations are identical with asymmetric information.

Theorem 2. *PBE allocations of the dynamic abstract economy are REE allocations and vice versa.*

Proof. First, we show that an REE, $(p^*(\cdot), x^*(\cdot)) \in \Re_+^{lS} \times \Re_+^{lnS}$, is a PBE allocation. To this end, consider strategy profile $(p(\cdot), \{x_i(\cdot)\}_{i=1}^n)$ where $p(s) = p^*(s)$, $x_i(\cdot)$ is measurable, $x_i(s,p) = x_i^*(s)$ if $p = p(s)$, and $x_i(s,p)$ is any affordable bundle at price vector if $p \neq p(s)$, for all $s = 1, \cdots, S$, $p \in \Re_+^l$, and $i = 1, \cdots, n$. Then, by construction, the market auctioneer's *interim* payoff at strategy profile $(p(\cdot), \{x_i(\cdot)\}_{i=1}^n)$ is 0, which is the highest *interim* payoff for him. It follows that the market auctioneer's strategy $p(\cdot)$ is optimal given the strategies of the other agents. On the other hand, for $i \geq 1$, consumer i's bundle $x_i(\cdot)$ is utility maximizing; hence, it is optimal for consumer i given the strategies of the other agents.

Now let strategy profile $(p^*(\cdot), x^*(\cdot))$ be a PBE. We show that $(p(s), x(s))$ with $p(s) = p^*(s)$ and $x_i(s) = x_i^*(s, p^*(s))$ is a REE. This direction is more involved. Fortunately, the measurability constraint helps to simplify the proof. Notice first that the affordability and measurability of $x_i^*(\cdot)$ as consumer i's PBE strategy automatically implies the afforability with respect to price function $p(\cdot)$

and measurability of $x_i(\cdot)$ as consumer i REE state-contingent bundle. Since the optimality of $x_i(\cdot)$ as consumer i REE state-contingent bundle is required conditional on information sets in the first line on the right hand side of (8), it follows from (10) that the sequentiality of $x_i^*(\cdot)$ as consumer i's PBE strategy implies the optimality of $x_i(\cdot)$ as consumer i's REE state-contingent bundle.

By measurability, for $s = 1, 2, \cdots, S$, $p(s') = p(s)$ for $s' \in S_0(s)$ and $x_i(s', p) = x_i(s, p)$ for $s' \in S_i(s, p)$, $p \in \Re^l_+$. Since $S_0(s) \supseteq \bigcap_{i=1}^n S_i(s, p)$, we have $x_i(s', p) = x_i(s, p)$ for all $s' \in S_0(s)$. Thus, by (7) and (9), the *interim* utility of the auctioneer conditional on state s can be simplified to:

$$\begin{aligned}
&\sum_{s' \in S_0(s)} \sum_{h=1}^{l} p_h(s') \min\{\sum_{i=1}^{n} (x_{ih}(s', p(s')) - \omega_{ih}(s')), 0\} \cdot \pi_0(s' \mid \sigma(s)) \\
&= \sum_{h=1}^{l} p_h(s) \min\{\sum_{i=1}^{n} (x_{ih}(s) - \omega_{ih}(s)), 0\} \cdot \sum_{s' \in S_0(s)} \pi_0(s' \mid \sigma(s)) \\
&= \sum_{h=1}^{l} p_h(s) \min\{\sum_{i=1}^{n} (x_{ih}(s) - \omega_{ih}(s)), 0\}
\end{aligned}$$

Using the last expression together with the sequential rationality of the agents' strategies, a similar reasoning as with the complete information case shows that the price vector $p(s)$ clears the spot markets. □

The notion of rational expectations equilibrium is a natural extension of Walrasian equilibrium to allow for information asymmetry. Theorem 2 shows that the equivalence result in Theorem 1 is robust with respect to information asymmetries.

3.4 Conclusion

This paper contributes to the literature on game-theoretic foundations of general equilibrium theory. We considered a dynamic variation of the abstract economy in Arrow and Debreau (1954) to more closely capture the timing of moves of the Walrasian model of market exchange. Our dynamic abstract economy yields a well defined game in extensive form. As such, various game-theoretic solution concepts can be applied to the dynamic abstract economy. Indeed, we showed that the set of SPE allocations coincides with the set of WE allocations with complete information; the set of PBE allocations coincides with the REE allocations with asymmetric information. These coincidence results provide useful tools for analyzing and refining WE and REE allocations.

3.5 Appendix

Proof for Example 2

The orginal setup:

For a state $s \in S$, if $\sigma_1(s) = c_1$, i.e., when $s \in \{(1,0),(3,1)\}$. Then the utility maximization problem for agent 1 is:

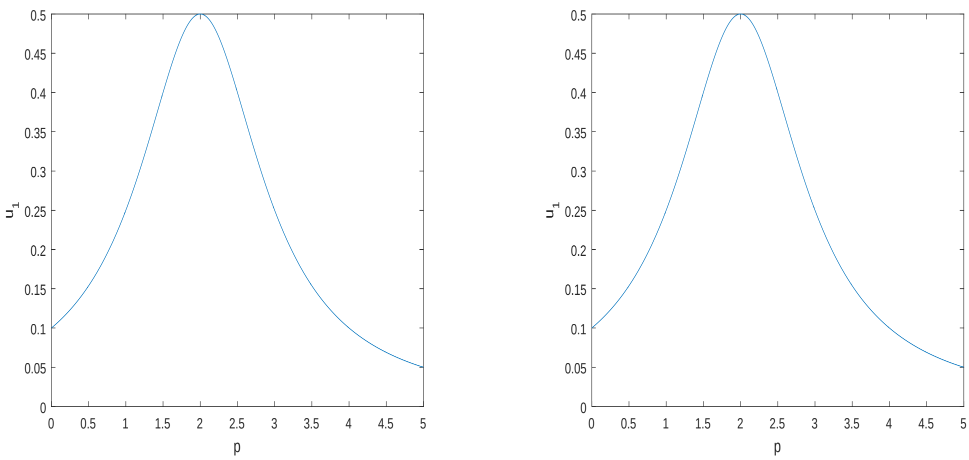
$$\max_{x_1} \frac{1}{2}\left[u_1\Big((1-p)x_1\Big) + u_1\Big((3-p)x_1 + 1\Big)\right]$$

Then one obtains $x_1(\sigma_1(s)) = -\frac{1}{(1-p)^2+(3-p)^2}$ when $\sigma_1(s) = c_1$. If $\sigma_2(s) = c_2$, i.e., when $s \in \{(1,1),(3,0)\}$, then the utility maximization problem for agent 1 becomes:

$$\max_{x_1} \frac{1}{2}\left[u_1\Big((1-p)x_1+1\Big)+u_1\Big((3-p)x_1\Big)\right]$$

One obtains $x_1(\sigma_1(s)) = \frac{1}{(1-p)^2+(3-p)^2}$ when $\sigma_1(s) = c_2$.

The following is the utility of agent 1 as a function of p when receiving either type of signal:



(a) $\sigma(s) = c_1$ (b) $\sigma(s) = c_2$

Figure 3.1

We can see that in both cases (receiving both types of signals), the utility of agent 1 is maximized at price $p = 2$. Since agent 2 receives no signal, when a state $s \in S$ occurs, the utility maximization for agent 2 is:

$$\max_{x_2} \frac{1}{4}\left[u_2\Big((1-p)x_1\Big) + u_2\Big((3-p)x_1+1\Big) + u_2\Big((1-p)x_1+1\Big) + u_2\Big((3-p)x_1\Big)\right]$$

Since u_2 is linear, $x_2 \in \text{argmax}_{x_2}\Big\{(8-2p)x_2+2\Big\}$. $p=2$ is also optimal for him because he will surely get a positive utilty no matter how much x_2 he demands. Therefore, $p^* = 2$ is a unique REE price.

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