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Essays on Credit and the Labor Market

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by

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Abstract

Essays on Credit and the Labor Market

Zachary Austin Bethune

This dissertation consists of three essays on credit and the labor market. The first essays studies the aggregate, business cycle relationship between consumer credit and unemployment. Using micro-level data I show there is a consistent *negative* effect of unemployment on both a household's use of and access to consumer credit. I find that upon job loss, households increase applications for credit, get denied more frequently, and experience significant reductions in both debt outstanding and average monthly charges. I interpret these effects as an increase in credit constraints for the unemployed and examine how this relationship impacts macroeconomic variables over the business cycle. To do so, I extend the canonical Mortensen and Pissarides (1994) model of unemployment to include a goods market with search and financial frictions. Households have limited commitment in repaying debt and face borrowing constraints that are disciplined by the ability of lenders to enforce financial contracts. The model predicts that job loss is followed by a contraction in borrowing constraints. In the aggregate, this channel leads to a strategic complementarity between (un)employment and firm hiring incentives as a higher fraction of unemployed consumers decreases the expected revenue from a labor match. I calibrate the model to match the estimated fall in credit upon job loss and examine how these individual unemployment-related credit shocks affect aggregate business cycles. I examine the response of unemployment to aggregate productivity and financial shocks. I find that productivity shocks do a poor job

of generating the co-movement of credit and unemployment we observe in the data. However, I find that aggregate financial shocks contribute significantly to the observed dynamics of both real and financial variables.

The second essay, co-authored with Guillaume Rocheteau and Peter Rupert, studies the long-term, aggregate relationship between consumer credit and unemployment. The model is similar to that developed in Chapter 1, however, we depart from the assumption that households' borrowing constraints are driven by an enforcement technology and instead allow enforcement to arise endogenously based on lenders' ability to monitor household repayment. As a result borrowing limits are endogenous and depend on the sophistication of the financial system, the frequency of liquidity shocks, and the rate of return on (partially) liquid assets that households can accumulate for self insurance. Moreover, firms' expected productivity is endogenous and depends on firms' market power in the goods market and the availability of unsecured credit to consumers. As a result of the complementarity between credit and labor markets, multiple steady states might exist. Across steady states unemployment and debt limits are negatively correlated. We calibrate the model to the U.S. labor and credit markets and illustrate the effects of an expansion in unsecured debt similar to that seen in the U.S. from 1978 to 2008. Under the baseline calibration, the rise in unsecured credit can account for approximately three quarters of the decline in the long-term average unemployment rate.

The third essay, co-authored with Tai-Wei Hu and Guillaume Rocheteau, studies the set of equilibria in a pure credit economy with limited commitment and endogenous debt limits, similar to that studied in Chapter 2. We show that the set of equilibria derived under "not-too-tight" solvency constraints, as in Chapter 2, is of measure zero in the whole set of Perfect Bayesian Equilibria. There exist a continuum of endogenous credit cycles of any periodicity and a continuum of sunspot equilibria, irrespective of the assumed trading mechanism. Moreover, any equilibrium allocation of the corresponding monetary economy is an equilibrium allocation of the pure credit economy but the reverse is not true. On the normative side, we establish conditions under which constrained-efficient allocations cannot be implemented with "not-too-tight" solvency constraints.

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Chapter 1

Consumer Credit, Unemployment, and Aggregate Labor Market Dynamics

1.1 Introduction

Between the late 1970s and 2007, the U.S. experienced a rapid increase in households' use of debt to finance consumption. At its peak, borrowing on consumer credit accounted for nearly twenty percent of personal consumption expenditures.[1] This trend was abruptly reversed during the 2007-2009 Great Recession, which featured a large contraction in the consumer credit market, coinciding with a dramatic decline in consumption spending and historically high unemployment.[2]

[1]Source: Federal Reserve Board's Consumer Credit G.19 Release and NIPA Table 2.1. Consumer credit is mostly comprised of credit cards, auto loans and student debt.

[2]The fall in consumption during the Great Recession has been extensively documented in the literature. For instance, see de Nardi *et al.* (2012) and Petev and Pistaferri (2012).

A growing body of research suggests that consumer debt is an important channel through which shocks to households get amplified leading to large and persistent responses in consumption. Empirically, this literature finds considerable cross-sectional evidence that regions of the U.S. that had the largest declines in household borrowing during the Great Recession also experienced the largest declines in consumption and employment (Mian and Sufi (2010), Midrigan and Philippon (2011), Mian *et al.* (2013), Mian and Sufi (forthcoming)). Recent theoretical work has examined the role of shocks to aggregate credit and liquidity constraints as a mechanism that prevents households from smoothing consumption.[3]

In this paper, I empirically and theoretically examine the effects of credit constraints for a salient group during downturns, job losers. Credit constraints are likely to be relevant for this group as these households value access to credit the most. Additionally, employment status and income are key criteria used by lenders in evaluating the credit-worthiness of borrowers (Crossley and Low, 2012). First, using household-level data from the 2007-2009 panel of the Survey of Consumer Finances, I identify a consistent *negative* effect of entering into unemployment on both a household's use of and access to consumer credit. I find that upon job loss, households increase demand for credit, get denied more frequently, and experience significant reductions in both debt outstanding and average monthly charges compared to households that maintained employment between 2007 and

[3]These include Midrigan and Philippon (2011), Hall (2011), Guerrieri and Lorenzoni (2011), among others. Additionally, there is considerable evidence that households are credit or liquidity constrained. Early work by Zeldes (1989) finds that households in the Panel Study of Income Dynamics with low liquid assets are indeed those households in which the test of the permanent income hypothesis fails. Others include Jappelli *et al.* (1998), Japelli (1990), Gross and Souleles (2002), and Agarwal *et al.* (2007). See Jappelli and Pistaferri (2010) for a review of this literature.

2009. This effect is particularly pronounced for borrowing on credit cards. While I cannot directly observe if the decline in credit for these households translated into a fall in consumption, I do find that there is a significant positive effect of unemployment on the likelihood of having zero liquid assets. This suggests that these households are limited in replacing their lost income by dis-saving.[4]

I interpret the effect of the fall in consumer debt as an increase in credit constraints for the unemployed and examine if this effect can explain the aggregate co-movement of debt, employment and consumption over the business cycle. To do so, I develop a model that features search in labor and goods markets in which credit is constrained by the ability of lenders to enforce financial contracts. The starting point is the canonical model of equilibrium unemployment by Mortensen and Pissarides (1994), hereafter MP, in which both firms and workers must go through frictional hiring before production can occur. However, in this framework firms are considered to sell their output seamlessly in a competitive, frictionless environment and there is no inherent role for credit in the goods market. I describe a household's need for liquidity (through credit) by incorporating search and matching frictions in the goods market in the style of Diamond (1990), Shi (1996), and Lagos and Wright (2005). A firm matched with a worker in the labor market produces intermediate output that it can either sell in a decentralized, frictional goods market or in a competitive, frictionless market as in MP. These two markets open sequentially and I assume that households have quasi-linear preferences over consumption in the decentralized and competitive market. This feature, combined with the fact that all labor income (wages and unemployment

[4] I use a broad measure of liquid assets and consider balances in checking, savings, and CD accounts as well as any treasury bills. See Section 1.2.2.

benefits) is received after the decentralized market closes, generates a need for credit on the side of households.[5]

The key friction of the model is that households lack commitment to repay debt. The amount of borrowing within the period depends on the ability of lenders to enforce debt contracts. I assume enforcement constraints are a function of both aggregate credit market conditions, similar to those used in the literature on firm financial constraints (i.e. Jermann and Quadrini (2012) and Monacelli *et al.* (2011)), as well as idiosyncratic household income. Similar to MP, a worker that enters into unemployment experiences a fall in their income. This fall causes the enforcement constraint to become tighter which leads to a fall in borrowing. In the model, firm revenues depend on the extent to which households are credit constrained. A fall in borrowing in the event of a job loss, decreases the demand for the output of a labor match. In equilibrium, this causes a lower number of firms to post vacancies and an increase in unemployment. Household credit constraints generate strategic complementarities, which if strong enough will lead to multiple equilibria as in Kaplan and Menzio (2014) or Bethune *et al.* (2015).

Finally, I calibrate the model to match the fall in credit for the unemployed between 2007-2009 and examine the importance of this channel in amplifying the response of macroeconomic variables to productivity and aggregate financial shocks. In order to discipline the extent of household financial shocks, I use an approach outlined Jermann and Quadrini (2012) with regards to firm credit. Using data on consumer credit and household income from the Flow of Funds, I construct a time series for household financial shocks using the model's enforcement

[5] This motivation for liquidity can also be found in studies of bank runs a la' Diamond and Dybvig (1983).

constraint under the assumption that it is always binding. This methodology is analogous to the standard approach of identifying productivity shocks using Solow residuals from the production function. I compare the response of unemployment and other labor market variables in the model to shocks to labor productivity and financial conditions. As first pointed out in Shimer (2005), and more recently in Hall (2014), productivity shocks in the context of the MP model do a poor job of generating sufficient movement in labor market variables. Similarly, I find that the credit effect of unemployment does not improve the performance of the model in this dimension. However, I do find that aggregate financial shocks contribute significantly to the observed dynamics of the labor market. Financial episodes are particularly pronounced in the Great Recession and the recession of the early 1980s.

This paper is closely related to the literature on financial frictions and unemployment. Wasmer and Weil (2004), Monacelli *et al.* (2011), Petrosky-Nadeau and Wasmer (2013), and Petrosky-Nadeau (2014) consider how financial frictions facing firms affects hiring and unemployment in the context of the MP framework. I differ in that my focus is on credit to households and financial frictions arise as a consequence of limited commitment and enforcement constraints, whereas in these papers financial frictions are in the form of search frictions. On the household side, recent empirical and theoretical work (including Mian and Sufi (2010), Keys (2010), Mian *et al.* (2013), Hsu *et al.* (2014), Haltenhof *et al.* (2014), Gropp *et al.* (2014), Athreya *et al.* (forthcoming), Mian and Sufi (forthcoming), among others) stresses the importance of the household debt channel in accounting for movements in the labor market, particularly during the Great Recession. This paper is the first to connect how credit constraints depend on an individual's job status and show that these constraints have implications on the labor market during business cycles.[6]

In terms of empirical evidence of credit constraints among the unemployed, this paper is closest to work by Sullivan (2008) and Crossley and Low (2012). Sullivan (2008) finds that low-asset households, or those in the bottom decile of the asset distribution, do not borrow from unsecured credit markets in response to job loss.[7] Using Canadian data, Crossley and Low (2012) find that a quarter of recent job losers could not borrow to increase consumption. I further this work by showing similar patterns for the U.S. as well as by quantifying the aggregate effects of the credit-unemployment channel. Finally, this paper is complementary to recent work by Herkenhoff (2013) that considers the impact of consumer credit access on unemployment. In his framework, greater access to unsecured credit decreases the consumption decline upon job loss, increases reservation wages, and leads to longer and deeper labor market recoveries. The current paper differs in that I consider the reverse, the impact of unemployment on consumer credit access. I show that an income shock, in the form of a job loss, is also a significant, negative credit shock which, in the aggregate, also leads to longer and deeper labor market recoveries.

[6] Additionally there is a growing literature that shows that job loss is associated with long-term earnings losses and consumption declines (Jacobson *et al.* (1993), Farber (2005), Stephens (2001), Browning and Crossley (2008), Davis and von Wachter (2011)). Krebs (2007) illustrates that cyclical variation in long-term earnings losses can lead to large welfare costs of business cycles. This paper differs in that, in the context of the MP model without credit frictions, a job loss is a temporary shock to earnings. I show that in the presence of credit constraints, these temporary losses lead to consumption declines as well as further increases in the severity of the credit constraint.

[7] Sullivan (2008) finds that households in the second and third deciles do replace lost income, however they only do so by 11.5 to 13.4 percent.

This paper proceeds as follows. Section 1.2 analyzes the relationship between consumer credit use and unemployment both in the aggregate and at the micro-level. Section 1.3 outlines the model. Section 1.4 gives a theoretical characterization of the equilibrium. Section 1.5 discuses the calibration, which includes identifying household financial shocks and shows the results of the quantitative experiments. Finally, Section 1.6 concludes.

1.2 Evidence on Household Credit and Unemployment in the Data

In this section, I use aggregate and household-level data to analyze the relationship between consumer credit use, unemployment, vacancies, and output over the business cycle. First, using aggregate time series I establish a strong business cycle correlation between consumer credit and macroeconomic variables. The co-movement is most pronounced during the Great Recession and the recessions in the early 1980s. Next, using household-level data I identify a consistent negative unemployment effect on both the access to and use of consumer credit. When a household looses their job, it is more difficult to obtain credit precisely the time when the demand for credit should increase.

1.2.1 Aggregate Data on Consumer Credit, Firm Entry, and Unemployment

Figures A.1 and A.2 plot the dynamics of consumer credit outstanding (dashed red line) in relation to key macroeconomic aggregates (solid blue lines). Each series has been logged and de-trended using a Hodrick-Prescott filter with a smoothing parameter of 100,000.[8] The time series of consumer credit comes from the Flow of Funds Accounts. Consumer credit includes both revolving accounts, such as credit cards, as well as non-revolving accounts, such as automotive and education loans.[9] It does not include loans secured by real estate, such as home equity lines of credit. In, Table A.1, I report the contemporaneous correlation between consumer credit and standard macroeconomic variables. For reference, I also report the standard deviation of the variable of interest, along with its relative standard deviation to consumer credit.

[8]This parameter is commonly used in the macroeconomic labor literature, for instance see Shimer (2005). In addition to allowing comparison to this literature, there are two other reasons to depart from the standard smoothing parameter used in the real business cycle literature of 1,600. First, as pointed out in Bethune *et al.* (2012), the standard HP filter is very sensitive to movements at the ends of data series and has a tendency to underestimate the trend component during large contractions. Since I am interested the dynamics of aggregate time series following the Great Recession, this is problematic. One option is to use a band-pass filter as suggested in Christiano and Fitzgerald (2003). The band-pass filter allows the user to pull out a particular frequency component of a data series, such as the business cycle, and because it is a one-sided filter it has less error at the end points. Another option is to use a simple linear trend. The method chosen in this paper is somewhere in the middle of these two approaches. Secondly, as Reinhart and Rogoff (2009) point out, the duration of financial crises are considerably longer than the period considered in most business cycle studies. This suggests the need to analyze key financial and macroeconomic variables over a longer frequency.

[9] See Appendix A.0.5 for a more detailed description of the data sources.

Consumer credit is pro-cyclical; it tends to decrease relative to trend during recessions and increase during expansions.[10] The magnitude of the fall in credit during the Great Recession was not unique. Similar declines occurred during the recessions of the early 1980s and the time period surrounding the 1991 recession. We see that consumer credit is also highly correlated with movements across labor aggregates. Unemployment and consumer credit have a negative correlation of 61.7%. The co-movement of these two variables is the particularly pronounced during the Great Recession and the recessions in the early 1980s. Additionally, declines in the demand for labor, measured as changes in the amount of job vacancies, and employment also coincide with those in credit. Consumer credit has a 51% correlation with the rate of vacancy creation and a 64.6% correlation with the employment to population ratio.

Additionally, consistent with the fall in labor demand and increase in unemployment, I find evidence of a decline in the demand for firms' output. For instance, the declines in retail trade sales almost identically track the declines in consumer credit. The contemporaneous correlation between sales and credit is 64.3%. Household consumption expenditures, while as a whole are not as volatile as consumer credit, experience the same turning points. Focusing on consumer durable purchases, the two series follow nearly identical deviations from trend, with a correlation of 77.1% and a relative standard deviation of 0.92.

The high correlation between consumer credit, labor, and macroeconomic aggregates illustrates the need for a better understanding of the relationship between

[10] The notable exception is the recession in 2001 following the stock market bubble when consumer credit accelerated relative to trend. The acceleration of debt during this time period is also evident in other forms of borrowing, namely mortgages.

household finances, unemployment, and real activity. In the next section, I provide household level evidence that, at least during the Great Recession, consumer credit was more difficult to obtain, limits on credit cards fell, and households decreased borrowing. I finish this section by showing that these patterns are considerably more pronounced if a household lost their job.

1.2.2 Micro Evidence on Consumer Credit and Unemployment

This section presents evidence on the relationship between consumer credit and unemployment at the household level. The goal is to identify the effect of unemployment on a household's use of and access to consumer credit. To do so I use data from the Federal Reserve Board's Survey of Consumer Finances (SCF). The SCF is the preeminent data source used to study household finances in the United States. Regarding consumer credit, it contains detailed information about levels of debt outstanding, credit limits, whether a household applied for credit, and whether they were denied. While the SCF is typically a triennial cross-sectional survey, respondents from the 2007 survey were re-interviewed in 2009, creating a two-period panel data set which allows me to observe changes in both employment and credit at a household level.

Sample Selection and Experimental Design In order to examine how unemployment affects a household's access to credit, I use a difference-in-difference approach to compare changes in credit for households that entered into unemployment over the 2007 to 2009 period versus those that remained employed. This time period corresponds to the largest recession in the U.S. since the Great Depression. The aggregate unemployment rate increased from 4.6% to 9.3%. The rate of monthly job layoffs increased from 1.3% of total employment to 1.7%. This equates to an additional five-hundred-thousand jobs lost per month due to involuntary reasons.[11] The timing of the survey allows me to identify both the effect of the Great Recession on all households' credit access and use as well as the differential effect from entering into unemployment.

The SCF contains observations at a household level. Since the goal of the experiment is to identify the effect of a change in an individual's labor market state, I first restrict the sample to single households defined as those that reported not having a spouse or partner as well as not sharing finances with any other person.[12] Additionally, I only consider households that stayed single throughout the survey. Doing so mitigates the effects of any changes in debt or earnings due to changes in the number of earners in a household and also makes classifying observations into employment states easier.

I use a broad definition of employment and classify households as employed if they reported that they were either currently working, accepted a job and waiting to start, or were on sabbatical or extended leave and expected to go back to work.[13] My treatment group consists of households that were employed at the

[11]Data from the Job Openings and Labor Turnover Survey (JOLTS). JOLTS defines layoffs and discharges as separations initiated by the employer. These include layoffs with no intent to rehire, formal layoffs lasting or expected to last more than 7 days, discharges resulting from mergers, downsizing or closings, firings or other discharges for cause, terminations of permanent or short-term employees, and terminations of seasonal employees.

[12]The SCF defines a household as a primary economic unit (PEU). The PEU is defined as the core individual or core couple in a household plus any minor children or other financially interdependent individuals with the core individual or couple. See Bricker *et al.* (2011) for more details on the design of the 2007-2009 panel survey.

[13]The survey question asked households about their job status at the date of the interview.

2007 survey date but reported being unemployed at the 2009 survey date. I compare this group to those households that reported being employed in both the 2007 and 2009 surveys and also reported not having any unemployment spell in the year previous to the 2009 survey date. This limits the potential for respondents that were unemployed between 2007 and 2009, but found work close to the 2009 survey date.[14] Finally, I restrict the analysis to households in which the head is between 20 and 70 years of age. This results in a sample of 3,820 households.

Consumer Credit and Household Labor, 2007-2009 First, in Table A.2, I describe the change in credit and labor market variables for the sample as a whole during the initial two years of the Great Recession. The mean of the variables of interest are reported for 2007 and 2009 in columns (1) and (2), respectively. Column (3) reports the difference in means between the two time periods. In regard to certain variables, the experiences of the sample between 2007 and 2009 coincide with the patterns we observe in the aggregate, discussed in Section 1.2.1. For instance, I find significant reductions in several measures of credit card use. The fraction of households with at least one credit card fell by 4.3 percentage points, average monthly charges on credit cards fell by $53, and the average debt limit fell by $1,240. Average credit card debt outstanding also fell by $212, though not statistically significant. Surprisingly, opposed to the behavior in the aggregate, automotive loans increased for the sample between 2007 and 2009.

[14]The SCF only asks respondents if they experienced any unemployment spell in the previous 12 months. It is possible that I classify some households as having not entered unemployment but that experienced an unemployment spell between the two survey dates but longer than a year before the last survey date. I drop all households that were employed in 2009, but reported having an unemployment spell in the previous year.

It is difficult to conclude from the evidence in Table A.2 if household credit was systematically more difficult to obtain in 2009 compared to 2007 or if households simply decreased their demand for debt. Perhaps the cleanest measure the SCF provides that helps differentiate the two channels is by directly asking respondents (i) if they applied for any credit in the two years previous to the survey and (ii) given they applied for credit, if they were denied. On average, households both applied for less credit and got denied more frequently during the Great Recession. The fraction of households applying for credit fell from 63% to 43%. While we still can not be sure how much of the fall in the rate of credit applications is driven by demand, given that a household might withhold applying for credit if they think they will be denied, we can conclude that those households that wanted credit experienced a fall in their access to it. Conditional on applying for credit, the likelihood of being denied increased from 19% to 25%.[15]

The final two rows of Table A.2 show that households' total income was decreasing, as well as average weekly hours. During a time when income and hours of work are falling, a consumption smoothing motive would suggest that borrowing should increase. However, this is the opposite of what we see in the Great Recession, as consumer debt declined. In the next section, I ask if the large increase in the number of unemployed households can provide any clarification of the trends we observe in Table A.2.

The Effect of Unemployment on Household Credit, Assets, and Income

This section illustrates the difference in credit, asset, and labor market outcomes

[15] I classify a household as having been denied credit if they reported ever being denied a credit application *and* if they never received that loan upon future applications.

for households that entered into unemployment during the initial two years of the Great Recession versus those that did not. To do so, I estimate the following difference-in-difference model:

$$y_{it} = \beta_0 + \beta_1 \mathrm{EU}_i + \beta_2 \mathbb{I}\{t = 2009\} + \beta_3 \mathrm{EU}_i \times \mathbb{I}\{t = 2009\} + \beta_{\mathbf{4}} \mathbf{X}_{it} + \epsilon_{it} \quad (1.1)$$

The variable EU_i indicates if the household entered into unemployment between 2007 and 2009. The variable $\mathbb{I}\{t = 2009\}$ is a dummy for the year 2009. The vector $\mathbf{X}_{it}$ includes observable household characteristics such as age, education, race and sex. The coefficient of interest is β_3. It identifies the effect of unemployment on changes in the variable of interest.

The identification relies on the parallel-trends assumption, or that without entering into unemployment, the changes in outcomes for the treatment group would coincide with that for the control group. I argue that the parallel-trends assumption is likely to be valid for two reasons. First, I show in Table A.3 the results of a balancing test which suggests the two groups have similar individual characteristics and pre-treatment outcomes. The individuals in the sample who lost their jobs between 2007 and 2009 are no more likely to be male or black, and are weakly more educated than those who maintained employment. Additionally, there is no evidence that either debt outstanding or the incidence of credit use differed between the two groups in 2007. Labor income is also comparable, around \$28,000. The notable difference is that those in the treatment group are younger, with an average age of 43 compared to 48 for the control group. This age gap partially explains the differences in total household income, weekly hours worked, and credit card debt limits as younger workers tend to have less non-labor income, work longer hours and have lower credit card limits.[16] Secondly, the time period under consideration consisted of a large, exogenous aggregate increase in an individuals' likelihood of being unemployed. The rate of monthly job layoffs increased from 1.3% to 1.7%, or around five-hundred-thousand jobs lost each month for involuntary reasons. This implies that any unobserved characteristics that are correlated with both credit and labor risk are mitigated.

Tables A.4 through A.7 report the estimated coefficient, $\hat{\beta}_3$, for different outcome variables. Table A.4 shows the results for consumer credit variables. There is a strong negative unemployment effect for changes in total consumer debt outstanding. For households that lost their job, credit fell by $2,809. This represents a fall of 60% of debt, on average. This decline is dominated by a fall in credit card debt which decreased by $2,504 more for the treatment group. Table A.5 further illustrates the effect of unemployment on credit cards. There is a consistent negative unemployment effect for both the likelihood of having a credit card and the likelihood of using it in a given month. Further, average monthly charges on credit cards fell by $250 more for households that entered into unemployment. These effects are in addition to the evidence that credit use was falling for all households during this time period. There was no significant unemployment effect on the amount of debt outstanding on automotive loans.

Table A.6 shows that not only was there a significant negative unemployment effect on the use of consumer credit debt, but that these decreases cannot be

[16] One might worry that younger borrowers have higher credit risk as they are the least experienced financially and so would have greater changes in credit constraints during the Great Recession. However, Debbaut *et al.* (2013) show that young borrowers are among the *least likely* to experience a serious credit card default.

explained by a fall in demand. I find that there was a positive unemployment effect on the demand for credit. Households who lost their jobs increased their rate of credit applications by 14% relative to those that maintained employment. However, we see that the rate at which households were being denied for credit increased significantly more for the unemployed. Consumer credit became more difficult to acquire, precisely for the group that should value it the most.

I further test for the possibility of a selection effect driving the rate of credit denials by examining the reason given to the borrower in the event they were denied credit. If we believe the reduction in credit was based on characteristics other than a change in employment status, for instance negative credit history, and if the probability of a household being in the treatment group is correlated with these characteristics, then I could be potentially identifying the selection of households into this group. Columns (3) and (4) of Table A.6 show the rate of credit denials for what I term 'credit-related' reasons, including having a low credit score and having a history of bankruptcy, and 'income/employment-related' reasons, which include lacking a job or insufficient income.[17] The rate of denials for credit related reasons showed no differential response for the treatment group. However, there is a positive unemployment effect on the rate of denials for income or employment related reasons. This result, combined with the fact that there are were no differences in any pre-treatment credit denial outcomes, suggests that households who lost their jobs in the Great Recession decreased their credit use as a direct result of facing higher constraints and the primary reason for the increased constraints was (un)employment related.

[17] See Appendix A.0.5 for further details.

Finally, Table A.7 reports the effect of unemployment on income and assets. First, households who lost their jobs were not able to smooth their income using forms other than labor. Total income for these households fell by $20,000 more than for the control group.[18] This decline was only partially offset by an increase unemployment benefits of $1,188. Secondly, there is consistent evidence that these households were dis-saving as a result of losing their job. Total liquid assets, measured as all balances in checking, savings, and CD accounts as well as any treasury bills, fell by $5,292 for the unemployed, around half of the 2007 average. Additionally, the unemployment effect explains the entire increase in the fraction of households that were liquidity constrained. I consider a strong measure of liquidity constraints as those households reporting having zero liquid assets at the time of the survey.[19] Consistent with the evidence in Kaplan *et al.* (2014), there is a considerable amount of hand-to-mouth households in the sample. In 2007, 13% of the control group had no liquid wealth and the difference for the treatment group was not significant. During the Great Recession the fraction of all households that had no liquid wealth doubled to 26%, which is entirely explained by those households in the sample that lost their jobs.

[18]Total income is defined similarly to the aggregate series in Section 1.2.1 and includes wages and salaries, income from sole proprietorships, and interest and dividend income.

[19]Holdings of currency is not reported in the SCF. However, the 2010 Survey of Consumer Payment Choice from the Federal Reserve Bank of Boston, suggests that the average amount of cash holdings is $340 and the median is $70. See Foster *et al.* (2013).

1.3 A Model of Consumer Credit and Unemployment

In this section, I present a model of consumer credit and unemployment. In the model, firms and workers meet in a decentralized labor market with search and matching frictions in the style of Mortensen and Pissarides (1994). These firm-worker pairs, then, sell a fraction of their output in a decentralized goods market with search frictions similar to Diamond (1990). I follow Diamond (1990) in assuming that trades in the decentralized goods market occur with pairwise credit. The key friction of the model is that households lack commitment to repay debt. The amount of borrowing depends on the ability of lenders to enforce debt contracts. I assume enforcement constraints are a function of both aggregate credit market conditions, similar to those used in the literature on firm financial constraints (i.e. Jermann and Quadrini (2012) and Monacelli *et al.* (2011)), as well as idiosyncratic income. Similar to Mortensen and Pissarides (1994), a worker that enters into unemployment experiences a fall in their income. This fall causes the enforcement constraint to become tighter which leads to a fall in borrowing.

In the model, firm revenues depend on the extent to which households are credit constrained. A fall in borrowing in the event of a job loss, decreases the demand for the output of a labor match. In equilibrium, this causes a lower number of firms to post vacancies and an increase in unemployment. Household credit constraints generate strategic complementarities, which if strong enough will lead to multiple equilibria as in Kaplan and Menzio (2014) or Bethune *et al.* (2015).

1.3.1 Environment

The model is in discrete time that continues forever. There exists a measure one of households and a large measure of firms. Each period is divided into three stages. In the first stage, households and firms trade indivisible labor services in a labor market (LM). In the second stage, they trade consumption goods with credit in a decentralized market (DM) with search frictions. Finally, in the last stage, wages are paid, debts are settled and trade occurs in a frictionless, competitive market (CM). The consumption good in the CM is treated as the numeraire.

Each household is endowed with one indivisible unit of labor and has expected, lifetime discounted utility of

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\ell(1-e_t)+\upsilon(y_t)+c_t\right], \tag{1.2}$$

where β is the period discount factor, $y_t \in \mathbb{R}^+$ is consumption in the DM, $c_t \in \mathbb{R}$ is consumption in the CM, $e_t \in \{0,1\}$ is time devoted to working and $\ell \in \mathbb{R}$ can be interpreted as a utility flow from leisure or home production. The utility function in the DM, $\upsilon(y)$, is twice continuously differentiated, strictly increasing, and concave. Further, υ is assumed to satisfy $\upsilon'(0)=\infty$ and $\upsilon(0)=0$.[20] Households earn wages, w_t, if employed ($e_t=1$) and income, b_t, if unemployed ($e_t=0$), both in units of the numeraire.

A firm is composed of one job and posses a technology to transform one unit of labor into $\bar{z}_t \in \mathbb{R}^+$ units of intermediate good in the LM. Production occurs at the end of the LM, after matching takes place. Intermediate goods can be costlessly

[20] The first assumption is sufficient to guarantee an interior solution to the bargaining problem in the DM. The second assumption is a normalization and helps simplify algebra.

transformed into $y_t \in [0, \bar{z}_t]$ units of the DM good (determined endogenously) and $\bar{z}_t - y_t$ units of the CM good.[21] In order to hire in the LM in period t, a firm must post a vacancy at cost $k > 0$, in units of the numeraire in period $t-1$.

The LM follows Mortensen and Pissarides (1994) in which households and firms match bilaterally to trade labor services. Let the measure of matches between s_t searching workers and o_t job openings be given by $m(s_t, o_t)$. I assume that the measure of job seekers in period t is equal to the measure of unemployed households at the end of period $t-1$, $s_t = u_{t-1}$. The matching function, $m(s, o)$, has constant returns to scale and is strictly increasing and strictly concave in both of its arguments. Moreover, $m(0, o) = m(s, 0) = 0$ and $m(s, o) < \min\{s, o\}$. Given these assumptions, a worker's job finding probability is defined as $m(s_t, o_t)/s_t = m(1, \theta_t) \equiv p(\theta_t)$, where $\theta_t = s_t/o_t$ is labor market tightness. Similarly, the job filling probability for firms is given by $m(s_t, o_t)/o_t = m(1/\theta_t, 1) \equiv f(\theta_t)$. Matches formed in the LM are exogenously destroyed at rate δ at the end of the CM.

The DM has a similar structure to the LM. A measure $n_t = 1 - u_t$ of retailers (productive firms) and a measure one of households form random bilateral meetings according to the matching technology $\alpha(n_t)$. Therefore, the probability a household meets a retailer in the DM is $\alpha(n_t)$ and the probability a retailer meets a household is $\alpha(n_t)/n_t$. The matching technology is assumed to satisfy $\alpha'(n) > 0$, $\alpha''(n) < 0$, $\alpha(n) \leq \min\{n, 1\}$, $\alpha(0) = 0$, and $\alpha(1) = 1$. DM matches are destroyed with probability one at the end of the period.

[21] For now, I maintain the assumption of no aggregate uncertainty and perfect foresight. In the numerical section, labor productivity will be assumed to follow an AR(1) process $ln(\bar{z}_t) = \rho_{\bar{z}} ln(\bar{z}_{t-1}) + \epsilon_{\bar{z},t}$, where $0 < \rho_{\bar{z}} < 1$ and $\epsilon_{\bar{z},t} \sim N(0, \sigma^2_{\bar{z}})$.

Households trade in the DM through borrowing, but lack commitment to repay their debt. In order to sustain credit relationships, the borrower must face a potential cost of default. I assume that lenders have access to an enforcement technology, which in the event of default allows them to recover up to a fraction ν of a household's current income.[22] In the model, ν represents aggregate financial conditions that affect all households regardless of employment status.[23] Therefore, in the model households are constrained by two dimensions: the ability of lenders to enforce debt contracts, ν, and the household's current income. Figure 1.1 shows the timing of the model.

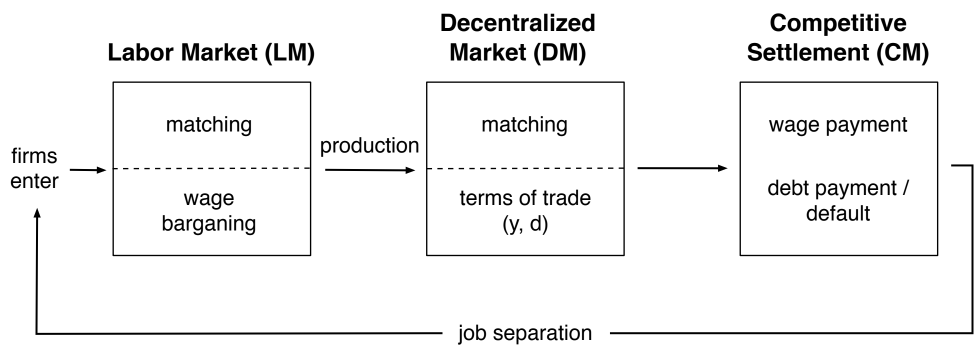

Figure 1.1: Timing

[22]It is equivalent to assume that ν is the probability that the lender can recover the entire amount of the loan. With probability $1 - \nu$, the recovery value is zero. For now I assume that ν is constant. In Section 1.5 I let ν be time-varying and follow an AR(1) process and consider aggregate financial shocks as innovations to that process.

[23]These could arise from many sources, for instance Herkenhoff and Ohanian (2012) stress the increase in congestion in the foreclosure process during the Great Recession.

1.3.2 Equilibrium

Centralized Market (CM) Consider a household entering the CM in period t. Let $W_t(d_t, e_t)$ be this household's value function. Upon entering the CM, a household's state is comprised of debt obligations d_t owed from trade in the previous DM and employment status e_t. Let $U_t(e_t)$ be their value function at the beginning of the LM in period t given by

$$W_t(d_t, e_t) = \max_{c_t} \; c_t + \ell(1 - e_t) + \beta U_{t+1}(e_t) \tag{1.3}$$

$$s.t. \; c_t + d_t = w_t e_t + b(1 - e_t) + \Delta. \tag{1.4}$$

Households maximize discounted lifetime utility choosing CM consumption, c_t, subject to their budget constraint which states that consumption and debt repayment must equal labor income plus any firm profits, Δ. Substituting the budget constraint (1.4) into (1.3), the household's value function becomes

$$W_t(d_t, e_t) = - d_t + \hat{w}_t(e_t) + \ell(1 - e_t) + \Delta + \beta U_{t+1}(e_t) \tag{1.5}$$

where $\hat{w}_t(e_t)$ represents labor income, $w_t e_t + b(1 - e_t)$. The use of linearity helps simplify the model in two important dimensions. First, notice from (1.5) that a household's lifetime utility is linear in debt, d_t. This will help simplify the credit contract in the DM since the surplus from trade will also be linear function of d_t. Secondly, linearity implies a household has no desire to smooth the repayment of debt over time and so with-in period debt contracts are weakly optimal in this environment.

The value function of a firm with a filled position at the beginning of the CM with x_t unsold inventories from the previous DM, d_t debt promises, and w_t wage obligations is given by

$$\Pi(x_t, d_t, w_t) = x_t + d_t - w_t + \beta J_{t+1} \tag{1.6}$$

where J_{t+1} is the value function of the firm at the beginning of the LM in period $t+1$.

Decentralized Market (DM) Trade Next, consider a match between a household and a firm in the DM. The terms of trade are given by the pair (y_t, d_t) which states the amount of DM good the firm transfers to the household, y_t, in exchange for d_t units of numeraire to be paid in the subsequent CM.

There are many ways to determine the terms of trade (i.e. proportional or Nash bargaining, Walrasian price setting, etc.). For the benchmark model, I assume that the solution is given by proportional bargaining which guarantees that trade is (pairwise) Pareto efficient and leads to an endogenous firm markup that is convenient in calibrating the degree of firm's market power.[24] The proportional

[24]Further, the proportional solution is monotonic in that each individual's surplus is increasing with the size of the total trade surplus. From Gu *et al.* (2013b), it is known that other, non-monotonic trading mechanisms (i.e. Nash or competitive pricing) can lead to endogenous credit cycles in limited commitment economies. With proportional bargaining, it is guaranteed that any endogenous cycles that arise are not due to the trading protocol. See Dutta (2012) for the strategic foundations of the proportional bargaining solution.

bargaining solution is given as the solution to the following problem

$$\max_{y_t, d_t} \quad \upsilon(y_t) + W_t(d_t, e_t) - W_t(0, e_t) \tag{1.7}$$

$$s.t. \;\; \upsilon(y) + W_t(d_t, e_t) - W_t(0, e_t) = \frac{\mu}{1-\mu}[\Pi_t(\bar{z} - y_t, d_t, w_t) - \Pi_t(\bar{z}, 0, w_t)] \tag{1.8}$$

$$d_t \leq \nu_t \hat{w}_t(e_t). \tag{1.9}$$

The maximization problem (1.7)-(1.8) above is given by Kalai (1977). The solution maximizes the household's surplus from trade while keeping fixed a proportional split of the total surplus between households and firms. The parameter $\mu \in (0, 1)$ can be interpreted as the household's bargaining power. Equation (1.9) is the enforcement constraint. Higher labor income or a better aggregate enforcement technology relaxes the constraint. Notice while $\hat{w}$ is endogenous, the household and the firm take it as given in the credit contract since wages are determined before the DM.

The bargaining problem (1.7) -(1.9) can be simplified by substituting in for W_t and Π_t from (1.5) and (1.6) and combining (1.7) and (1.8).

$$\max_{y_t} \mu[\upsilon(y_t) - y_t] \tag{1.10}$$

$$s.t. \;\; d_t = (1-\mu)\upsilon(y_t) + \mu y_t \leq \nu_t \hat{w}_t(e_t) \tag{1.11}$$

The maximand in (1.10) represents the household's share, μ, of the total surplus, $\upsilon(y_t) - y_t$ from DM trade. Equation (1.11) gives the pricing rule for the transfer from the household to the firm. It says that the wealth the household transfers to the firm is a non-linear function, $(1-\mu)\upsilon(y_t) + \mu y_t$, of the firm's DM output. Let

y^* be the first-best level of output defined as $\upsilon(y^*) = 1$. The solution to (1.10) - (1.11) is given by

$$y_t = y(e_t, w_t) = \begin{cases} y^* & \text{if } (1-\mu)\upsilon(y^*) + \mu y^* \leq \nu_t \hat{w}_t(e_t) \\ y_t & \text{s.t. } (1-\mu)\upsilon(y_t) + \mu y_t = \nu_t \hat{w}_t(e_t) \end{cases} \tag{1.12}$$

From (1.11) - (1.12), we can completely determine the terms of trade from knowledge of the household's payment capacity $\hat{w}_t(e_t)$, which depends on their current employment status, e_t, and equilibrium wage, w_t. If the payment capacity is above a certain threshold, $(1-\mu)\upsilon(y^*) + \mu y^*$, then the solution to the bargaining problem is to trade the first best level, y^*. Otherwise, households borrow up to their constrained limit and the terms of trade are given by $\{y(e_t, w_t), \hat{w}_t(e_t)\}$.[25] In order to simplify notation, I denote the DM consumption of employed and unemployed agents as $y^1 = y(1, w)$ and $y^0 = y(0, w)$, respectively.

Let $V_t(e_t)$ be the household's lifetime utility upon entering the DM at date t with employment status e_t. V_t satisfies

$$V_t(e_t) = \alpha(n_t)[\upsilon(y_t) + W_t(d_t, e_t)] + (1 - \alpha(n_t))W_t(0, e_t) \tag{1.15}$$

$$= \alpha(n_t)\mu[\upsilon(y_t) - y_t] + W_t(0, e_t) \tag{1.16}$$

[25]The bargaining contract must also satisfy the household and firm participation constraints given by

$$\upsilon(y_t) + W_t(d_t, e_t) \geq W_t(0, e_t) \tag{1.13}$$

$$\Pi_t(\bar{z}_t - y_t, d_t, w_t) \geq \Pi_t(\bar{z}_t, 0, w_t) \tag{1.14}$$

which never bind given the bargaining solution above.

where I use (1.5) and (1.12) to substitute in for $W_t(d_t, e_t)$ and d_t respectively. A household entering the DM gets matched with a firm with probability $\alpha(n_t)$, upon which they consume a fraction, μ of the total surplus from the bilateral relationship. With probability, $1 - \alpha(n_t)$, the household does not get matched and enters the CM without any debt. In (1.16), the household takes the terms of trade (1.11) - (1.12) as given. The value function of a firm at the beginning of the DM along the equilibrium path, F_t, is given by

$$F_t = \frac{\alpha(n_t)}{n_t}\Big[n_t\Pi_t(\bar{z}_t - y_t^1, d_t, w_t) + (1-n_t)\Pi_t(\bar{z}_t - y_t^0, 0, w_t)\Big] + \Big[1 - \frac{\alpha(n_t)}{n_t}\Big]\Pi_t(\bar{z}, 0, w_t) \tag{1.17}$$

$$= \frac{\alpha(n_t)}{n_t}(1-\mu)\big\{n_t[\upsilon(y_t^1) - y_t^1] + (1-n_t)[\upsilon(y_t^0) - y_t^0)]\big\} + \bar{z} - w_t + \beta J_{t+1} \tag{1.18}$$

The firm matches with a household in the DM with probability $\alpha(n_t)/n_t$ and trades y_t^e, $e = \{0, 1\}$. With probability n_t, they meet an employed household and with probability $1 - n_t$ they meet an unemployed household. Further, with probability $1 - \alpha(n_t)/n_t$, the firm doesn't meet a trading partner and carries the full amount of intermediate good, $\bar{z}_t$, into the CM.

Substituting Π_t from (1.6) and using the terms of trade (1.11) - (1.12), equation (1.18) gives F_t as the sum of the firm's total expected revenue from trade in the CM and DM in terms of the numeraire minus wages, w_t, plus the discounted continuation value of the firm in the following LM, J_{t+1}. Define the expected revenue as z_t, given by

$$z_t \equiv \frac{\alpha(n_t)}{n_t}(1-\mu)\big\{n_t[\upsilon(y_t^1) - y_t^1] + (1-n_t)[\upsilon(y_t^0) - y_t^0]\big\} + \bar{z} \tag{1.19}$$

Notice that z_t depends positively on the level of DM trade described by y^e, which is itself a function of wages, w_t.[26]

Labor Market (LM) Moving to the LM, the value function for a household with access to credit, given employment status, e_t, is given by

$$U_t(1) = (1-\delta)V_t(1) + \delta V_t(0) \tag{1.20}$$

$$U_t(0) = (1-p(\theta_t))V_t(0) + p(\theta_t)V_t(1). \tag{1.21}$$

If employed, with probability δ the household transitions to unemployment. Likewise, if unemployed, with probability $p(\theta_t)$, the household finds a job and transitions into employment. Substituting in for $V_t(e_t)$ in (1.20)-(1.21) from (1.16), yields

$$U_t(1) = \alpha(n_t)\mu[\upsilon(y_t^1) - y_t^1] + (1-\delta)W_t(0,1) + \delta W_t(0,0) \tag{1.22}$$

$$U_t(0) = \alpha(n_t)\mu[\upsilon(y_t^0) - y_t^0] + (1-p(\theta_t))W_t(0,0) + p(\theta_t)W_t(0,1) \tag{1.23}$$

Households have an expected surplus from DM trade equal to the first term in (1.22)-(1.23). Otherwise, the progression of a household through the labor market is similar to that in Mortensen and Pissarides (1994). If employed, with probability

[26] Sometimes I will make explicit the dependence of z_t on wages and refer to $z(w_t)$.

$(1-\delta)$ the household maintains their job or with probability δ they get separated. If unemployed, a household finds employment with probability $p(\theta_t)$ and with probability $1-p(\theta_t)$ they have to continue searching in the following period. The firm's value function follows similarly. Let J_t be the expected lifetime value of a firm with a filled vacancy at the beginning of the LM, given by

$$J_t = (1-\delta)F_t + \delta V_t \tag{1.24}$$

where V_t is the value function of a vacant firm. From (1.24), a firm gets exogenously destroyed with probability δ and must wait a period before searching for a worker. Otherwise, the firm enters the DM with expected value F_t. Free entry of firms to the matching process guarantees that the value of a vacancy must be zero for all t, $V_t = 0$. Substituting in for F_t from (1.18), we can write

$$J_t = z_t - w_t + \beta(1-\delta)J_{t+1} \tag{1.25}$$

Notice, z_t is a function of DM trades, (y_t^0, y_t^1), and the current measure of employed workers, n_t. A higher level of DM trade leads to higher expected firm revenue; $\partial z_t/\partial y_t^e \geq 0$. On the other hand, firm entry has an ambiguous effect on z_t. With constant returns to scale matching, the probability an individual firm is matched with a worker is decreasing in the measure of firms, n_t. Higher unemployment is good for vacant firms because it becomes more likely to find a match in the LM. However, higher unemployment is bad for firms that stay filled because a higher fraction of unemployed workers implies that firms are more likely to meet a consumer in the DM that is more credit constrained. Hence, the sign of $\partial z_t/\partial n_t$ is ambiguous.

Wage Determination I assume wages are chosen such that the surplus generated in an employment match is proportionally split between the household and firm according to exogenous shares λ and $1-\lambda$, respectively. That is, I assume wages in period t are given by

$$V_t(1) - V_t(0) = \frac{\lambda}{1-\lambda} J_t \tag{1.26}$$

The wage outcome in (1.26) is given as the solution to the proportional bargaining problem in Kalai (1977) where $V_t(1) - V_t(0)$ is the household's surplus from being employed and J_t is the firm's surplus from having a filled position. Using (1.16) and (1.18), Appendix A.0.2 derives the equilibrium wage equation

$$w_t = \lambda[z_t(w_t) + \theta_t k] + (1-\lambda)(b + \ell - \alpha(n_t)\mu[S^1(w_t) - S^0]) = \Gamma_t(w_t) \tag{1.27}$$

where $S^0 = \upsilon(y_t^0) - y_t^0$ and $S^1 = \upsilon(y_t^1) - y_t^1$ represent the joint surplus from a DM match with unemployed and employed households, receptively. Notice in (1.27), I make explicit the dependence of y_t^1 on wages through the loan contract (1.12). The wage is a weighted average of the firm's revenue augmented by average recruiting costs per vacancy, θk, and a household's flow utility from being unemployed augmented by the net utility cost of potentially losing access to credit. The equilibrium wage is a fixed point of $w_t = \Gamma_t(w_t)$.

A higher wage relaxes credit constraints for employed workers, which has two effects on $\Gamma_t(w_t)$. First, higher credit implies more trade in the DM between firms and employed workers, $\partial y^1/\partial w > 0$, which increases a firms' expected revenue, z_t. This effect leads to a larger surplus in a labor match which puts upward pressure on wages. Secondly, as credit expands for employed workers, the household's outside option in labor bargaining is negatively effected. Unemployment not only coincides with a fall in income, but also a shock to credit constraints. Which effect dominates depends on the relative bargaining power of households in labor and goods markets. To see this, we can substitute in for $z(w_t)$ using (1.19) and express $\Gamma_t(w_t)$ as

$$\Gamma_t(w_t) = \lambda\Big[\frac{\alpha(n_t)}{n_t}(1-\mu)[n_t S^1(w_t) + (1-n_t)S^0] + \bar{z}_t + \theta_t k\Big] + (1-\lambda)\Big[(\ell + b) - \alpha(n_t)\mu[S^1(w_t) - S^0]\Big] \tag{1.28}$$

$$= w_t^{DMP} + (\lambda - \mu)\alpha(n_t)[S^1(w_t) - S^0] + \lambda\frac{\alpha(n_t)}{n_t}(1-\mu)S^0 \tag{1.29}$$

where $w_t^{DMP} = \lambda(\bar{z}+\theta_t k)+(1-\lambda)(\ell+b)$, which is identical to the equilibrium wage in the Pissarides (2000) textbook model. It coincides with the equilibrium wage in this model if there were no credit (i.e. $\nu = 0$). The first and last term in (1.29) are constant with respect to the wage in the match. The second term depends on the surplus in the goods market between a firm and an employed worker. The sign depends on the bargaining power of the household as a worker in the LM relative to their bargaining power as a consumer in the DM. If λ is higher, then the first effect discussed above dominates and wages are inflated due to the positive effect on firm revenue. However, in the opposite case when a household's bargaining power is higher in the goods market, the second effect dominates which creates downward pressure on wages. If this negative effect is large enough, the wage could fall below w_t^{DMP}. Under this scenario, the introduction of household credit creates a 'liquidity discount' on wages equal to the additional value households place on employment by increasing their access to credit. Figure 1.2 illustrates the determination of wages under the two cases discussed above.

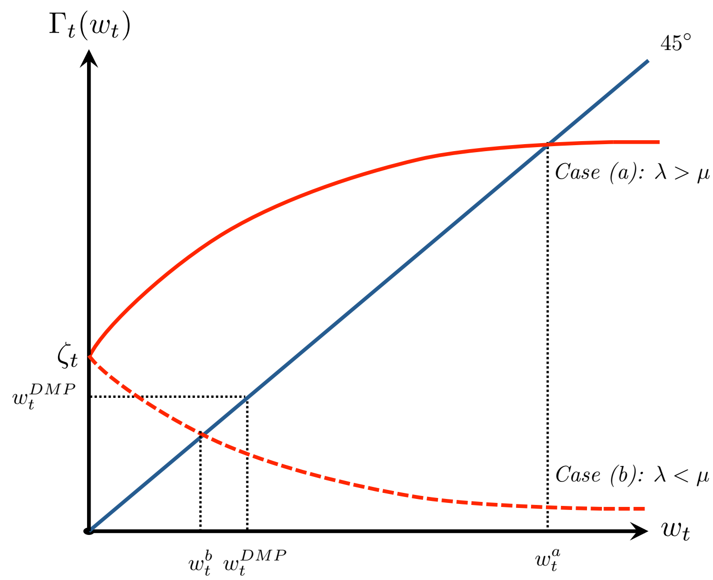

Figure 1.2: Wage Determination

If $w_t = 0$, then $\Gamma_t(0) = w_t^{DMP} + \left[\lambda \frac{(1-n_t)}{n_t}(1-\mu) + (1-\lambda)\mu\right]\alpha(n_t)S^0$, denoted as ζ_t in the figure. In Case (a), $\lambda > \mu$ and the equilibrium wage is higher than in an environment without credit, $w_t^a > w_t^{DMP}$. Case (b) illustrates the opposite when $\lambda < \mu$ and the equilibrium wage is lower than in an environment without credit, $w_t^b < w_t^{DMP}$.[27] Lemma 1 makes this precise.

[27]Notice it is also possible under case (b) that the third term in (1.29) is large enough such that wages are higher than w_t^{DMP}.

Lemma 1. *There exists a unique, positive solution to (1.27). Additionally,*

(a) If $\lambda > \mu$, then $w_t \in [\zeta_t, \bar{w}_t]$ where $\bar{w}_t = w^{DMP} + \frac{\alpha(n_t)}{n_t}(1-\mu)[\upsilon(y^*) - y^*]$. Hence, $w_t \geq w^{DMP}$ for any (u, θ) combination since $\zeta_t \geq w^{DMP}$.

(b) If $\lambda < \mu$, then $w_t \in [\underline{w}_t, \zeta_t]$ where $\underline{w}_t = w^{DMP} - (\mu - \lambda)\alpha(n_t)[u(y^*) - y^*]$. If $\zeta_t - w^{DMP} < (\mu - \lambda)\alpha(n_t)S^1(w^{DMP})$, then $w_t < w^{DMP}$. Otherwise, $w_t \geq w^{DMP}$ as in (a).

If household's have more bargaining power as workers than as consumers, then there is a positive credit externality on wages. In the other case, if households have more bargaining power as consumers than as workers, and the net surplus of credit to unemployed households in the labor match isn't too big, $\zeta_t - w^{DMP} < (\mu - \lambda)\alpha(n_t)S^1(w^{DMP})$, then there is a negative credit externality on wages.

Firm Entry and Unemployment Plugging in the wage from (1.27) into (1.25) we can derive the difference equation for the value of a filled job as

$$J_t = S^f(n_t, \theta_t) + \beta(1-\delta)J_{t+1} \tag{1.30}$$

Equation (1.30) gives a familiar law of motion for the value of a filled job. The function S^f represents the firms share the total surplus from a labor match equal to

$$\begin{aligned} S^f(n_t, \theta_t) = (1-\lambda)[\bar{z}_t - (b+\ell)] - \lambda\theta_t k + (1-\lambda)\frac{\alpha(n_t)}{n_t}(1-\mu)[n_t S^1 + (1-n_t)S^0] \\ + (1-\lambda)\alpha(n_t)\mu[S^1 - S_0] \end{aligned} \tag{1.31}$$

where S^1 is a function of n and θ through its dependence on the wage given by (1.27).[28] The first two terms in (1.31) are standard and equal to the firms share of exogenous output minus a worker's outside option in an environment with no credit, $b + \ell$, adjusted for the costs of vacancy creation. The last two terms are novel. The first is equal to the firm's share of the additional expected revenue of a labor match from operating in the DM. The second term represents rents the firm collects through wage bargaining, equal to their share of the the household's cost of loosing access to credit upon unemployment. Lemma 2 characterizes the comparative statics when $\lambda > \mu$.[29]

Lemma 2 (Comparative Statics of $S^f(n, \theta)$)**.** *Let $S^f(n, \theta)$ be given by* (1.31)*.*

(i) $\partial S^f / \partial \theta \leq 0$ *for all* n*.*

(ii) Suppose $S^0 = 0$ *(i.e.* $b = 0$*). Then* $\partial S^f / \partial n \geq 0$ *for all* n*.*

(iii) Suppose $S^0 \neq 0$*. Then the sign of* $\partial S^f / \partial n$ *is in general ambiguous, but must change sign from positive to negative.*

The value of the firm's labor surplus is weakly decreasing in labor market tightness. In the extreme case, if unemployed households are completely denied credit, $S^0 = 0$, the effect of higher employment on the labor match is always positive. This is because higher employment unambiguously leads to a larger surplus in the DM. If $S^0 > 0$, there are two forces at play when employment increases. First the

[28] S^0 is a function of non-employment income b and other exogenous parameters, such as aggregate financial conditions, ν. See equation (1.12).

[29] This case is informative for the quantitative section because under all of the calibrations considered, $\lambda > \mu$.

rate of finding a trading partner in the DM for the firm declines. The sign of $\partial(\alpha(n)/n)/\partial n$ is negative given a constant returns to scale matching function. However in general, and increase in n, increases wages which has a positive effect on S^f.

The law of motion for unemployment also follows the standard difference equation

$$u_{t+1} = (1 - p(\theta_t))u_t + \delta(1 - u_t). \tag{1.32}$$

From (1.32), the measure of unemployed households in period $t + 1$ is equal to the fraction of unemployed households in period t that did not get matched in the previous LM, $(1 - p(\theta_t))u_t$, plus the fraction of employed households that became separated from their job $\delta(1 - u_t)$ between periods. We are now ready to define the equilibrium for the perfect-foresight economy.

Definition 1. *A discrete-time perfect-foresight equilibrium is given by the sequence $\{u_t, J_t\}_{t=0}^{\infty}$ satisfying (1.30) and (1.32) such that u_0 is given and $\lim_{t\to\infty} J_t$ is finite.*

Given a series for J_{t+1}, we can determine labor market tightness, θ_t, as the solution to the free entry condition, $V_t = 0 \; \forall \; t$. This implies that $k = \beta f(\theta_t)J_{t+1}$ in every period, where $\partial\theta_t/\partial J_t > 0$. Wages are determined by (1.27) which, given b, pin down the level of trade in DM, (y^1, y_0).

Letting $w_t = w(u_t, J_t)$ represent the solution to (1.27) and the free entry condition $\theta(J_t) = f^{-1}(k/\beta J_t)$, we can characterize the

1.4 Equilibrium Characterization

1.4.1 Steady States in a Simplified Model

In this section I characterize all steady-state equilibria in the perfect foresight economy under the extreme case that unemployed households receive no credit (i.e. when $b = 0$). For convenience, I also assume that $\lambda > \mu$ as in case (a) in Lemma 1. A steady-state is a point in (u, J) space such that the unemployment rate and value of a firm are constant through time. Steady state unemployment is given as a function of J by (1.32) setting $u_t = u_{t+1}$:

$$u = \frac{\delta}{\delta + p(\theta(J))} \tag{1.33}$$

This is the standard Beveridge curve derived in Mortensen and Pissarides (1994). It equates the flow of households into and out of unemployment. Steady state unemployment is the ratio of total separations to total labor market 'churn', given by the total rate of separations and job finding. Let $\underline{u}$ denote the lower bound for unemployment given as $\underline{u} = \delta/(1 + \delta)$. Since for all values of J above βk, $\theta'(J) > 0$, in the limit as $J \to \infty$ we have that $\theta \to \infty$. Given the assumptions on $p(\cdot)$, $\theta \to \infty$ implies $p \to 1$. If $J < \beta k$, the labor market shuts down and $\bar{u} = 1$. In Figure 1.3, (1.33) is represented as the downward-sloping blue line. Using (1.30), we can characterize J as a function of $u = 1 - n$:

$$J = \frac{S^f(1 - u, \theta(J))}{1 - \beta(1 - \delta)} \tag{1.34}$$

Given u, the value of a filled job is the solution to the fixed point problem in (1.34). Lemma 3 this problem is well defined.

Lemma 3. *There exists a unique, positive solution to* (1.34)*,* J^**. Additionally, for any* $u \in [\underline{u}, \bar{u}]$*,* $J^* > J^{DMP}$*, where* $J^{DMP} = [(1-\lambda)(\bar{z}-\ell) - \lambda\theta k]/(1-\beta(1-\delta))$ *and* $\partial J^*/\partial u$ *takes on the sign of* $\partial S^f/\partial u$*.*

The right panel of Figure 1.3 illustrates (1.33) and (1.34) for three possible cases. In the first case, the top red dotted line, there exists a unique steady state. In this case, the credit effects of unemployment are not strong enough to generate multiplicity. In the second case, the solid green line, the complementarities between credit and hiring lead to two steady-state equilibria. Across these equilibria unemployment and credit (and vacancies) are negatively correlated. In the third case, the bottom dashed purple line, there also exists two steady-state equilibria, however as unemployment increases credit eventually shuts down and $J^* = J^{DMP}$. In the figure, this leads to labor market autarky and $u^* = 1$. The left panel of Figure 1.3 illustrates a case when the credit channel is strong enough to generate multiple equilibria, but in the event credit shuts down, the labor market is still active. Under this case, there are three steady-state equilibria.

Consider an exogenous increase in the output of a labor match, $\bar{z}$. The $\dot{J} = 0$ locus shifts up, for instance moving from the solid-green line to the dotted-red line in Figure 1.3. Unemployment decreases in the 'good' steady-state, or the one with the lowest unemployment rate, and the value of a filled job increases. If the increase in $\bar{z}$ is large enough, the multiplicity of steady-states vanishes. A similar effect occurs with an increase in aggregate financial conditions, ν. The $\dot{J} = 0$ nuclei shifts up for every value of u.

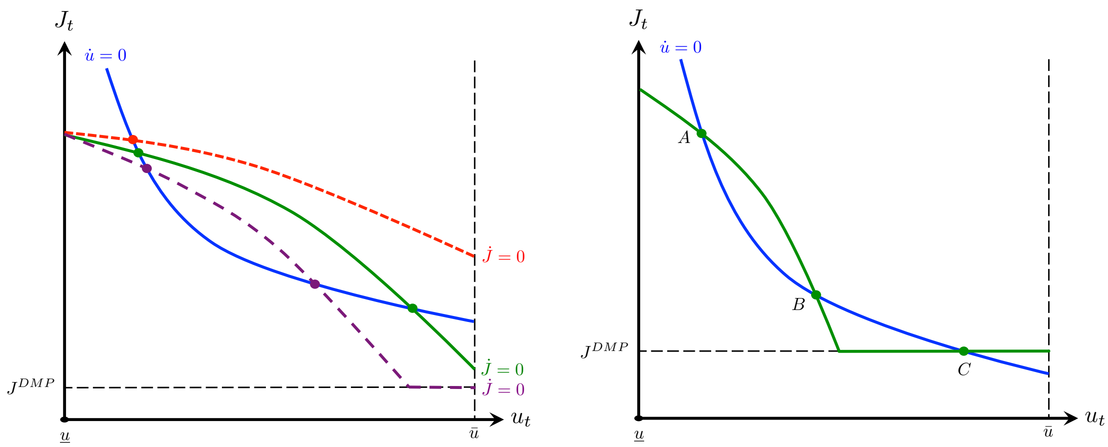

Figure 1.3: Steady States

1.4.2 Dynamics in a Simplified Model

In this section, I characterize the perfect-foresight dynamics in the simplified environment considered in Section 1.4.1. I first focus on properties of equilibria in a neighborhood of the set of stationary equilibria and then characterize the environment's global dynamics, or those dynamics beginning anywhere in the (u, J) domain. It is helpful to first transform the equilibrium conditions (1.30) and (1.32) from discrete time into continuous time.[30] The following defines all perfect-foresight equilibria in continuous time.

Definition 2. *A continuous-time, perfect-foresight equilibrium is given by the path* $\{u_t, J_t\}_t$ *such that*

[30] See Appendix A.0.4 for a detailed discussion on transforming the model from discrete into continuous time.

(i) For all $t \geq 0$, u_t *satisfies the law of motion*

$$\dot{u}_t = \delta - [p(\theta(J_t)) + \delta]u_t \tag{1.35}$$

(ii) For all $t \geq 0$, J_t *satisfies the no arbitrage condition*

$$\dot{J}_t = (r + \delta)J_t - S^f(1 - u_t, \theta(J_t)) \tag{1.36}$$

(iii) $\lim_{t\to\infty} J_t$ *are finite and* u_0 *is given.*

The functions S^f is defined in (1.31) and is independent of time, t. First, consider the dynamics in a neighborhood of the steady state. Approximating the system (1.35) -(1.36) by linearizing around a given steady state (u_i^*, J_i^*) yields

$$\begin{pmatrix} \dot{u}_t \\ \dot{J}_t \end{pmatrix} \approx \mathbf{M} \begin{pmatrix} u_t - u^* \\ J_t - J^* \end{pmatrix}, \tag{1.37}$$

where the $\mathbf{M}$ represents the Jacobian of the system (1.35) -(1.36) given by

$$\mathbf{M} = \begin{pmatrix} -[p(\theta(J^*)) + \delta] & -p'(\theta(J^*))\theta'(J^*)u^* \\ -S_1^f(1 - u^*, \theta(J^*)) & (r + \delta) - S_2^f(1 - u^*, \theta(J^*))\theta'(J^*) \end{pmatrix} \tag{1.38}$$

The system is characterized by one forward looking 'jump' variable, J_t, that is dynamically unstable and one backward looking 'predetermined' variable in u_t that is dynamically stable. The local dynamics can be characterized by solving for the sign of the two eigenvalues of $\mathbf{M}$. The top two elements in (1.38) are both negative. Part (i) of Lemma 2 guarantees that $S_2^f \leq 0$ for all u and hence the bottom-right element is positive in sign. Additionally, from part (ii) of Lemma 2 and the fact that $b = 0$ in the environment considered, the sign of $S_1^f(1-u_i^*, \theta(J_i^*)$ is weakly positive and hence the bottom-left element of (1.38) is weakly negative. Therefore, it is straightforward to show that the eigenvalues must be opposite in sign since $\det(\mathbf{M}) \leq 0$.

Figure 1.4 illustrates two examples of dynamic equilibria. The left panel shows the case in which the complementarities between household credit and firm revenue are not strong enough to generate multiple equilibria. In this case there is a unique saddle path that is downward-sloping in the (u, J) domain. This case is illustrative in comparison to the dynamics of the standard Mortensen-Pissarides model in which the saddle path is a horizontal line (i.e. Pissarides (2000), Section 1.7). Suppose unemployment starts above its steady state value. When the saddle path is flat, the value of a filled job (as well as labor-market tightness) remains constant as the economy converges to the steady state. As unemployment decreases, firms post less vacancies since lower unemployment implies a lower job-filling rate. In equilibrium, the decline in vacancy posting exactly corresponds to the decline in unemployment. However if the saddle path is downward-sloping as in the top-left panel of Figure 1.4, labor market tightness increases along the transition. In this case, vacancies must fall slower than unemployment. The reason is due to the credit effect of unemployment: higher unemployment is good for firms filling vacancies but lowers the value of a filled job.

The right panel provides an example with multiple steady-state equilibria. The dynamics are characterized by two saddle paths that cross through the steady states with the lowest unemployment rate (point A) and the highest unemployment rate (point C). The intermediate steady state is a sink. For any initial values, (u_0, J_0), within the grey-shaded region in Figure 1.4, the economy makes counter-clockwise cycles converging to point B. For any initial value of u_0, there exist an infinite, bounded measure of equilibria converging to the intermediate steady state and two unique equilibria converging to either the high or low steady state. This example illustrates that if the negative credit effect of unemployment is strong enough, expectations about future unemployment can have real macroeconomic consequences, as in endogenous credit-unemployment cycles.[31]

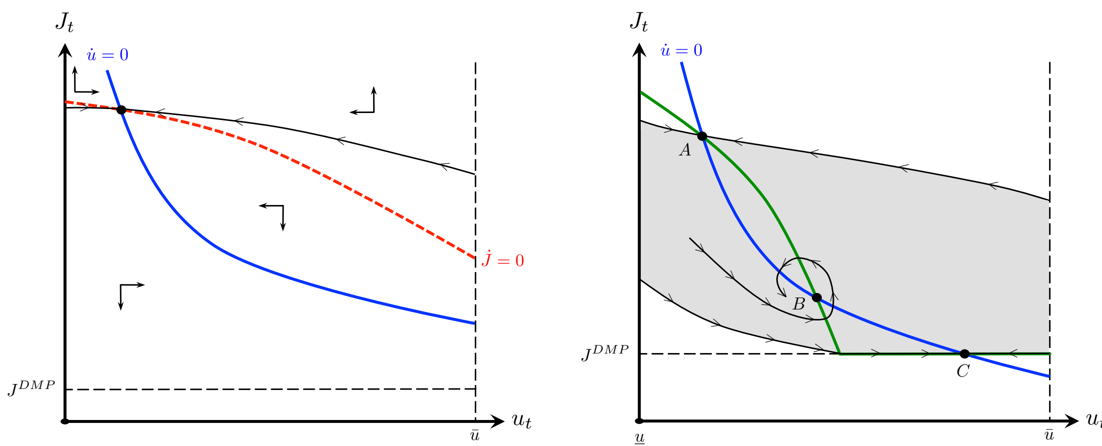

Figure 1.4: Perfect Foresight Dynamics

1.5 Quantitative Analysis

The primary experiment considered in this section examines if the negative effect of unemployment on consumer credit identified in Section 1.2.2 has an impact

[31] In Section 1.5 below, all the calibrations considered deliver a unique steady state, as in the left panel of Figure 1.4.

on business cycles in the aggregate. To do so, I consider two sources of exogenous fluctuations. The first are standard: shocks to aggregate labor productivity as in Shimer (2005). The second source are aggregate financial shocks that affect all households. These shocks have been stressed in the literature on credit frictions on the side of firms. For instance Jermann and Quadrini (2012) find that shocks to a firm's ability to raise funds through debt markets contributes significantly to the dynamics of macroeconomic aggregates. I consider a similar shock, though now on households' ability to finance consumption, through an exogenous change in lenders' ability to enforce financial contracts.

1.5.1 Calibration

The period in the model is set to a quarter. I set agent's discount factor to $\beta = 0.99$. All empirical targets represent quarterly averages over the time period 1978 Q1 to 2007 Q4. The model is first solved by a projection algorithm in which expectations are computed using Gauss-Hermite quadrature.[32] I then simulate the model to compute the moments for calibration.

Labor Market The calibration of the majority of the labor market parameters follows closely to the labor search literature following Shimer (2005), and more recently the literature on financial frictions and unemployment (i.e. Petrosky-Nadeau and Wasmer (2013), Petrosky-Nadeau (2014), and Monacelli *et al.* (2011)). First, I assume aggregate labor productivity fluctuates over time according to an

[32] As stressed in Petrosky-Nadeau and Zhang (2013), it is imperative to use global solution algorithm in quantifying the dynamics of the DMP model as log-linearization understates the mean and volatility of unemployment.

AR(1) process

$$ln(\bar{z}_{t+1}) = \rho_{\bar{z}_t} ln(\bar{z}_t) + (1-\rho_{\bar{z}}) ln(\mu_{\bar{z}}) + \epsilon_{\bar{z},t} \quad \text{s.t.} \quad \epsilon_{\bar{z},t} \sim N(0, \sigma^2_{\bar{z}}) \tag{1.39}$$

I normalize $\mu_{\bar{z}} = 1.0$. Using the Bureau of Labor Statistics series on output per worker, I estimate $\rho_{\bar{z}} = 0.889$ and $\sigma_{\bar{z}} = 0.0075$. For the matching technology, I use a constant returns to scale function as suggested in den Haan *et al.* (2000), $m(s,o) = so/(s^{\eta_L} + o^{\eta_L})^{1/\eta_L}$, which has the nice property of binding matching probabilities between zero and one. I set the curvature parameter, η_L, to match the average job finding probability. Following Shimer (2012), I first estimate the monthly job finding probability according to $p_t = 1 - (U^L_{t+1}/U_t)$, where U^L represents the number of workers with an unemployment duration above 5 weeks and U is the total number of unemployed workers.[33] I find that p=0.601. This implies that $\eta_L = 1.200$, consistent with the estimate in den Haan *et al.* (2000). The household's bargaining power corresponds to an egalitarian solution, $\lambda = 0.5$. Vacancy posting costs, k, are set to match 13% of quarterly wages as suggested by Silva and Toledo (2009). I set the exogenous job destruction rate, δ, to target an unemployment rate of 6.1%, the average over the sample considered.

The remaining two parameters associated with the labor market are the value of leisure, ℓ, and non-employment income, b. In an equilibrium with binding debt constraints, the fall in credit upon job loss exactly coincides with the fall in labor income. To see this, suppose (1.9) holds with equality for both employed and unemployed households. We can measure the proportional fall in credit as

[33] Data are from the monthly Current Population Survey (CPS).

$1-(d^0/d^1) = 1-(b/w)$. Non-employment income is crucial in disciplining the strength of the complementarities between credit and unemployment in the model. To set b, I use the evidence discussed in Section 1.2.2 and target a 60.4% decline in a households access to credit in the event of a job loss.I then set the value of leisure such that the total labor surplus is 71% of the average wage as in Hall and Milgrom (2008).

Credit and Goods Market In general, the frictions in the goods and credit market are designed to capture the inefficiencies of the process of getting produced products into the hands of consumers. Fluctuations in these inefficiencies are what induce fluctuations in unemployment. In the model, matching frictions, α, determine the frequency of a household's liquidity needs. Alternatively, credit frictions capture the difficultly households face in financing the purchases of goods once the liquidity shock is realized. To calibrate the parameters of the credit and goods market, I use data on household consumer credit use and firms market power in the retail sector.

The matching technology in the DM is also chosen to follow den Haan *et al.* (2000), $\alpha(n) = n/(n^{\eta_G}+1)^{1/\eta_G}$.[34] I set the curvature parameter, η_G, to target the ratio of consumer credit outstanding to output. During the sample period, this averaged 9.47%. In the model total debt is $D = \alpha(n_t)[nd_t^1 + (1-n)d_t^0]$. Total output, across the DM and CM, is $Y = n_t z_t - \theta_t u_t k$. Hence I set $D/Y = 0.097$, which results in $\eta_G = 1.27$. This implies an average matching rate of $\alpha = 0.56$, or that liquidity shocks occur every 1.8 quarters.

[34] This specification is also a general form of the matching technology used in the early New Monetarist literature, i.e.Kiyotaki and Wright (1993).

Utility over DM consumption is given by, $\upsilon(y) = y^{1-\gamma}/(1-\gamma)$. The elasticity parameter is set to target the marginal propensity to consume (MPC) out of an increase in debt limits of 14%, as given in Gross and Souleles (2002). In the model the MPC of an agent in employment state e is given by $MPC^e = \mu[\upsilon'(y^e) - y^e]/[(1-\mu)\upsilon'(y^e) + \mu]$. The aggregate MPC is then given by $MPC = nMPC^1 + (1-n)^M PC^0$. I set $MPC = 0.14$, which implies $\gamma = 0.4$. The household's bargaining weight, μ, is set to match a gross retail markup of 39%.[35]

To discipline aggregate financial conditions, ν_t, I follow the approach outlined in Jermann and Quadrini (2012). I first construct a series for ν_t using the enforcement constraint (1.9) under the assumption that is it always binding. That is, I assume

$$d_t^1 = \nu_t w_t \tag{1.40}$$

$$d_t^0 = \nu_t b \tag{1.41}$$

In the aggregate, this implies that total debt, D_t, is equal to

$$D = \alpha(n_t)[n_t d_t^1 + (1-n_t)d_t^0] = \nu_t \alpha(n_t)[n_t w_t + (1-n_t)b] = \nu_t \alpha(n_t) I \tag{1.42}$$

Replacing d_t^1 and d_t^0, we can express total debt as a fraction of total income as

$$\alpha(n_t)\nu_t = \frac{\alpha(n_t)[n_t d_t^1 + (1-n_t)d_t^0]}{[n_t w_t + (1-n_t)b]} \tag{1.43}$$

[35]Derived from retail gross margins. The average ratio of gross retail margins to sales from 1992-2007 was 0.28. Hence the average markup is calculated by $0.28/(1-0.28) = 0.39$. See https://www.census.gov/retail.

The numerator is equal to total borrowing in meetings with employed and unemployed agents, multiplied by the measure of agents taking loans, $\alpha(n)n$ and $\alpha(n)(1-n)$, respectively. The denominator is total labor income, $n_t w_t$, plus unemployment benefits, $(1-n_t)b$. Empirically, for the numerator I use the aggregate time series for consumer credit outstanding, minus education loans, in the household sector from the Flow of Funds Accounts.[36] For the denominator, I use the aggregate time series for disposable personal income from the BEA.[37] Given an estimate for η_L in the matching function, I am able to fully identify ν_t from (1.43). Notice that disciplining the financial shock in this way, assumes that movements in total consumer credit to disposable income in the data are generated from two sources, $\alpha(n_t)$ and ν_t. Feeding the constructed series of ν_t back into the model will, by itself, only generate the series of credit to income in the data if doing so also matches the series of u_t, which I don't calibrate to. After constructing the series for, ν_t, I estimate the autoregressive process

$$ln(\nu_{t+1}) = \rho_\nu ln(\nu_t) + (1-\rho_\nu)ln(\mu_\nu) + \epsilon_{\nu,t} \quad \text{s.t.} \quad \epsilon_{\nu,t} \sim N(0, \sigma_\nu^2) \tag{1.44}$$

The estimation yields $\mu_\nu = 0.928$, $\rho_\nu = 0.960$, and $\sigma_\nu = 0.0165$. The top two panels of Figure 1.5 show the constructed series for $\bar{z}_t$ and ν_t, respectively. The

[36]Total consumer credit covers most short and intermediate-term credit arrangements. However, data on student loans suggests many of these loans are deferred for several years. Since the objective of the quantitative exercise is to see how borrowing in the current quarter affects sales for same quarter, I exclude this type of debt from the exercise.

[37]See NIPA Table 2.1 Personal Income and Its Disposition. This measure of income is broader than that in the model. It includes income from other, non-labor, sources including receipts on assets, dividend or interest payments, and other transfer receipts besides unemployment insurance (i.e. social security). I could alternatively only use data on wages and salaries plus unemployment benefits. Doing so would only increase the estimated volatility in aggregate financial conditions.

bottom two panels show the innovations, $\epsilon_{\bar{z},t}$ and $\epsilon_{\nu,t}$. Since 1978, labor productivity has fluctuated around 3-4% of its long-run average. Those fluctuations mostly arise from the persistence parameter in the AR(1) process. However, similar to what Jermann and Quadrini (2012) find with respect to business credit, I find that household financial conditions are largely driven by innovations in the process. Table A.8 summarizes the choice of functional forms and Table A.9 gives the calibrated parameters.

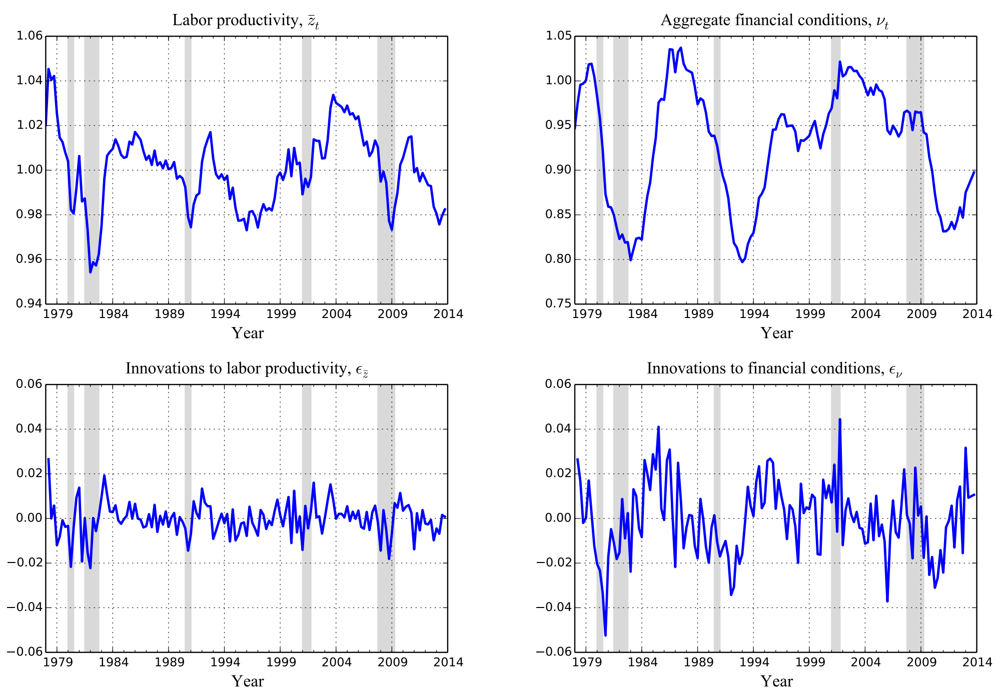

Figure 1.5: Stochastic processes of labor productivity and aggregate financial conditions.

1.5.2 Quantitative Results: Productivity versus Credit Shocks

In this section, I use the calibrated model to examine if the fall in credit upon job loss is enough to explain the co-co-movement of credit and unemployment in the aggregate. I consider two exogenous sources of fluctuations: aggregate productivity, $\bar{z}_t$ and aggregate financial conditions, ν_t. For each case, I feed the estimated shock process, $\epsilon_{\bar{z},t}$ or $\epsilon_{\nu,t}$ into the model while keeping constant the other variable at its unconditional mean, $\mu_{\bar{z}}$ or μ_{ν}. The model implied series are then logged and de-trended using a Hodrick-Prescott filter with smoothing parameter of 100,00 for comparison to the data shown in Section 1.2.1.

Productivity Shocks Figure 1.6 illustrates the effects of productivity shocks on the unemployment rate, vacancies, output, and total consumer credit. The model series are in red and their counterpart in the data is represented in blue. First, productivity shocks have a limited effect on the cyclical movement of unemployment, as has been well documented since Shimer (2005). Only during the 1982 recession does the model predict that unemployment increases close to that in the data. The recessions in 1990, 2001, and 2007 only feature a slight increase in unemployment and a quick reversal back below trend. The predictions for vacancy creation are slightly more in line with the data, though still under represent the magnitude of the deviations. For both series, the model completely misses the expansion between 2004 and 2007. Movements in labor productivity alone imply an increasing unemployment rate and falling vacancy creation between this time period.

A similar story follows with regards to output and consumer credit outstanding. The model implied series and the data coincide in sign for the recessions of the early 1980s, though again misses on the magnitude of the deviations. Each of the recessions post-1990 are not only shallow, but also short lived. For instance, the data suggest that after the 1990 recession, GDP stagnated relative to trend until 1996. The movement in labor productivity in the model suggests that there should not have been any stagnation. Debt also shows no significant deviations. Again, the model suggests that movements in labor productivity miss the contraction of consumer credit in both the mid-1990s and after the Great Recession. This leads to the conclusion that productivity shocks cannot independently explain either the movement in labor market variables or the movement of consumer credit.

Credit Shocks Figure 1.7 plots the unemployment rate, vacancies, output, and consumer credit for exogenous fluctuations in aggregate financial conditions, ν_t. The model implied series are represented in green and their empirical counterpart in blue. Household financial shocks in the model come much closer to explaining movements in the data. The aggregate unemployment rate is in line with the model predictions, particularly for the 1990 recession. Surprisingly, the model has a difficult time explaining the movements of unemployment during the Great Recession. In the data, unemployment started to increase in the early part of 2007. Movements in household financial conditions lead to an increase only beginning in 2009, after the majority of the change in the data occurred. In magnitude,

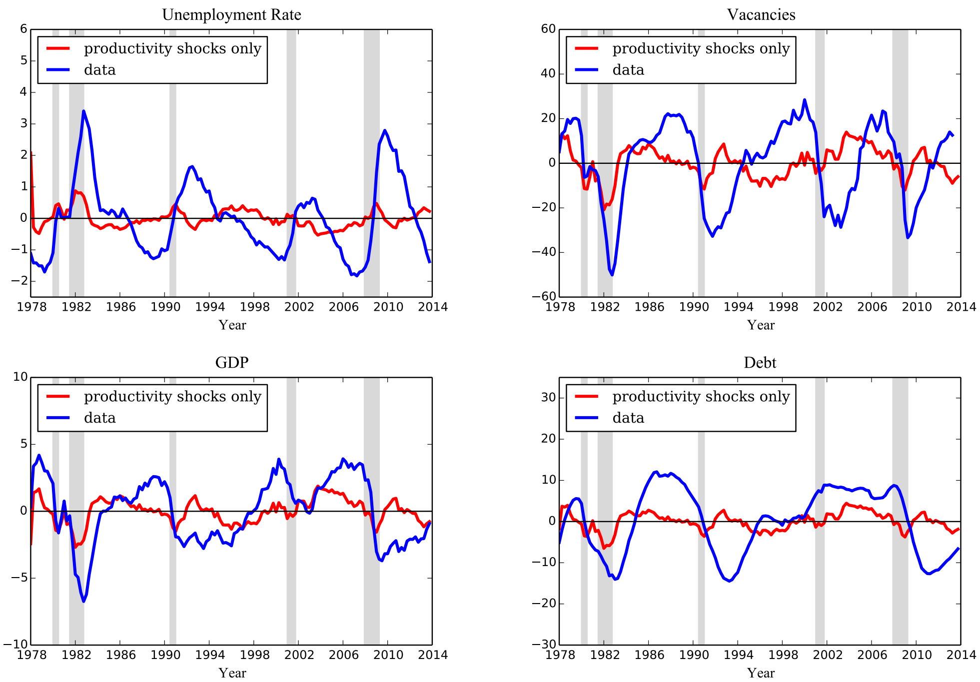

Figure 1.6: Productivity Shocks

changes in consumer credit can explain 30 percent of the total change during the Great Recession.

Credit shocks also do a better job explaining the demand for labor. The model predicts the entire fall in vacancies during the early 1980s and 1990s. Regarding the Great Recession, vacancies also fall by the same magnitude though the model predicts that fall starting only in 2009. An additional dimension credit shocks improve on are the persistence of labor market variables, particularly after the 1990s. For instance the slow decline of the unemployment rate in the model, coincides with the data.

The changes in labor market variables as a result of credit shocks also coincide with output and consumer debt. The model predicts a similar fall in output during the 1990s and improves on the length of the contraction. As with the labor market variables, credit shocks miss the beginning of the Great Recession by nearly two years. However, once the recession begins, the model does a good job in predicted the pace of the recovery. Finally, as could be expected, credit shocks do a good job fitting the movements in household debt over the cycle.

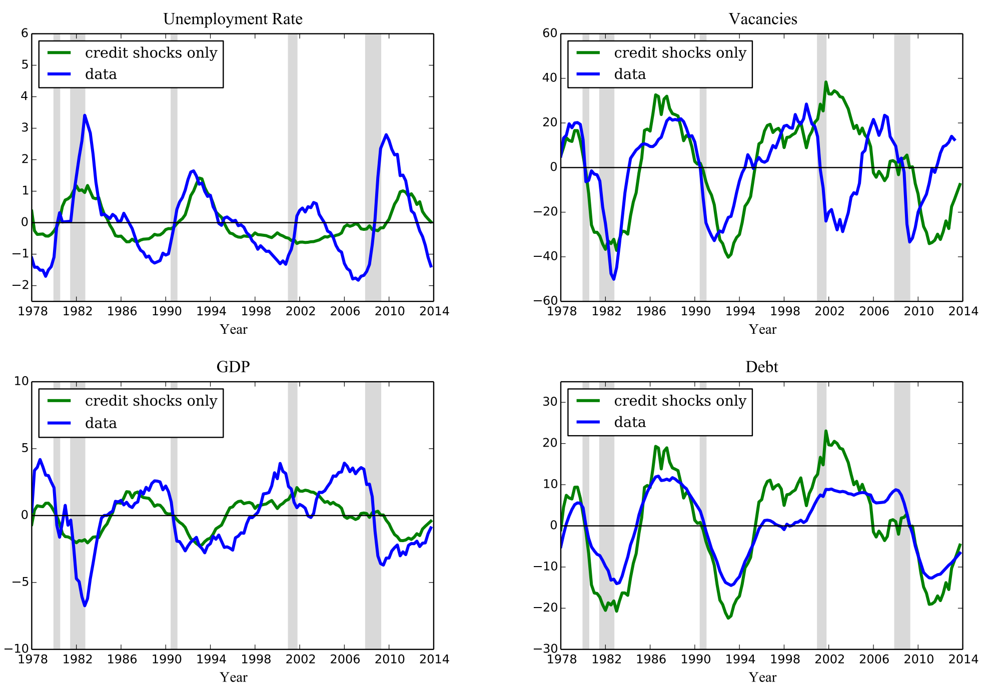

Figure 1.7: Credit Shocks

1.6 Conclusion

There is consistent evidence that households face constraints in financing consumption in the face of income shocks. Understanding which groups in the population these constraints affect the most is important in linking the observed movements in debt, employment and consumption during financial crises. The starting point for this paper was illustrating the fact that an income shock, in the form of a job loss, is also a significant credit shock. Credit constraints increase precisely for the group that values credit access the most, the unemployed.

I proposed a model that generates an increase in credit constraints for the unemployed that is both analytically tractable, easily quantifiable, and nests into the workhorse model of unemployment, Mortensen and Pissarides (1994). This easily allows comparison to other studies in the literature. I calibrate the model to match the fall in credit for the unemployed and explore to what extent this mechanism generates the observed business cycle co-movement of consumer credit, unemployment, and other macroeconomic variables. I show that productivity shocks do a poor job of generating the patterns in the data. However, I show that shocks to aggregate household financial conditions amplify the drop in credit upon job loss and contribute significantly to the dynamics of both real and financial variables.

Chapter 2

Aggregate Unemployment and Household Unsecured Debt

2.1 Introduction

Average household balances on unsecured loans more than tripled from 1980 to 2007, from roughly 3 to 10 percent of consumption (see Figure 2.1).[1] In 2007, more than 73 percent of all U.S. households had at least one credit card and roughly 50 percent of all households carried outstanding balances on these accounts.[2] Evidence suggests that unsecured debt has become easier to obtain and limits on credit cards have become increasingly more generous. The expansion of unsecured credit over this time period coincides with a decrease in the share of liquid assets

[1]Unsecured debt or non-collateralized debt refers to loans that are not tied to any asset. Unsecured debt primarily consists of revolving accounts, such as credit card loans, see Sullivan (2005).

[2]Source: 2007 Survey of Consumer Finances.

among all assets held by households and a long-term decrease in the unemployment rate.[3]

These trends were abruptly reversed following the 2007-08 financial crisis: the unemployment rate increased from about 4.5 to 10 percent while households use of unsecured credit and liquid assets returned to their 1995 level. These recent changes have led many commentators to speculate about the relationship between the recent credit crunch and the slow recovery and high levels of unemployment following the recession.[4]

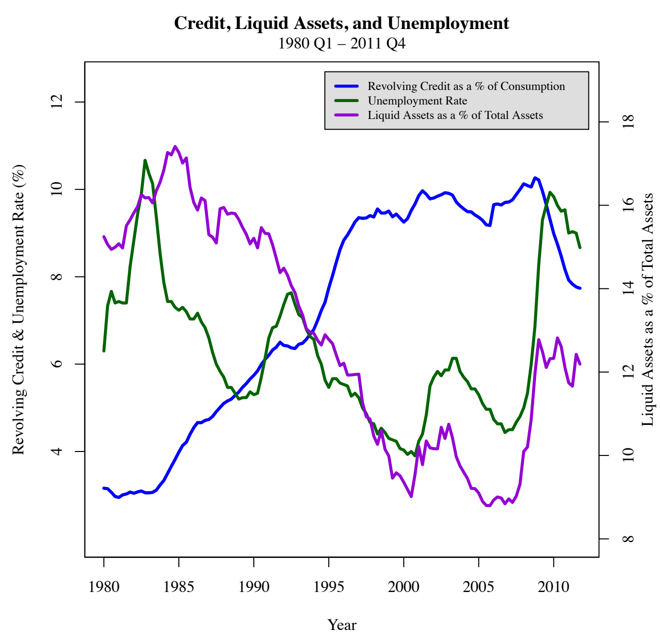

Figure 2.1: Unemployment, Revolving Credit, and Liquid Assets

[3]We define liquid assets as M2 plus treasury securities held by households as a percentage of their total assets given by the Federal Reserve Board's Z.1 Flow of Funds. The unemployment rate is the civilian unemployment rate reported by the Bureau of Labor Statistics.

[4]See, e.g., the article in the New-York Times of October 29, 2008, titled "As U.S. economy slows, credit card crunch begins" or the article in the Wall Street Journal of March 10, 2009, titled "Credit cards are the next credit crunch."

The objective of this paper is to provide a tractable dynamic general equilibrium model with trading frictions in which to analyze the relationship between household unsecured debt, liquid assets, and unemployment and the joint behavior of labor and credit markets, both qualitatively and quantitatively. Our starting point is the canonical model of equilibrium unemployment by Mortensen and Pissarides (1994)—MP hereafter. While this model is explicit about the search-matching frictions that prevail in the labor market, trades in the goods market are assumed to be seamless: firms' output can be sold instantly, households have no need for borrowing (and if they do, repayment can be enforced), and there is no role for liquidity. In contrast, in this paper, we describe household unsecured credit and its relation to labor market outcomes by incorporating a retail goods market with search frictions and limited commitment—along the lines of Diamond (1987a), Diamond (1990) , and Shi (1996).

The model assumes that frictional labor and goods markets open sequentially, as in Berensten *et al.* (2011). As in MP firms that enter the labor market post vacancies and unemployed workers look for jobs according to a time-consuming process. The output produced by firms can then be sold in a decentralized goods market where retailers and households are matched bilaterally, and households use liquid assets and unsecured debt to finance their purchases. The matching shocks in the decentralized goods market are analogous to liquidity shocks in banking models, except that the frequency of these shocks is endogenous in our analysis. Unsold inventories are traded in a frictionless competitive market where households have linear utility, as in the original MP model. Households value consumption in the decentralized retail market more than they value consumption in the competitive market and firms have some market power that allows them to charge a price higher than their marginal cost.

Following Kehoe and Levine (1993), households in the decentralized goods market face endogenous borrowing constraints because they cannot commit to repay their debt—the repayment of the debt must be self-enforcing. In order for unsecured credit arrangements to be incentive feasible some form of punishment must take place if an agent defaults on its obligations. If agents are anonymous and their trading histories are private information, agents cannot be punished from reneging on their debt. Therefore, we will assume that an imperfect record-keeping technology is available that keeps track of defaulting individuals and that makes this information publicly available. If a household defaults, and if default is publicly recorded, then the household is excluded permanently from credit arrangements. The endogenous debt limit that results from this threat increases as households become more patient, as the frequency of trade increases, but it decreases if firms have a higher market power.

We will assume that households are heterogeneous in terms of their access to unsecured credit. Some households' histories of default can be publicly recorded, and therefore these households can be trusted to repay their debt. Other households cannot be monitored or are seen as untrustworthy: those households are not able to borrow. Irrespective of their access to credit, we give the possibility to all households to accumulate liquid assets—assets that can be used in the decentralized goods market as means of payment or collateral to secure one's debt. However, these assets are costly to hold in the sense that their rate of return is less than the rate at which households discount future utility.

We show that the use of liquid assets as means of payment can coexist with the use of unsecured credit. So our model addresses the challenge in monetary theory of generating coexistence of money—which requires lack of record-keeping—and credit—which requires some record-keeping. For such equilibria to exist the rate of return of liquid assets cannot be too high—so that credit is a better option than money and the punishment from defaulting is sufficiently severe—and it cannot be too low—so that some households have incentives to accumulate liquid assets. Generically, households specialize in their methods of payment. Households with access to unsecured credit do not hold liquid assets and finance their purchases with credit only. Households without access to credit only hold liquid assets as a way to insure themselves against idiosyncratic spending shocks in the retail goods market.

Our model generates the following two-way interaction between the credit and labor markets. First, the aggregate state of the labor market affects the debt limit through the frequency of trade in the retail goods market. Indeed the number of firms participating in the labor market determines the number of retailers in the decentralized goods market. If the labor market is tight—there are a lot of active firms producing output to be sold in the retail market—then households receive frequent opportunities to consume, which makes exclusion from unsecured credit very costly. As a result, an increase in aggregate employment tends to raise households's debt limits. Second, firms' decisions to open vacancies depend on expected sales in the decentralized goods market, which itself depends on the availability of unsecured credit to households. If households can borrow large amounts, then firms can sell a larger fraction of their output in the decentralized goods market where they can charge a positive markup, which raises their expected revenue. Therefore, an increase in debt limits promotes job creation and reduces unemployment.

In the spirit of Kehoe and Levine (2001) we study analytically two limiting economies: one where households have no access to unsecured credit but can accumulate liquid assets to insure themselves against idiosyncratic trading opportunities in the decentralized market; a second one without liquid assets but where agents can borrow up to some endogenous limit. The former case can be interpreted as the pure monetary economy of Berensten *et al.* (2011). In accordance with Rocheteau and Wright (2005) and Rocheteau and Wright (2013), such an economy can have multiple steady-state equilibria because of the complementarity between households' choice of liquid assets and firms' participation decisions. Moreover, an increase in the rate of return of liquid assets raises output and decreases unemployment at the highest equilibrium.

The second case, which corresponds to a pure credit economy, also has multiple steady-state equilibria. Across equilibria, unemployment and credit limits are negatively correlated. In one equilibrium there is a higher market tightness, i.e., a lower unemployment rate, as well as a higher credit limit. Intuitively, if there are a lot of producers in the market, then the punishment for not repaying one's debt, i.e., the exclusion from the goods market, is large since households would have to forgive a large number of trading opportunities in the future. As default is more costly, the household debt limit increases and, as a result, firms can expect large sales in the decentralized goods market. By a symmetric logic the second equilibrium is one with a high unemployment rate and a low credit limit. Therefore a high unemployment equilibrium with a credit crunch is self-fulfilling.

We calibrate a version of the model to match the U.S. economy pre-Great Recession. We illustrate the equilibrium effects of a reduction in the availability of unsecured credit that match the empirical facts in Figure 2.1. From 1980 to 2010 unsecured debt increased from 2% to 10% of total consumption spending. Under the baseline calibration, the model predicts that steady state unemployment was 1.7 percentage points higher in 1980. This matches approximately 70% of the movement in trend unemployment between these two time periods. We additionally consider the effects of the credit crunch between 2007 and 2010 in which unsecured debt fell from 10% of consumption to 8%. The model predicts an increase in steady state unemployment of .4 percentage points, from 5.13% to 5.53%. This corresponds to approximately 10% of the total increase in unemployment from 2007 to 2010. The results suggest that the prevalence of unsecured credit can explain long-term movements in the efficiency of the labor market, but are not enough, by itself, to explain short term fluctuations as that seen since 2007.

2.1.1 Literature

Pairwise credit in a search-theoretic model was first introduced by Diamond (1987a), Diamond (1987b), and Diamond (1990). The environment is similar to Diamond (1982), where agents are matched bilaterally and trade indivisible goods. The punishment for not repaying a loan is permanent autarky. The role of record-keeping technologies to sustain some forms of credit arrangements has been emphasized by Kocherlakota (1998). Kocherlakota and Wallace (1998) consider the case of an imperfect record-keeping technology where the public record of individual transactions is updated after a probabilistic lag. Nosal and Rocheteau (2011), Chapter 2 describe pure credit economies in quasi-linear environments similar to the one in this paper. Sanches and Williamson (2010) were the first to introduce limited commitment to study the coexistence of money and pairwise credit in the Lagos-Wright environment. Gomis-Porqueras and Sanches (2013) study optimal monetary policy in a version of the Sanches-Williamson model focusing on incentive-feasible schemes where all trades are voluntary, including the ones with the government. Gu *et al.* (2013b) study banking and endogenous credit cycles in this type of environment.

Our paper is also closely related to the literature on unemployment and money, e.g., Shi (1998). Our model has a similar structure as in Berensten *et al.* (2011) that extends the quasi-linear environment of Lagos and Wright (2005) to include a frictional labor market. In Rocheteau *et al.* (2007) only the goods market is subject to search frictions but unemployment emerges due to indivisible labor. In all these models credit is not incentive feasible. In contrast we introduce an imperfect record keeping technology to make unsecured credit incentive feasible and to allow for the coexistence of liquid assets and credit. Liquid assets are formalized via a storage technology as in Lagos and Rocheteau (2008).

There is a related literature studying unemployment and financial frictions that affect the financing of firms. Wasmer and Weil (2004) add a credit market with search-matching frictions in the Mortensen-Pissarides model. They assume that firms search for investors in order to finance the cost of opening a vacancy.

Versions of this model have been calibrated by Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau (2014). Other work by Monacelli *et al.* (2011) study a labor search model in which firms issue debt under limited enforcement. In their framework, debt serves as a mechanism to decrease the bargaining position of workers, thereby lowering the wage bill. A contraction of credit is then followed by an increase in wages, lower entry, and higher unemployment. Our model differs from this literature in that credit frictions are on the side of households, they take the form of limited commitment instead of search frictions between lenders and borrowers, and a decentralized goods market where unsecured credit is formalized explicitly.

Finally, our paper is related to recent work by Herkenhoff (2013) that focuses on the role of revolving credit as a self-insurance mechanism against unemployment shocks. This mechanism delivers a positive aggregate relationship between unemployment and credit as easy credit conditions lead to a smaller consumption decline upon job loss, higher reservation wages, and therefore higher unemployment. In contrast, we focus on household's self insurance against shocks occurring in the goods market.[5]

[5]While we do not assume that the frequency of shocks is linked to one's labor market status, it would be straightforward to allow for such an extension in our model. For instance, suppose the frequency of shocks is larger for the unemployed. Then an increase in credit will increase wages and unemployment. Similarly our model could easily be extended to have ex-post heterogeneity, as in Pissarides (2000), Chapter 6 to explain how credit availability makes households more or less choosy.

2.2 Environment

The set of agents consists of a $[0, 1]$ continuum of households and a large continuum of firms. Time is discrete and goes on forever. Each period of time is divided into three stages. In the first stage, households and firms trade indivisible labor services in a labor market (LM) subject to search and matching frictions. In the second stage, they trade consumption goods in a decentralized retail market (DM) with search frictions. In the last stage, debts are settled, wages are paid, and households and firms can trade assets and goods in a competitive market (CM). We take the good traded in the CM as the numéraire good. The sequence of markets in a representative period is summarized in Figure 2.2.

Figure 2.2: Timing

The lifetime utility of a household is given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[\ell(1-e_t)+\upsilon(y_t)+c_t\right], \tag{2.1}$$

where $\beta = 1/(1+r) \in (0, 1)$ is a discount factor, $y_t \in \mathbb{R}_+$ is the consumption of the DM good, $c_t \in \mathbb{R}$ is the consumption of the numéraire good (we interpret $c < 0$ as production), and $e_t \in \{0, 1\}$ represents the (indivisible) time devoted to work in the first stage, so that ℓ can be interpreted as the utility of leisure or home production.[6] The utility function in the DM, $\upsilon(y_t)$, is twice continuously differentiable, strictly increasing, and concave. Moreover, $\upsilon(0) = 0$ and $\upsilon'(0) = \infty$. (We need υ to be bounded below to have a well-defined bargaining problem in the DM.)

Each firm is composed of one job. In order to participate in the LM at t, a firm must advertise a vacant position, which costs $k > 0$ units of the numéraire good at $t - 1$.[7] The measure of matches between vacant jobs and unemployed households is given by $m(s_t, o_t)$, where s_t is the measure of job seekers and o_t is the measure of vacant firms (job openings). The measure of job seekers in t is equal to the measure of unemployed households at the end of $t - 1$, $s_t = u_{t-1}$. The matching function, m, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $m(0, o) = m(s, 0) = 0$ and $m(s, o) \leq \min(s, o)$. The job finding probability of an unemployed worker is $p_t = m(s_t, o_t)/s_t = m(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. We assume that $\lim_{\theta_t \to +\infty} m(1, \theta_t) = 1$, i.e., the job finding probability approaches one when market tightness goes to infinity. The vacancy filling probability for a firm is $f_t = m(u_t, o_t)/o_t = m(1/\theta_t, 1)$. We assume that $\lim_{\theta_t \to 0} m(1/\theta_t, 1) = 1$, i.e., the vacancy filling probability approaches one when market tightness goes to zero.

[6] One can interpret a negative consumption, $c < 0$, as borrowing across CMs with perfect enforcement. Alternatively, one can make assumptions to guarantee that $c \geq 0$ always holds. For instance, if households receive exogenous endowments at the beginning of each CM, then one can choose the size of these endowments so that consumption is non-negative.

[7] We take the view of a recruiting technology that is not labor intensive so that k does not need to be linked to equilibrium wages.

An existing match is destroyed at a beginning of a period with probability $\delta \in (0, 1)$. A household who is employed in the LM receives a wage in terms of the numéraire good, w, paid in the subsequent CM. A household who is unemployed in the LM receives income in terms of the numéraire good, b, interpreted as unemployment benefits.

Each filled job produces $\bar{z} > 0$ units of goods in the first stage. These $\bar{z}$ units are divided between some (endogenous) amount $y_t \in [0, \bar{z}]$ sold in the DM and the rest, $\bar{z} - y_t$, sold in the CM. This makes y the opportunity cost of selling y in the DM. Let y^* solve $v'(y^*) = 1$. We assume that $y^* \in (0, \bar{z})$. The assumption $v'(y) > 1$ for all $y \in [0, y^*)$ captures the notion that households value the opportunities to consume early in the DM more than CM consumption. Therefore, in line with the banking literature, one can interpret an opportunity to consume in the DM as a liquidity shock.

The DM goods market has a similar structure as the LM in that it involves bilateral random matching between retailers (firms) and consumers (households).[8] The matching probabilities for households and firms are $\alpha(n_t)$ and $\alpha(n_t)/n_t$, respectively, where $n_t = 1 - u_t$ is the measure of participating firms. So $\alpha(n_t)$ is a measure of the frequency of the liquidity shocks in the DM. We assume $\alpha'(n) > 0$, i.e., a tight labor market implies a high frequency of trading opportunities in the

[8] The description of the goods market with search frictions follows the model in Diamond (1990), except that we assume a constant-return-to-scale matching function. The framework has been adopted in the monetary literature since Kiyotaki and Wright (1989). This description has several advantages for our purpose. First, the notion of credit seems more natural in the context of a market with bilateral relationships. Second, search frictions rationalize the presence of unsold goods (inventories). Third, search frictions generate an endogenous frequency for random consumption opportunities that depends on the state of the labor market. This frequency will play an important role in the determination of credit limits.

goods market. Moreover, $\alpha''(n) < 0$, $\alpha(n) \leq \min\{1, n\}$, $\alpha(0) = 0$, $\alpha'(0) = 1$ and $\alpha(1) \leq 1$.

Households in the DM lack commitment. Therefore, firms are willing to extend credit to households only if the repayment of debt in the subsequent CM is self-enforcing.[9] If a household defaults on its debt, such default is recorded publicly with probability $\rho \in [0, 1]$. The parameter ρ can be interpreted as a measure of the sophistication of the financial system.[10] We assume that only a fraction, ω, of households can be monitored and have access to credit. The remaining households cannot borrow.[11] Moreover, as in Wallace (2005), a monitored household can choose to become unmonitored at any point in time.

Finally, there is a technology allowing households to store goods: each unit of the numéraire good invested in the storage technology at t yields $R < 1 + r$ units of the numéraire good at $t+1$.[12] Equivalently, we can think of stored goods as Lucas trees in fixed supply, A, that yield a dividend normalized to one before the CM opens and are traded at the ex-dividend price ϕ in the CM. (See, e.g., Rocheteau and Wright (2013), in a related model.) The gross rate of the return of those Lucas trees is then $R = 1 + 1/\phi$. Stored goods (or claims on these goods)

[9] Even though we assume that firms lend directly to households, one can think of firms as consolidating the roles of a producer and a financial intermediary such as a bank.

[10] Sanches and Williamson (2010) adopt a similar assumption whereby a fraction of sellers has no monitoring potential. Our assumption treats firms/sellers symmetrically. Williamson (2011) captures the imperfection of the record-keeping mechanism by assuming that sellers have access to a buyer's history only with a given probability. Uninformed sellers might find it optimal to extend credit to buyers, which allows for the possibility of default in equilibrium.

[11] The fact that a group of households does not have access to credit can also be an equilibrium outcome even if those households can be monitored. For instance, if firms believe that some households will not repay their debts, then they won't lend to those households, and it becomes optimal for them not to repay their debts since they are already excluded from future credit transactions.

[12] For a similar storage technology, see Lagos and Rocheteau (2008)

are perfectly divisible and portable and can be carried into the DM. If a household has a units of stored goods in the DM, it can transfer up to a fraction $\nu \in [0, 1]$ as means of payment. Alternatively, the household can obtain a collateralized loan to be repaid in the next CM for an amount equal to νRa.[13] Stored goods are consistent with different interpretations, including currency, demand deposits, shares of mutual funds, and even home equity. The opportunity cost of holding liquid assets is $R - (1 + r)$.

2.3 Equilibrium

In the following we focus on steady-state equilibria in which labor market outcomes and loan contracts in the DM are constant over time. We characterize an equilibrium by moving backward from the description of households's choices in the centralized market (CM), to the determination of prices and quantities in the retail goods market (DM), and finally the entry of firms and the determination of wages in the labor market (LM).

2.3.1 Settlement and competitive markets (CM)

Let $W_e(d, a)$ denote the lifetime expected utility of a monitored household in the CM with debt d from the previous DM, in units of the numéraire good, and a units of liquid assets, where $e \in \{0, 1\}$ indicates the labor market status ($e = 0$ if the household is unemployed and $e = 1$ otherwise). We assume that the debt

[13] The partial acceptability of assets due to private information frictions is formalized in Rocheteau (2011), Lester *et al.* (2012), and Li *et al.* (2012). Sanches and Williamson (2010) describe an economy with money and credit. In contrast to our model, engineering a positive return on money requires taxation, and agents can default on their tax liabilities.

issued in the DM is repaid in the CM and is not rolled over across periods.[14] Similarly, let U_e be the household's value function in the LM. Then we have

$$W_e(d,a) = \max_{c,a' \geq 0} \{c + (1-e)\ell + \beta U_e(a')\} \tag{2.2}$$

$$\text{s.t.} \quad c + d + a' = ew + (1-e)b + Ra + \Delta - T. \tag{2.3}$$

The first two terms between brackets in (2.2) are the utility of consumption and the utility of leisure, the third term is the continuation value in the next period. Thus, from (2.2)-(2.3) the household chooses its net consumption, c, and liquid assets, a', in order to maximize its lifetime utility subject to a budget constraint. The left side of the budget constraint, (2.3), is composed of the household's consumption, the repayment of its debt, and its purchase of liquid assets, a'. The right side is the household's income associated with its labor status (w if employed and b if unemployed), the gross return of its beginning-of-period liquid assets (Ra), and the profits of the firms (Δ), minus taxes (T). We substitute c from (2.3) into (2.2) to obtain

$$W_e(d,a) = Ra - d + ew + (1-e)(\ell + b) + \Delta - T + \max_{a' \geq 0} \{-a' + \beta U_e(a')\}. \tag{2.4}$$

From (2.4) the value function of the household in the CM is linear in its wealth, $Ra - d$. Moreover, from the linear preferences in CM, the choice of liquid assets

[14] We can extend our theory to allow for debt to be rolled over across periods as follows. A debt contract is defined by a CM payment, Δ, and a probability, $1-\varrho$, that debt is exogenously extinguished. So ϱ can be interpreted as the probability that debt is revolved. Also, if the household gets hit with a liquidity shock, with probability α, the debt contract is terminated. The expected discounted value of this contract is $(1+r)\Delta / [1 + r - \varrho(1-\alpha)]$. Setting this value equal to d, we obtain the CM payment, $\Delta = [1 - \varrho(1-\alpha)/(1+r)]\, d$.

for the next period, a', is independent of the assets held at the beginning of the period, a.

Consider next the value function of a household with no access to credit—either the household defaulted on its debt in the past and this default was publicly recorded, or it cannot be monitored. We focus on equilibria where the household is excluded from credit permanently. Firms have no incentive to lend to this household as they anticipate that it would default on its loan; and it is rational for the household to default on its loan as it doesn't expect to get any loans in the future. The value of a household with no access to credit is

$$\tilde{W}_e(a) = Ra + ew + (1-e)(\ell + b) + \Delta - T + \max_{a' \geq 0} \left\{ -a' + \beta \tilde{U}_e(a') \right\}, \qquad (2.5)$$

where $\tilde{U}_e(a')$ is the LM value function of a household with no access to credit. We assume and verify later that the wage paid to a household is independent of its access to credit.

Finally, the expected discounted profits of a firm in the CM with x units of inventories, d units of household's debt, a units of liquid assets, and a promise to pay a wage w, are

$$\Pi(x, d, a, w) = x + d + Ra - w + \beta(1-\delta)J. \qquad (2.6)$$

A firm with x units of inventories can sell x units of numéraire good; the d units of household's debt are worth d units of numéraire good (since there will be no default in equilibrium), and the a units of liquid assets are worth Ra units of numéraire good. So $x+d+Ra$ is the real value of the sales made by the firm within a period across the DM and CM. If the firm remains profitable, with probability $1-\delta$, then the expected profits of the firm at the beginning of the next period are J.

2.3.2 Retail goods market (DM)

Consider a match between a firm and a household who holds a units of liquid asset in the DM goods market. A contract is a triple, (y, d, τ), that specifies the output produced by the firm for the household, y, the unsecured debt to be repaid by the household in the next CM, d, and the transfer of liquid assets, τ. The terms of the contract are determined by Kalai (1977) proportional bargaining solution with $\mu \in [0, 1]$ denoting the household's share.[15] This trading mechanism guarantees that the trade is (pairwise) Pareto efficient and it generates an endogenous markup (if $\mu < 1$). The solution is given by:

$$(y, d, \tau) = \arg\max_{y,d,\tau} [\upsilon(y) - d - R\tau] \tag{2.7}$$

$$\text{s.t.} \quad \upsilon(y) - d - R\tau = \frac{\mu}{1-\mu} (d + R\tau - y) \, . \tag{2.8}$$

According to (2.7)-(2.8) the terms of the contract are chosen so as to maximize the household's surplus subject to the constraint that this surplus is equal to $\mu/(1-\mu)$ times the surplus of the firm. The surplus of the household is defined as its utility from consumption, $\upsilon(y)$, net of the payment to the firm, $d + R\tau$. The surplus of

[15] The proportional bargaining solution has several desirable features. First, it guarantees the value functions are concave in the holdings of liquid assets. Second, the proportional solution is monotonic (each player's surplus increases with the total surplus), which means households have no incentive to hide some assets. These results cannot be guaranteed with Nash bargaining (Aruoba, Rocheteau, and Wright (54). Dutta (2012) provides strategic foundations for the proportional bargaining solution.

the firm is defined in a similar way. The problem (2.7)-(2.8) is subject to the debt constraint, $d \leq \bar{d}$, i.e., the household cannot borrow more than a limit, $\bar{d}$, arising from households's lack of commitment, and the feasibility constraint, $\tau \leq \nu a$, i.e., the household cannot transfer more than a fraction of its (partially-)liquid assets. Alternatively, we can think of the household as negotiating a loan of size $d + \nu R\tau$ where only the part $\nu R\tau$ is secured with collateral.

The bargaining problem can be simplified further by substituting $d + R\tau$ from (2.8) into (2.7) to obtain

$$y = \arg\max_{y} \mu \left[\upsilon(y) - y \right] \tag{2.9}$$

$$\text{s.t.} \quad d + R\tau = (1-\mu)\upsilon(y) + \mu y \leq \bar{d} + R\nu a. \tag{2.10}$$

According to (2.10) the transfer of wealth from the household to the firm is a non-linear function, $(1-\mu)\upsilon(y) + \mu y$, of the output produced by the firm. The price of one unit of DM output is $1 + (1-\mu)\left[\upsilon(y) - y\right]/y$, where the second term can be interpreted as the average markup over cost. Given this non-linear pricing rule, output is chosen to maximize the household's surplus, which is a fraction of the total surplus of the match. The solution to the bargaining problem is $y = y^*$ if $(1-\mu)\upsilon(y^*) + \mu y^* \leq \bar{d} + R\nu a$ and $(1-\mu)\upsilon(y) + \mu y = \bar{d} + R\nu a$ otherwise. So provided that the household has enough payment capacity, agents trade the first-best level of output, y^*. If the payment capacity of the household is not large enough, the household borrows up to its limit. If the household does not have access to credit, because it cannot be monitored or has a recorded history of default, then $d = \bar{d} = 0$.

The expected discounted utility of a household in the DM is

$$\begin{aligned} V_e(a) &= \alpha(n)\left[\upsilon(y) + W_e\left(d, a-\tau\right)\right] + \left[1-\alpha(n)\right] W_e(0,a) \\ &= \alpha(n)\mu\left[\upsilon\left(y\right) - y\right] + W_e(0,a), \end{aligned} \tag{2.11}$$

where the terms of trade, (y, d, τ), depend on the household's debt limit and holdings of liquid asset as indicated by the bargaining problem, (2.9)-(2.10). According to the first equality in (2.11), the household is matched with a firm with probability $\alpha(n)$, in which case the household purchases y units of output against d units of debt and τ units of liquid asset. With probability, $1-\alpha(n)$, the household does not have any trading opportunity in the DM, and hence it enters the CM without any debt. The second equality in (2.11) is obtained by using the linearity of W_e. It says that if the household is matched, with probability $\alpha(n)$, then it enjoys a fraction μ of the match surplus. Similarly, the expected lifetime utility of a household with no access to credit is given by

$$\tilde{V}_e(a) = \alpha(n)\mu\left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right] + \tilde{W}_e(a). \tag{2.12}$$

According to (2.12) if the household does not have access to unsecured credit, then it can only spend its liquid assets and consume the quantity, $\tilde{y}$, obtained from the bargaining problem, (2.9)-(2.10), with $\bar{d}=0$. Here, we have assumed that a household who defaulted on its debt in the past can choose to be nonmonitored and therefore cannot be excluded from pure monetary trades.

2.3.3 Labor market (LM)

Households Consider a household who is employed at the beginning of a period. Its lifetime expected utility is

$$U_1(a) = (1-\delta)V_1(a) + \delta V_0(a). \tag{2.13}$$

With probability, $1-\delta$, the household remains employed ($e=1$) and offers its labor services to the firm in exchange for a wage in the next CM. With probability, δ, the household loses its job and becomes unemployed ($e=0$), in which case it will not have a chance to find another job before the next LM in the following period. Substituting V_1 and V_0 by their expressions given by (2.11),

$$U_1(a) = \alpha(n)\mu\left[\upsilon(y) - y\right] + (1-\delta)W_1(0,a) + \delta W_0(0,a). \tag{2.14}$$

The household enjoys an expected surplus in the goods market equal to the first term on the right side of (2.14). The last two terms are the household's continuation values in the CM depending on its labor status.

The expected lifetime utility of a household who is unemployed at the beginning of the period is

$$U_0(a) = (1-p)V_0(a) + pV_1(a), \tag{2.15}$$

where p is the job finding probability. Substituting V_1 and V_0 by their expressions given by (2.11),

$$U_0(a) = \alpha(n)\mu\left[\upsilon(y) - y\right] + (1-p)W_0(0,a) + pW_1(0,a). \tag{2.16}$$

By a similar reasoning the value functions for households with no access to credit, $\tilde{U}_e$, solve

$$\tilde{U}_1(a) = \alpha(n)\mu\left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right] + (1-\delta)\tilde{W}_1(a) + \delta\tilde{W}_0(a) \tag{2.17}$$

$$\tilde{U}_0(a) = \alpha(n)\mu\left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right] + (1-p)\tilde{W}_0(a) + p\tilde{W}_1(a), \tag{2.18}$$

where $\tilde{y}$ is the DM consumption of a household with no access to credit, $\bar{d} = 0$.

Firms Free entry of firms implies that the cost of opening a vacancy must be equal to the probability of filling the vacancy in the next LM times the discounted value of a filled job, $k = \beta f J$ (assuming there is entry), where $J = \mathbb{E}\left[\Pi(x, d, \tau, w)\right]$ is the expected discounted profits of a filled job. It satisfies

$$J = z - w + \beta(1-\delta)J, \tag{2.19}$$

where z is the firm's expected revenue in both the DM and CM expressed in numéraire good,

$$z = \bar{z} + \frac{\alpha(n)}{n}(1-\mu)\left\{\omega\left[\upsilon\left(y\right) - y\right] + (1-\omega)\left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right]\right\}. \tag{2.20}$$

From (2.19) the value of a filled job is equal to the expected revenue of the firm net of the wage plus the expected discounted profits of the job if it is not destroyed, with probability $1-\delta$. When writing the revenue of the firm in (2.20) we have conjectured that the level of output traded in a match is identical across households with the same debt limit irrespective of their labor status. If a firm is successful in selling some of its output in the retail market, with probability $\alpha(n)/n$, then it receives a payment (d, τ) but forgoes y in the CM. Therefore, its received payments increase by $d + R\tau - y$, which from (2.10) is equal to $(1-\mu)\left[\upsilon\left(y\right)-y\right]$. Solving for J we obtain

$$J = \frac{z - w}{1 - \beta(1-\delta)}. \tag{2.21}$$

The value of a job is equal to the discounted sum of the profits where the discount rate is adjusted by the probability of job destruction.

Wage The wage is determined by bargaining between the household and the firm. As is standard in the literature, we adopt Nash/Kalai bargaining as our solution. The wage is set to divide the match surplus according to the following rule, $V_1(a) - V_0(a) = \lambda J/(1-\lambda)$, where $\lambda \in [0,1]$ is the household's bargaining power in the labor market. The firm's surplus, J, is given by (2.21). We conjecture that employed and unemployed workers face the same debt limit and hold the same quantity of assets. As a result, from (2.11) the surplus of a household from being employed, $V_1(a) - V_0(a) = W_1(0,0) - W_0(0,0)$, is independent of the household's asset holdings or borrowing capacity. Therefore, we will assume that the household holds its optimal level of liquid assets and we will omit this argument in the value functions. Skipping some algebra (available on request) the expression for the wage is given by

$$w = \lambda z + (1-\lambda)\left(\ell + b\right) + \lambda\theta k. \tag{2.22}$$

The expression for the wage, (2.22), is identical to the one in Pissarides (2000). The wage is a weighted average of firm's revenue, z, and household's flow utility from being unemployed, $\ell + b$, augmented by a term proportional to firms' average recruiting expenses per vacancy, $\upsilon k/u$. By the same reasoning as above the same wage is paid to households with no access to credit.

Market tightness

The ratio of vacant jobs per unemployed worker is determined by the free-entry condition according to which $k = \beta f J$ where J is given by (2.21). Substituting w by its expression from (2.22) into (2.21) and using that $\beta = 1/(1+r)$,

$$\frac{(r+\delta)\,k}{f} = (1-\lambda)\,(z-\ell-b) - \lambda\theta k. \tag{2.23}$$

If $(r+\delta)k > (1-\lambda)(z-\ell-b)$, (2.23) determines a unique $\theta > 0$ for a given z. The financial frictions affect firms' entry decision through z, their expected revenue. If credit is more limited, then households have a lower payment capacity, sales in the DM goods market, y, fall, which reduces z (provided that $\mu < 1$). As z is reduced, fewer firms find it profitable to enter the market.

2.3.4 Liquidity

A household's optimal holdings of liquid assets is obtained by substituting $U_e(a)$ given by (2.14)-(2.16) into (2.4), i.e., a solves

$$\max_{a\geq 0}\left\{-\,(1-\beta R)\,a + \beta\alpha(n)\mu\left[\upsilon\,(y) - y\right]\right\}, \tag{2.24}$$

where y is the solution to the DM bargaining problem, (2.9)-(2.10), i.e., $(1-\mu)\upsilon(y)+\mu y=\bar{d}+R\nu a$ if the solution is less than y^* and $y=y^*$ otherwise. According to (2.24) households choose their holdings of liquid assets in order to maximize their expected surplus in the DM net of the cost of holding assets, which is approximately equal to the difference between the gross rate of time preference, β^{-1}, and the gross rate of return of liquid assets, R. The problem in (2.24) is independent of the labor status of the household, which establishes that both employed and unemployed households will hold the same quantity of liquid assets conditional on facing the same debt limit.

From the bargaining solution $dy/da=\nu R/\left[(1-\mu)\upsilon'(y)+\mu\right]$ if $\bar{d}+R\nu a<(1-\mu)\upsilon(y^*)+\mu y^*$. Therefore, the first-order condition associated with (2.24) is

$$-(1+r-R)+\alpha(n)\mu\nu R\left[\frac{\upsilon'(y)-1}{(1-\mu)\upsilon'(y)+\mu}\right]\leq 0, \tag{2.25}$$

with equality if $a>0$. The first term on the left side of (2.25) is the opportunity cost of holding liquid assets. The second term on the left side of (2.25) is the liquidity premium of the asset, i.e., the expected marginal benefit from holding liquid assets in the DM. This expected marginal benefit is computed as follows. With probability, $\alpha(n)$, the household has an opportunity to spend its marginal unit of asset in the DM; this marginal unit buys dy/da units of DM output, which is valued at the marginal surplus of the household in a DM meeting, $\mu\left[\upsilon'(y)-1\right]$.

A household with no access to credit solves a similar portfolio problem as in (2.24) where y is replaced with $\tilde{y}$ and $(1-\mu)\upsilon(\tilde{y})+\mu\tilde{y}=R\nu\tilde{a}$. After rearranging terms, it becomes:

$$\max_{\tilde{y}\geq 0}\left\{\left[\alpha(n)\nu\mu-\left(\frac{1+r}{R}-1\right)(1-\mu)\right]\upsilon\left(\tilde{y}\right)-\left[\frac{1+r}{R}-1+\alpha(n)\nu\right]\mu\tilde{y}\right\}. \tag{2.26}$$

Using that $\upsilon'(0)=\infty$, a necessary and sufficient condition for $\tilde{y}>0$ is that the first term between squared brackets in (2.26) is positive, i.e., $\alpha(n)\nu\mu > \left[(1+r)/R-1\right](1-\mu)$. It is optimal for households with no access to credit to hold liquid assets if the holding cost of those assets, $1+r-R$, is not too large. Moreover, the higher the frequency of trades, $\alpha(n)$, the higher the household's bargaining power, μ, and the more likely it is that households will accumulate liquid assets.

2.3.5 Borrowing constraint

Consider a household with debt level, d, and labor status, e, in the CM. The incentive compatibility constraint for the repayment of the household's debt is

$$-d+W_e(0,a)\geq \rho\tilde{W}_e(a)+(1-\rho)W_e(0,a), \tag{2.27}$$

where $\tilde{W}_e$ is the value of a household who is excluded permanently from credit transactions. The left side of (2.27) is the expected lifetime utility of the household if it does not default: the household pays back its debt and enters the CM with a units of liquid asset and future access to credit. The right side is the expected lifetime utility of the household if it defaults. With probability, ρ, the identity of the defaulting household is publicly known, and as a result the household is banned from future credit but it can keep trading with liquid assets (because he can choose to be non-monitored).[16] Its continuation value is $\tilde{W}_e(a)$. If the default is not recorded, with probability $1-\rho$, then the household's public trading history shows no event of default, which allows the household to keep its line of credit. In this case its continuation value is $W_e(0,a)$.

Using the linearity of W_e and $\tilde{W}_e$, from (2.4) and (2.5), the household credit constraint, (2.27), can be reexpressed as

$$d \leq \bar{d} \equiv \rho \left[W_e(0,0) - \tilde{W}_e(0) \right]. \tag{2.28}$$

For repayment to be incentive compatible the household's debt cannot be greater than the expected cost from defaulting, which is equal to the probability of losing access to credit, ρ, times the difference between the lifetime utility of a household with access to credit, W_e, and the lifetime utility of a household with no access to credit, $\tilde{W}_e$. From (2.28) $\bar{d}$ is independent from the quantity of assets held by the household when entering the CM. Using (2.4) and (2.5), the debt limit can be rewritten as

$$\bar{d} = \rho \left\{ \max_{a \geq 0} \left[-a + \beta U_e\left(a\right) \right] - \max_{\tilde{a} \geq 0} \left[-\tilde{a} + \beta \tilde{U}_e\left(\tilde{a}\right) \right] \right\}. \tag{2.29}$$

[16] Our assumption is consistent with the one in Kehoe and Levine (1993) according to which an agent who defaults on a contract cannot be excluded from spot markets trading. It is also consistent with Wallace (2005) who assumes that monitored people who defect can join the ranks of the nonmonitored people and suffer no further punishment Similarly, in Aiyagari and Williamson (2000) an agent who defaults can trade with money in the future. In contrast, Sanches and Williamson (2010) assume that if a buyer defaults, then sellers will refuse to take his money.

From (2.29) the possibility offered to households to self-insure against idiosyncratic shocks in the DM by holding liquid assets will affect debt limits.

In order to characterize the debt limit it is useful to introduce the following two thresholds for the gross rate of return of liquid assets:

$$\bar{R} \equiv \frac{\rho\,(1+r)}{r\nu+\rho} \tag{2.30}$$

$$\underline{R} \equiv \frac{(1-\mu)(1+r)}{\alpha(n)\nu\mu+1-\mu}. \tag{2.31}$$

We show in the next Lemma that $\bar{R}$ is an upper bound for the gross rate of return on liquid assets above which the repayment of credit is not incentive compatible. From (2.26) the threshold, $\underline{R}$, is the gross rate of return below which households do not want to accumulate liquid assets.

Proposition 1. ***(Endogenous debt limit)*** *For given n, the debt limit, $\bar{d}$, is a solution to*

$$r\bar{d} = \Gamma(\bar{d}), \tag{2.32}$$

where

$$\begin{aligned}\Gamma(\bar{d}) \equiv\ & \rho \max_{a\geq 0}\left\{-\left(1+r-R\right)a+\alpha(n)\mu\left[\upsilon\left(y\right)-y\right]\right\} \\ & -\rho \max_{\tilde{a}\geq 0}\left\{-\left(1+r-R\right)\tilde{a}+\alpha(n)\mu\left[\upsilon\left(\tilde{y}\right)-\tilde{y}\right]\right\}.\end{aligned} \tag{2.33}$$

There exists a $\bar{d}>0$ solution to (2.32) if and only if $r<\rho\alpha(n)\mu/(1-\mu)$ (i.e., $\underline{R}<\bar{R}$) and $R\leq\bar{R}$. Moreover, if $R<\bar{R}$, then this solution is unique; if $R=\bar{R}$, then any $\bar{d}\in[0,R\nu\tilde{a}]$ is a solution.

As conjectured earlier, the debt limit is independent of the household's employment status.[17] The determination of the debt limit, $\bar{d}$, is represented in Figure 2.3. The line, $r\bar{d}$, is the return to the household from having access to a line of credit of size $\bar{d}$. The curve, $\Gamma(\bar{d})$, represents the flow cost from defaulting on one's debt if the debt limit for future DM trades is equal to $\bar{d}$. This cost is equal to the probability of being caught, ρ, times the loss from not being eligible for a loan in the future. The first term on the right side of (2.33) is the expected DM surplus of a household with access to credit net of the cost of holding liquid assets. The second term gives a similar expression for households with no access to credit. The punishment from defaulting increases with the size of the credit line, i.e., Γ is upward sloping. Moreover, $\Gamma(0) = 0$. If a household anticipates that it will not have access to credit in the future, then there is no cost from defaulting. As a consequence, there always exists an equilibrium with no unsecured credit.[18]

For a further characterization of Γ we distinguish two cases. First, if the size of the credit line, $\bar{d}$, is less than the payment capacity of a household with no access to credit, $R\nu\tilde{a}$, then Γ is linear. Indeed, as shown in the bottom panels of Figure 2.3, households with a credit limit of $\bar{d} < R\nu\tilde{a}$ choose asset holdings so as equalize their payment capacity with the one of households with no credit line, $R\nu a + \bar{d} = R\nu\tilde{a}$. This result comes from the observation that both types of households face the same trade-off at the margin, captured by (2.25), in terms of the cost and benefit from holding liquid assets. As a result, the cost from defaulting is equal to the probability, ρ, times the quantity of liquid assets that the household has to accumulate to replace the credit line, $\tilde{a} - a = \bar{d}/R\nu$, where

[17]This does not rule out the existence of other equilibria where $\bar{d}$ would be a function of e.
[18]For a similar result, see Sanches and Williamson (2010).

the cost of holding one unit of liquid asset is equal to $1 + r - R$. Hence, the slope of Γ is $\rho(1 + r - R)\bar{d}/R\nu$.

Second, households with a credit limit, $\bar{d} > R\nu\tilde{a}$, find it optimal to hold no liquid assets (from (2.25)), and Γ is strictly concave (provided that $\bar{d}$ is not too large). To see this, notice from (2.10) that $\partial y/\partial\bar{d} = 1/\left[(1-\mu)\upsilon'(y) + \mu\right]$, and hence

$$\frac{\partial\left[\upsilon(y) - y\right]}{\partial\bar{d}} = \frac{\upsilon'(y) - 1}{(1-\mu)\upsilon'(y) + \mu} = \frac{1}{1 - \mu + 1/\left[\upsilon'(y) - 1\right]}, \tag{2.34}$$

which is decreasing in y when $y < y^*$. So the surplus from a DM trade, $\upsilon(y) - y$, is strictly concave in $\bar{d}$.

For unsecured debt to emerge the slope of Γ at $\bar{d} = 0$ must be greater than r. The expression for $\Gamma'(0)$ depends on whether households with no access to credit find it optimal to hold liquid assets. If $R \leq \underline{R}$, then households with no access to credit choose not to accumulate liquid assets, $\tilde{a} = 0$. In that case, from (2.33) and (2.34) $\Gamma'(0) = \rho\alpha(n)\mu/(1-\mu)$, and hence a necessary condition for credit to be sustainable is $r < \rho\alpha(n)\mu/(1-\mu)$, as indicated in Proposition 1. Households must be sufficiently patient and care enough about the future punishment in case of default for the repayment of debt to be self-enforcing. The threshold for the rate of time preference below which unsecured credit is incentive compatible increases with the probability of being punished in case of default, ρ, with the frequency of liquidity shocks, α, and with the household's market power in the DM, μ.

If $R > \underline{R}$, households with no access to credit accumulate liquid assets, $\tilde{a} > 0$. This possibility of self-insurance lowers the cost of defaulting, and hence the condition for credit to be incentive incompatible is more stringent. In that case

$\Gamma'(0) = \rho(1+r-R)/R\nu$ so that $r < \Gamma'(0)$ can be expressed as $R < \bar{R}$. So unsecured credit can be sustained in equilibrium if the rate of return of liquid assets is not too close to the rate of time preference, $R/(1+r) < \rho/(r\nu+\rho)$. Graphically, the curve representing Γ intersects the curve representing $r\bar{d}$ from above. See the left panel of Figure 2.3. In contrast, if R is greater than $\bar{R}$, then the cost of defaulting is too small to sustain unsecured credit and Γ is located underneath the line $r\bar{d}$. See right panel of Figure 2.3. Finally, there is a knife-edge case where $r\bar{d}$ and $\Gamma(\bar{d})$ coincide so that there are a continuum of debt limits, $\bar{d} \in [0, R\nu\tilde{a}]$. For any $\bar{d} \in [0, R\nu\tilde{a}]$, the flow value of credit is $r\bar{d}$ and the flow cost from defaulting is $r\bar{d}$, which makes the debt limit indeterminate.

The following Corollary provides a condition on the rate of return of liquid assets for the coexistence of credit and money.

Corollary 1. ***(Coexistence of money and credit)*** *A necessary condition for the coexistence of money and credit is $R \in (\underline{R}, \bar{R}]$. If $R > \bar{R}$, then unsecured credit is not incentive compatible. If $R < \underline{R}$, then agents do not hold liquid assets.*

As shown in Proposition 1 if $R > \bar{R}$, then unsecured credit is not incentive compatible and, as a result, all households choose the same holdings of liquid assets, $a = \tilde{a}$. The economy in this case corresponds to a pure monetary economy. Households' payment capacity, $R\nu a = R\nu\tilde{a}$, increases with R. See the upward-sloping dashed lines in Figure 2.4 for all $R > \bar{R}$. When R is exactly equal to $1+r$, there is no cost of holding liquid assets (which is equivalent to a Friedman-rule outcome) and households accumulate enough liquidity to finance y^*, i.e., $R\nu a \geq (1-\mu)\upsilon(y^*) + \mu y^*$. In Figure 2.4 the dashed lines become vertical at $R = 1+r$.

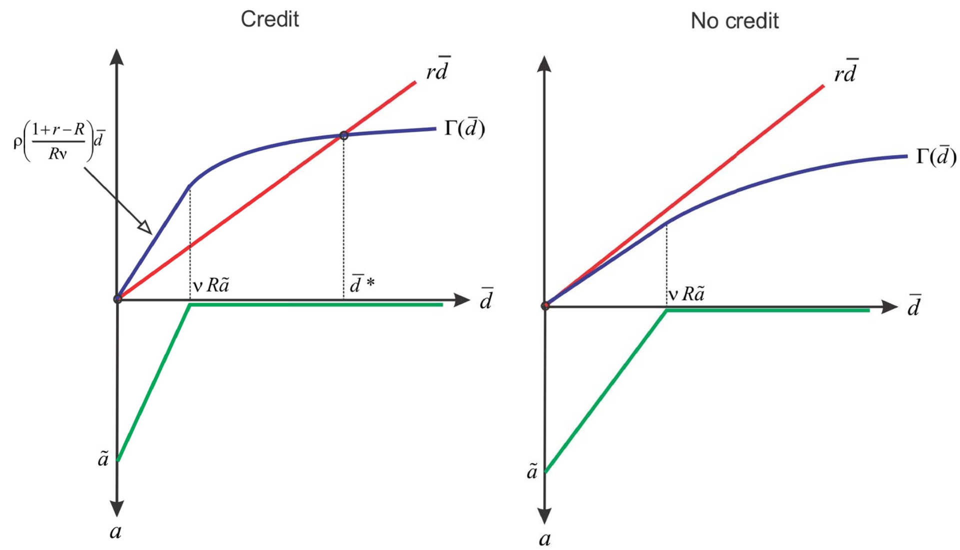

Figure 2.3: Debt Limit

If $R < \bar{R}$, then unsecured credit is incentive compatible. As R increases, the punishment from being excluded from credit is lower since defaulters can accumulate liquid assets at a lower cost. Graphically, the curve representing Γ in Figure 2.3 shifts downward and the debt limit decreases. In Figure 2.4, the plain line representing $\bar{d}$ is downward sloping. For all $R \in \left(\underline{R}, \bar{R}\right)$ there is coexistence of money and credit: some households pay with liquid assets only while other households pay with credit only.[19] Moreover, households with access to credit have a larger payment capacity than households who trade with liquid assets only

[19]In order to have households hold money and use credit one could assume that there is a probability that a firm in the retail market is not able to identify a consumer and therefore cannot extend credit for him. For a related assumption, see Sanches and Williamson (2010).

(graphically, the plain curve is located above the dashed one). Finally, from (2.26) if $R < \underline{R}$, then the cost of holding liquid assets is so high that even households with no access to credit choose not to hold any liquidity. In Figure 2.4 the dashed lines are horizontal at 0. The economy is a pure credit economy.

The following Corollary shows how the aggregate state of the labor market affects credit limits.

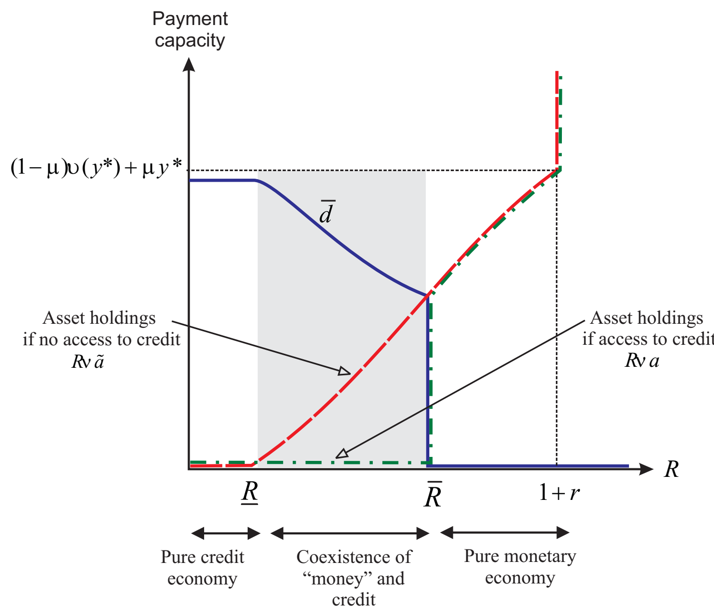

Figure 2.4: Debt Limit vs. Rate of Return

Corollary 2. ***(Credit limits and the labor market)*** *Assume $r < \rho\alpha(n)\mu/(1-\mu)$ and $R < \bar{R}$. The solution, $\bar{d} > 0$, to (2.32) is increasing with n.*

The state of the labor market affects debt limits through the frequency of consumption opportunities in the DM goods market, α. If employment is high, then there are a large number of firms participating in the retail market and, as a result, households have frequent trading opportunities. Such frequent trading opportunities imply that the cost from defaulting on one's debt is high, and therefore the debt limit is high. In Figure 2.3, as n increases, the curve representing Γ moves upward for all $\bar{d} > R\nu\tilde{a}$.

2.3.6 Steady states with unsecured credit

We consider steady-state equilibria with unsecured debt, $\bar{d} > 0$. As we have shown above, in any such equilibrium households who have access to credit will not accumulate liquid assets, $a = 0$ (except in the knife-edge case where $R = \bar{R}$), because their payment capacity is greater than the one of households with no access to credit, $y > \tilde{y}$, where

$$(1-\mu)\upsilon(y) + \mu y = \min\left[(1-\mu)\upsilon(y^*) + \mu y^*, \bar{d}\right]. \tag{2.35}$$

From (2.25) the choice of liquid assets by households with no access to credit solves

$$\begin{aligned} \alpha(n)\mu\nu\left[\frac{\upsilon'(\tilde{y}) - 1}{(1-\mu)\upsilon'(\tilde{y}) + \mu}\right] &= \frac{1+r-R}{R} \text{ if } \frac{1+r-R}{R} < \frac{\alpha(n)\nu\mu}{1-\mu} \\ \tilde{y} &= 0 \text{ otherwise,} \end{aligned} \tag{2.36}$$

where

$$\tilde{a} = \frac{(1-\mu)\upsilon(\tilde{y}) + \mu\tilde{y}}{\nu R}. \tag{2.37}$$

The steady-state level of unemployment in the LM, u, is such that the flow in unemployment is equal to the flow out of unemployment, i.e., $pu = \delta(1-u)$, which gives

$$u = \frac{\delta}{m(1,\theta)+\delta}. \tag{2.38}$$

From (2.20) and (2.23), labor market tightness (assuming it is positive) solves

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\theta},1\right)} + \beta\lambda\theta k = (1-\lambda)\left\{\frac{\alpha(1-u)}{1-u}(1-\mu)\left\{\omega\left[\upsilon\left(y\right)-y\right] + (1-\omega)\left[\upsilon\left(\tilde{y}\right)-\tilde{y}\right]\right\} + \bar{z}-\ell-b\right\}. \tag{2.39}$$

Definition 3. *A steady-state equilibrium with unsecured credit is a list* $(u,\theta,y,\tilde{y},\tilde{a},\bar{d})$ *that solves (2.32)-(2.39) with* $\bar{d}>0$.

Assuming $\bar{z} > \ell + b + \frac{(r+\delta)k}{1-\lambda}$, let $\theta_0 > 0$ be the unique solution to

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\theta_0},1\right)} + \beta\lambda\theta_0 k = (1-\lambda)\left(\bar{z}-\ell-b\right).$$

Denote $n_0 = m(1,\theta_0)/\left[m(1,\theta_0)+\delta\right]$ the associated level of employment. The following proposition provides sufficient conditions for the existence of an equilibrium with credit.

Proposition 2. ***(Existence of an equilibrium with unsecured credit)*** *If* $\bar{z} > \ell + b + (r+\delta)k/(1-\lambda)$, $R < \bar{R}$, *and* $r < \rho\alpha(n_0)\mu/(1-\mu)$, *then there exists a steady-state equilibrium with unsecured credit,* $\bar{d} > 0$.

A sufficient condition for the labor market to be active is that labor productivity is sufficiently high relative to workers' utility of leisure and unemployment benefits. This is a standard existence condition for an active equilibrium in the MP model. For unsecured credit to emerge, it must also be the case that the rate of return on liquid assets is not too high since otherwise the threat of exclusion from credit transactions would not be strong enough to sustain households' incentives to repay unsecured credit.

Notice that Proposition 2 only gives sufficient conditions for existence. An equilibrium might exist even if $\bar{z} < \ell + b + (r + \delta)k/(1 - \lambda)$ because effective productivity, z, is endogenous and depends on the availability of credit. Moreover, in contrast to the MP model, the equilibrium might not be unique because of the complementarity between the credit limit in the goods market and aggregate employment. We will illustrate this last point in the following section.

Finally, if we want to interpret liquid assets as Lucas trees in fixed supply we add the following market-clearing condition,

$$\phi A = \omega a + (1 - \omega)\tilde{a}. \tag{2.40}$$

The asset price, ϕ, is endogenous and it determines the gross rate of return on liquid assets, $R = 1 + 1/\phi$. Equivalently, one can rewrite the asset-market clearing condition as $A = (R - 1)[\omega a + (1 - \omega)\tilde{a}]$.

2.4 Two limiting economies

Before we turn to a quantitative exploration of the model we will characterize the set of equilibria and some comparative statics for two limiting economies: (i) A pure monetary economy where households do not have access to unsecured debt

($\omega = 0$ or $\rho = 0$); (ii) A pure credit economy where all households have access to unsecured credit ($\omega = 1$) but there is no liquid asset ($R = 0$ or $\nu = 0$).

2.4.1 Pure monetary economies

Suppose that there is no record keeping technology that can keep track of households' defaults ($\omega = 0$ or $\rho = 0$). Because of lack of commitment, credit is not incentive feasible as households would always renege on their debts. Therefore, households must hold liquid assets in order to trade in the DM. We assume that households can transfer all their liquid assets in a match, $\nu = 1$.[20]

A steady-state monetary equilibrium can be reduced to a pair, $(\tilde{y}, \theta)$, that solves (2.36) and (2.39) with $\omega = 0$ and $n = 1 - u$, i.e.,

$$\frac{1+r}{R} = 1 + \alpha(n)\mu\left[\frac{\upsilon'(\tilde{y}) - 1}{(1-\mu)\upsilon'(\tilde{y}) + \mu}\right] \tag{2.41}$$

$$\frac{(r+\delta)k}{m\left(\frac{1}{\theta}, 1\right)} + \beta\lambda\theta k = (1-\lambda)\left\{\frac{\alpha(n)}{n}(1-\mu)\left[\upsilon(\tilde{y}) - \tilde{y}\right] + \bar{z} - \ell - b\right\}, \tag{2.42}$$

where $n(\theta) = m(1,\theta)/[m(1,\theta)+\delta]$. We study each equilibrium condition in turn.

Let us start with the equilibrium condition for the choice of liquid assets, (2.41). If $\alpha\mu/(1-\mu) \leq (1+r-R)/R$, where α is evaluated at $n = 1/(1+\delta)$, then for all $\theta \geq 0$ there is no $\tilde{y} > 0$ solution to (2.41). So a necessary condition for a monetary equilibrium to exist is that households have enough bargaining power

[20] This limiting economy is analogous to the pure currency economy studied in Berensten *et al.* (2011) except that the medium of exchange takes the form of capital in our model, as in Lagos and Rocheteau (2008), instead of fiat money.

in the DM,

$$\mu > \frac{1+r-R}{1+r-R\left[1-\alpha\left(\frac{1}{1+\delta}\right)\right]}. \tag{2.43}$$

Assume (2.43) holds and define $\underline{\theta}> 0$ as the solution to

$$\frac{\mu\alpha\left[n(\underline{\theta})\right]}{1-\mu} = \frac{1+r-R}{R}. \tag{2.44}$$

According to (2.41) and (2.44), for all $\theta \leq \underline{\theta}$, $\tilde{y} = 0$ and, for all $\theta > \underline{\theta}$, $\tilde{y}$ is an increasing function of θ. Indeed, as $n(\theta)$ increases, the frequency of trading opportunities in the DM increases, which raises the non-pecuniary return of liquid assets and gives households higher incentives to accumulate them. As market tightness tends to infinity, $n(\theta)$ approaches $1/(1+\delta)$, and y approaches some upper bound, $\bar{y}$. Finally, as R tends to $1+r$, i.e., the cost of holding liquid assets approaches 0, $\underline{\theta}$ tends to 0 and $\tilde{y}$ tends to y^* for all θ. Indeed, if liquidity is costless, all households want to hold $Ra \geq \mu y^* + (1-\mu)\upsilon(y^*)$ irrespective of the frequency at which trading opportunities in the DM occur. In the left panel of Figure 2.6 we represent the equilibrium condition, (2.41), by LD (demand for liquidity).

Let us turn to the vacancy supply condition, (2.42). First, we assume that

$$(1-\lambda)\left\{(1-\mu)\left[\upsilon\left(y^*\right)-y^*\right]+\bar{z}-\ell-b\right\} > (r+\delta)k. \tag{2.45}$$

Condition (2.45) guarantees that if households have enough liquid assets to buy the first-best level of output in the DM, $\tilde{y} = y^*$, then the labor market is active, $\theta > 0$. If, in addition, $(1-\lambda)(\bar{z}-\ell-b) < (r+\delta)k$, then the labor market is active only if there is a minimum amount of trade in the DM. Define $\underline{y} \in (0, y^*)$

as the solution to

$$(1-\lambda)\left\{(1-\mu)\left[\upsilon\left(\underline{y}\right)-\underline{y}\right]+\bar{z}-\ell-b\right\}=(r+\delta)k. \tag{2.46}$$

For all $y \leq \underline{y}$, $\theta=0$; for all $y>\underline{y}$, $\theta>0$ is an increasing function of y for all $y<y^{*}$, and it reaches a maximum at $y=y^{*}$. Under these conditions there is an inactive equilibrium with $\tilde{y}=\theta=0$, and if an active equilibrium exists, then there is an even number of active equilibria (except for degenerate cases where the equilibrium is unique). In the left panel of Figure 2.6 we represent the equilibrium condition, (2.42), by VS (vacancy supply).

If $(1-\lambda)(\bar{z}-\ell-b)>(r+\delta)k$, then the firm's productivity is high enough to cover its entry cost even if the DM is inactive. Denote $\underline{\underline{\theta}}>0$ the solution to (2.42) with $\tilde{y}=0$, i.e.,

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\underline{\underline{\theta}}},1\right)}+\beta\lambda k\underline{\underline{\theta}}=(1-\lambda)\,(\bar{z}-\ell-b)\,. \tag{2.47}$$

This is the market tightness that would prevail if the DM shuts down. Any solution to (2.41)-(2.42) is such that $\theta\geq\underline{\underline{\theta}}$. Let $\hat{R}<1+r$ be the solution to $\underline{\underline{\theta}}=\underline{\theta}(\hat{R})$. If $R>\hat{R}$, then $\underline{\underline{\theta}}>\underline{\theta}(R)$ and any equilibrium is such that $\tilde{y}>0$. We recapitulate our characterization of the pure monetary economy in the following proposition.

Proposition 3. ***(Pure monetary equilibrium)*** *Consider an economy with no unsecured credit ($\omega=0$ and/or $\rho=0$) and suppose (2.43) and (2.45) hold.*

1. *If* $\bar{z} - \ell - b < (r+\delta)k/(1-\lambda)$*, then there is an inactive equilibrium,* $(\theta, \tilde{y}) = (0,0)$*. If* R *is sufficiently close to* $1+r$*, there are also an even number of equilibria with both an active DM,* $\tilde{y} > 0$*, and an active LM,* $\theta > 0$*.*

2. *If* $\bar{z} - \ell - b > (r+\delta)k/(1-\lambda)$*, then all equilibria have an active LM,* $\theta > 0$*. Moreover, if* $R > \hat{R}$*, then all equilibria have an active DM.*

The left panel of Figure 2.6 provides a graphical representation of part 1 of Proposition 3. There is an inactive equilibrium and two active equilibria. The logic for the multiplicity of equilibria is based on strategic complementarities between firms' entry decision and households' choice of liquid assets.[21] If households accumulate a lot of liquid assets, then firms' expected profits in the DM are high and a lot of firms participate. But if aggregate employment is high, the frequency of consumption opportunities is also high, and households have incentives to accumulate a large quantity of liquid assets. By the same logic, there is an equilibrium with a small number of firms and with few liquid assets held by households. So the aggregate liquidity in the economy and unemployment are negatively correlated across equilibria.

From part 2 of Proposition 3 we learn that the labor market is active in any equilibrium provided that firms' productivity is sufficiently high. Moreover, the retail goods market will also be active if the rate of return of liquid assets is sufficiently close to the rate of time preference.

Finally, we can study the effects of monetary policy on aggregate liquidity and the labor market, where monetary policy is described by the choice of R.

[21] For a similar result in the context of an economy with fiat money and free entry of sellers, see Rocheteau and Wright (2005).

From (2.41) as the rate of return on liquid assets increases, households increase their asset holdings for a given θ. Graphically, in Figure 2.6, the curve LD moves to the right. At the highest equilibrium both market tightness and DM output increase. Consequently, a higher interest rate leads to lower unemployment and higher output.

2.4.2 Pure credit economies

We now assume that all households have access to credit, $\omega = 1$, and there is no liquid asset, $R = 0$ (goods cannot be stored) or $\nu = 0$ (capital goods are not portable or cannot be transferred in the DM, let say, because they can be counterfeited at no cost). From Proposition 1 the credit limit is a solution to $r\bar{d} = \rho\alpha(n)\mu\left[\upsilon(y) - y\right]$ where $(1-\mu)\upsilon(y) + \mu y = \min\left[(1-\mu)\upsilon(y^*) + \mu y^*, \bar{d}\right]$. If the credit limit binds, then y solves

$$y = \left\{1 - \frac{r}{\left[r + \rho\alpha(n)\right]\mu}\right\}\upsilon(y). \tag{2.48}$$

Provided that $r < \rho\alpha\mu/(1-\mu)$, then there is a unique $y > 0$ solution to (2.48). If the solution to (2.48) is greater than y^*, then $\bar{d} > (1-\mu)\upsilon(y^*) + \mu y^*$ and $y = y^*$. Market tightness is determined by

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\theta}, 1\right)} + \beta\lambda\theta k = (1-\lambda)\left\{\frac{\alpha(n)}{n}(1-\mu)\left[\upsilon\left(y\right) - y\right] + \bar{z} - \ell - b\right\}. \tag{2.49}$$

We can reduce an equilibrium to a pair, $(\theta, y) \in \mathbb{R}_+ \times [0, y^*]$, that solves the conditions above.

We first describe an equilibrium where households' borrowing constraint in the DM is not binding, which implies $y = y^*$. It requires the right side of (2.48) to be greater than the left side when both are evaluated at $y = y^*$, i.e.,

$$\left[\frac{\rho\alpha(n^*) - r(1-\mu)/\mu}{r + \rho\alpha(n^*)}\right]\frac{\upsilon(y^*)}{y^*} \geq 1, \tag{2.50}$$

where $n^* = n(\theta^*)$ is the employment level given by (2.49) when $y = y^*$. Assuming (2.45) holds, $\theta^* > 0$ and $n^* > 0$. From (2.50) credit is abundant if the match surplus in the DM is sufficiently large, if households have sufficient market power in the goods market (i.e., μ is high), and if the financial system is sufficiently sophisticated (i.e., ρ is high). A necessary condition for the debt constraint not to bind is $\rho\alpha\left(n^*\right) > r(1-\mu)/\mu$. In Figure 2.5, we describe a case with multiple equilibria where the high equilibrium is such that the debt limit is not binding. Notice that the curve representing the credit limit as a function of market tightness, labelled CL, becomes vertical when θ is above a threshold. Indeed, for high values of θ, $\bar{d} > (1-\mu)\upsilon(y^*) + \mu y^*$ and households have enough borrowing capacity to finance the purchase of y^*. In this case the CL curve intersects the vacancy-supply curve, VS, at its maximum.

We now study equilibria where borrowing constraints do bind. We analyze each equilibrium condition in turn starting with the condition for the DM output level, (2.48). Define $\underline{\theta} < \infty$ as the solution to $\rho\alpha(n)\mu = r(1-\mu)$, i.e.,

$$\frac{m(1,\underline{\theta})}{m(1,\underline{\theta}) + \delta} = \alpha^{-1}\left[\frac{r}{\rho}\left(\frac{1-\mu}{\mu}\right)\right], \tag{2.51}$$

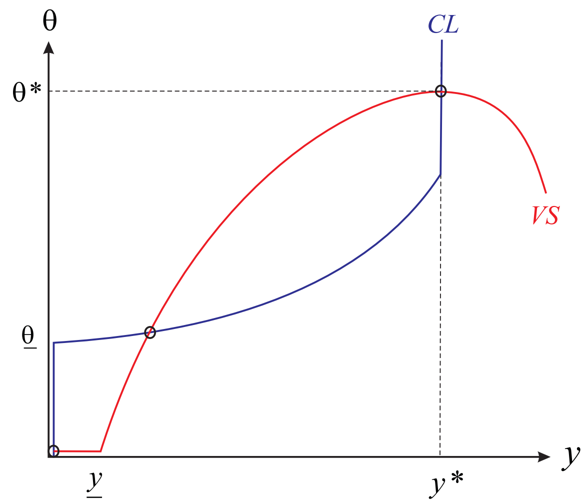

Figure 2.5: Multiple equilibria in which the debt limit at the high steady state is not binding.

if $r < \rho\alpha\left[1/(1+\delta)\right]\mu/(1-\mu)$, and $\underline{\theta}= +\infty$ otherwise. For all $\theta \leq \underline{\theta}$, $y = 0$. Below a threshold for market tightness, unsecured credit cannot be sustained. Above $\underline{\theta}$, y increases with θ because $\alpha(n)$ is an increasing function of θ. As θ tends to infinity, y approaches $\bar{y}$, where $\bar{y}$ solves (2.48) with $n = 1/(1+\delta)$. In Figure 2.6 we represent the equilibrium condition, (2.48), by a curve labelled CL (credit limit).

Let us turn to the vacancy-supply condition, (2.49), represented by the curve labelled VS in Figure 2.6. It gives a positive relationship between θ and y for all $y \leq y^*$. Intuitively, if households have a higher payment capacity, they buy more output in the DM, and firms have more incentives to participate. Assume $(1-\lambda)(\bar{z}-\ell-b) < (r+\delta)k$. For all $y \leq \underline{y}$, where $\underline{y}$ is defined in (2.46), $\theta = 0$. Under this condition, there is an inactive equilibrium with $\theta = y = 0$ and an even number of equilibria (possibly zero) across which $\theta > 0$ and $y > 0$ are positively correlated. If $(1-\lambda)(\bar{z}-\ell-b) > (r+\delta)k$, then $\theta > 0$ for all $y \geq 0$. In this case any equilibrium has an active labor market, even if the DM is inactive. Define $\underline{\underline{\theta}} > 0$ as the solution to (2.47), i.e., the market tightness when the DM is inactive, and

$$\hat{\rho}(\mu) = \frac{r(1-\mu)}{\mu\alpha\left[\frac{m(1,\underline{\underline{\theta}})}{m(1,\underline{\underline{\theta}})+\delta}\right]} \tag{2.52}$$

$$\hat{\mu} = \frac{r}{r+\alpha\left[\frac{m(1,\underline{\underline{\theta}})}{m(1,\underline{\underline{\theta}})+\delta}\right]}. \tag{2.53}$$

From (2.52) the quantity, $\hat{\rho}(\mu)$, is defined as the level of monitoring such that $\underline{\theta} = \underline{\underline{\theta}}$. For all $\rho > \hat{\rho}(\mu)$, credit can be sustained if $\theta = \underline{\underline{\theta}}$. From (2.53) the threshold, $\hat{\mu}$, is defined such that $\hat{\rho}(\hat{\mu}) = 1$. Therefore, for all $\mu > \hat{\mu}$ and $\rho > \hat{\rho}(\mu)$, $\underline{\theta} < \underline{\underline{\theta}}$ and any equilibrium has both an active labor market and an active retails goods market since $\theta > \underline{\underline{\theta}} > \underline{\theta}$.

Proposition 4. ***(Pure credit equilibrium)*** *Suppose (2.45) holds.*

1. *If (2.50) holds, then there exists an equilibrium with $y = y^*$ and $\theta = \theta^*$.*

2. *If $\bar{z}-\ell-b < (r+\delta)k/(1-\lambda)$, then there is an inactive equilibrium with $(\theta, y) = (0,0)$. If r is sufficiently close to 0, there are also an even number of active equilibria with both an active DM, $y > 0$, and an active LM, $\theta > 0$.*

3. *If $\bar{z}-\ell-b > (r+\delta)k/(1-\lambda)$, then any equilibrium has an active LM, $\theta > 0$. Moreover, if $\mu > \hat{\mu}$ and $\rho > \hat{\rho}(\mu)$, then all equilibria have an active DM, $y > 0$.*

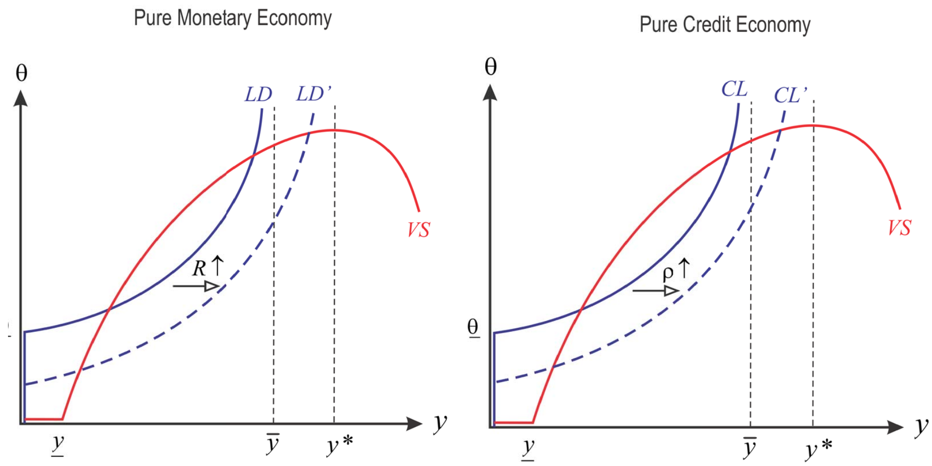

Figure 2.6: Limiting economies and multiple equilibria

In Figure 2.6 we represent the determination of an equilibrium under the condition in Part 2 of Proposition 4. There is an inactive equilibrium where households do not have access to credit and firms do not participate in the labor market, and two active equilibria. Credit limits and unemployment rates are negatively correlated across equilibria. Intuitively, if there are a lot of firms in the retail market, then the punishment for not repaying one's debt, i.e., the exclusion from the DM market, is large since households would have to forgive a large number of trading opportunities. As a result, households can borrow a large amount and firms can expect large profits in the DM. In Figure 2.5 the high equilibrium is such that liquidity is abundant in the sense that borrowing constraints are not binding. By a symmetric logic there is also an equilibrium with a high unemployment rate and a low credit limit. So our model provides a theory of high unemployment due to a self-fulfilling credit crunch.

As in the case of a pure monetary economy, the labor market is active in any equilibrium provided that productivity is sufficiently high. Moreover, the retail goods market is active in any equilibrium if the record-keeping technology is sufficiently effective and households are sufficiently patient. An improvement in the sophistication of the financial system (i.e. an increase in ρ) affects the credit limit condition (2.48) indicated as a rightward shift in the CL' curve in Figure 2.6. As ρ increases, the expected loss from default is higher which tightens household's incentive compatibility constraint for all levels of unemployment and labor market tightness. This leads to higher levels of DM trade, increasing labor market tightness, and a decrease in steady state unemployment.

2.5 Quantitative Analysis

We now turn to a quantitative evaluation of the theory. To start, the model is calibrated to match features of the U.S. economy between 2000 and 2008, characterized by high levels of unsecured credit and low unemployment. We then exogenously reduce the availability of credit in accordance with the observed data between 1978 and 1986 in order to evaluate to what extent the expansion of credit can explain the long-term decrease in unemployment from the mid 1980s to the early 2000s. We also analyze a similar experiment in which we reduce unsecured credit in accordance with the observed data in 2011 and evaluate its contribution to the high levels of unemployment during the Great Recession.

2.5.1 Calibration

The model period is chosen to be a month. Accordingly, we set $r = .003$, implying $\beta = 0.997$. All empirical targets represent monthly averages over the time period 2000-2008.

Labor market. The calibration of the labor market follows Shimer (2005) and Berensten *et al.* (2011) in the context of a monetary model. The matching function is Cobb-Douglas, $m(s, o) = As^{\eta}o^{1-\eta}$. We set the elasticity with respect to job seekers to $\eta = 0.5$, in the middle of the suggested range from empirical surveys (see, e.g., Petrolongo and Pissarides, 2001). The households' bargaining power corresponds to an egalitarian solution, $\lambda = 0.5$. It is also the value consistent with constrained-efficiency (Hosios, 1990) in an economy with non-binding borrowing constraints.

The matching efficiency parameter, A, the job destruction rate, δ, and the vacancy posting cost, k, are jointly determined by targeting the job finding probability, the unemployment rate, and labor market tightness. The unemployment rate comes from the Current Population Survey (CPS) and averaged 5.1% between 2000-2008, $u = 0.051$. We estimate the job finding probability according to $p_t = 1 - (U^L_{t+1}/U_t)$, where U^L is the number of workers unemployed over 5 weeks and U is the total number of unemployed workers from the CPS.[22] We find that $p = 0.36$. Labor market tightness is given by the Job Openings and Labor Turnover Survey (JOLTS).[23] From 2000-2007, there were 0.523 job openings for

[22] See Shimer (2012) for the structural foundation of this equation.

[23] It is standard to normalize either the measure of vacancies or labor market tightness to one (see Shimer (2005) or Berensten *et al.* (2011)). Since our time period allows the use of

every unemployed worker, $\theta = 0.51$. Using the moments above, labor market matching efficiency is given by $A = p\theta^{\eta-1} = 0.50$. From (2.38) the job destruction rate is $\delta = pu/(1-u) = 0.019$. Finally, k is given by the vacancy supply condition, (2.49), which yields vacancy costs as a percentage of monthly wages to be 9.2%.

The remaining two parameters associated with the labor market are the value of leisure, ℓ, and unemployment income, b. We interpret b to represent unemployment benefits in the US and so we set the replacement ratio to one half, i.e., $b = 0.5w$. The value of leisure, ℓ, is an important parameter to pin down the response of equilibrium unemployment to productivity. Shimer (2005) sets $\ell = 0$ and finds that the unemployment rate is not responsive enough to changes in productivity. Alternatively, Hagedorn and Manovskii (2008) show that if this parameter is calibrated to match hiring costs, then $b + \ell$ should be set to 95% of wages, which results in the response of unemployment to productivity that is more in line with data. Since the effects of credit on unemployment in our model are channeled through changes in productivity, z, we follow Hagedorn and Manovskii (2008) and set $b + \ell/w = .95$, or $\ell = 0.45w$.[24]

Credit and Goods Market The DM matching function takes the form $\alpha(n) = \epsilon\sqrt{n}$, which corresponds to a Cobb-Douglas specification where households and firms have an equal contribution to the matching process. The household's bar-

JOLTS, we instead use some measure of vacancies to job seekers. This choice does not affect the quantitative results.

[24] An important difference between our methodology and that of Hagedorn and Manovskii (2008) is that we let unemployment benefits b vary with wages in the experiments we consider. Doing so dampens the response of unemployment to productivity changes and is more in line with the movements in the two variables over the long run.

gaining weight, μ, is set to match the retail markup of 40% as discussed in Faig and Jerez (2005).

We focus on revolving unsecured credit, which primarily consists of credit and charge cards, as debt limits for such loans are observable in the data. To target the eligibility of households to unsecured credit, we choose ω to represent the fraction of US households that had a non-zero debt limit. The Survey of Consumer Finances (SCF) reports that the average percentage of households having at least one unsecured credit account with a non-zero limit. Over 2000-2007, this averaged 74.7%. Therefore, $\omega = 0.747$.

The efficiency parameter of the DM matching function, ϵ, is chosen to match the average credit utilization rate, defined as the fraction of the unsecured debt limit outstanding. In the model, average debt outstanding is given by $\omega\alpha(n)\bar{d}$ while the average debt limit is $\omega\bar{d}$, which implies a credit utilization rate is equal to $\alpha(n)$. Mian and Sufi (2010) report a median credit card utilization rate of 23.4% in the fourth quarter of 2006. Hence, we target $\alpha = 0.23$. This implies that households receive a liquidity shock on average every 4.3 months.

The monitoring probability, ρ, is chosen to target the average amount of monthly consumption financed through unsecured credit. In the model, aggregate consumption is given by $C = \alpha(n)(1-\mu)\{\omega[\upsilon(y)-y]+(1-\omega)[\upsilon(\tilde{y})-\tilde{y}]\}+n\bar{z}-\theta uk$ and consumption out of unsecured credit is $\omega\alpha(n)\bar{d}$. We target $\omega\alpha(n)\bar{d}/C$ as the ratio of new monthly charges on credit and charge cards given by the SCF and total monthly household consumption expenditures as reported in the Consumer Expenditure Survey (CEX). Over 2000-2008, the average household had \$845 of new charges on credit cards and total consumption expenditures of \$3,681 during a month. Therefore, we set $\omega\alpha(n)\bar{d}/C = 0.23$, which results in $\rho = 0.30$.

We take M2 as our measure of liquid assets and the rate of return on liquid assets, R, is chosen to target the real user cost of M2 monetary services as published by the St. Louis Federal Reserve. The real user cost is defined as the discounted interest rate spread between a benchmark rate and the own rate of return on assets included in M2.[25] In the model the real user cost is given by $(1 + r - R)/(1 + r)$. Over 2000-2008, the real user cost averaged 0.055%. This implies $R = 1.0025$.

Utility over DM consumption is given by, $\upsilon(y) = \upsilon_0 y^{1-\gamma}/(1 - \gamma)$. The level and elasticity parameters in the utility function, (υ_0, γ), are set such that the relationship between the demand for liquid assets, $R\tilde{a}$, and the opportunity cost of holding them, $(1 + r - R)/(1 + r)$, matches the data. In the model, aggregate holdings of liquid assets normalized by consumption are given by $(1 - \omega)[(1 - \mu)\upsilon(\tilde{y}) + \mu\tilde{y}]/\nu C$, where aggregate consumption C is defined above. The demand for liquid assets depends on the opportunity cost $(1 + r - R)/(1 + r)$ through $\tilde{y}$ given by (2.37). We set υ_0 to match average M2/C and γ to match the empirical elasticity of M2/C to the real user cost, which we estimate to be -0.17. Together these imply $(\upsilon_0, \gamma) = (1.42, 0.03)$. Figure 2.7 shows the estimated demand for liquid assets.[26]

[25]The benchmark rate is given as the upper envelope of a set of rates including the Baa corporate bond yield and other short-term money market rates plus a small liquidity premium. See Anderson and Jones (2011) for a review of the Monetary Services Index (MSI).

[26]The estimated money demand curve fits the empirical relationship well until 1991, when there was an apparent shift down in the demand for liquid assets.

The targets discussed above are sufficient to pin down all but one parameter, ν, which determines the acceptability of liquid assets as a means of payment in the DM. We first solve for a range of ν that is consistent with the coexistence of liquid assets and credit in equilibrium. We find ν must range between .001 and .1. In the baseline calibration, we set $\nu = .05$ and show a sensitivity analysis around this target.[27] Table 2.1 reports the calibrated parameters.

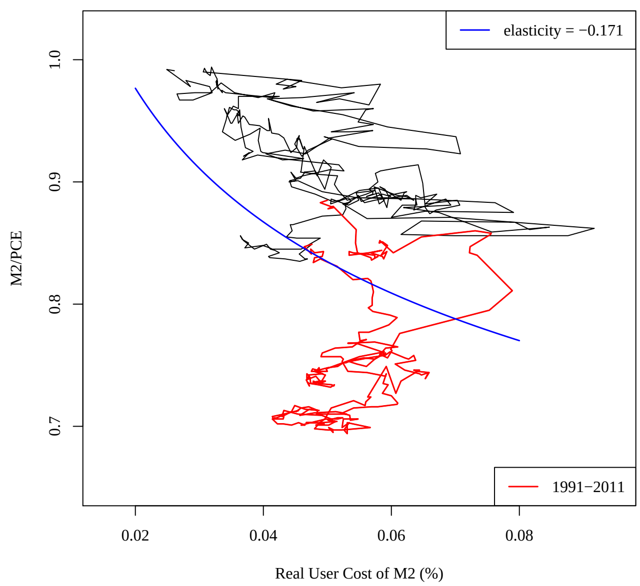

Figure 2.7: Demand for Liquid Assets

[27]At first, the suggested range for ν may seem to be too small; however, it is important to consider that this parameter captures various costs that are not directly taken into account in the model. For instance, Sanches and Williamson (2010) emphasize the possibility of theft in anonymous transactions.

Description	Value	Source/Target
Labor Market		
Unemployment benefits, b	0.53	$b = .5w$
Value of leisure, ℓ	0.48	$(b+\ell)/w = .95$, Hagedorn and Manovskii (2008)
Job destruction rate, δ	0.019	Unemployment rate, CPS
Vacancy cost, k	0.10	Job finding probability Shimer (2012), CPS
Elasticity of LM matching function, η	0.50	Petrolongo and Pissarides (2001)
LM bargaining, λ	0.50	Hosios (1990) condition
LM matching efficiency, A	0.50	Vacancy rate, JOLTS
Credit & Goods Market		
DM production, $\bar{z}$	1	Normalization
Access to unsecured credit, ω	0.74	Fraction that hold at least 1 credit card, SCF
DM bargaining, μ	0.13	Retail Markup 40% , Faig and Jerez (2005)
Elasticity of DM matching function, ψ	0.50	Equal contribution to goods market matching
DM matching efficiency, ϵ	0.24	Credit card utilization rate, Mian and Sufi (2010)—
Detection Rate, ρ	0.30	Debt to consumption, SCF & CEX
Return on Liquid Assets, R	1.0025	Real user cost of M2, St. Louis Federal Reserve
Utility level parameter, υ_0	1.42	M2 to consumption, FRB H.6 & NIPA
Risk aversion, γ	0.03	Elasticity of M2 to real user cost

Table 2.1: Parameter Values

2.5.2 Quantitative Results

In this section, we use the calibrated model to analyze the effects of an exogenous change in unsecured credit on the aggregate labor market. Our experiments will focus on two episodes: (i) the expansion of credit between the late 1970s and early 2000s and (ii) the sharp drop in credit availability during the Great Recession from 2008 to 2011.[28]

1978-2008 The 1970s and early 1980s consisted of the passage of several regulatory acts that affected the household credit market. Beginning with the Fair Credit Reporting Act of 1970, these regulations essentially standardized the reporting and use of consumer credit histories.[29] We interpret the result of these acts, combined with the well documented IT revolution, as a long-term improvement in monitoring technology. We will vary two parameters ω and ρ. In the model, ω controls the measure of households that have access to the unsecured credit market while ρ affects the endogenous debt limit for those households who maintain access to credit.

Compared to the benchmark time period 2000-2008, unsecured credit as a percentage of consumption was 70.6% lower during 1978-1968. We engineer this reduction in credit by first decreasing ω to 65%, the fraction of households in the 1983 SCF that reported having at least one credit card. We then decrease ρ by 5.1% to generate the total fall in the ratio of unsecured credit to consumption of 70.6%. The results are presented in Table 2.2.

[28]See Figure 2.1.

[29]The series of acts were the Fair Credit Reporting Act of 1970, the Community Reinvestment Act of 1977, and the Equal Credit Opportunity Act of 1974.

The difference in the availability of unsecured credit between 1978 and 2008 is largely attributable to changes in borrowing limits. As indicated in the first row of Table 2.2, the fall in the fraction of households with any access to credit, ω, generates only 13.3% of the total 70.6% difference in the credit to consumption ratio. The remainder is generated by differences in borrowing limits.

Holdings of liquid assets increase as a result of the restriction of credit. The model slightly over accounts for the total difference in holdings of M2 as a percentage of consumption between 1978 and 2008, as indicated in the last two columns of row 2, but in general captures the trade-off between the utilization of credit and money in the aggregate. Measured productivity decreases as the absence of credit leads to lost consumption opportunities. The model predicts that productivity decreases by 4.45%.

The difference in credit conditions has significant implications for unemployment in the long run. The model predicts that credit differences can account for 71% of the total difference in the average unemployment rate between 1978-1986 and the benchmark time period of 2000-2008. A counter-factual prediction of the model is the movement in the average job finding rate. The model predicts that lower credit limits generate a fall in worker's job finding rate as firms entry declines. However, the data suggest that the average job finding rate was 16.1% higher during the time period of 1978-1986 compared to the benchmark.[30]

The time period under consideration also consisted of long-term movements in inflation, or in the context of our model can be considered as movements in the return on liquid assets, R. The cost of assets plays a crucial role in disciplining

[30] For a discussion and evidence of worker job finding rates, see Shimer (2012).

credit in the model. As assets become relatively more expensive, the cost of defaulting on credit increases so debt constraints are relaxed. Inflation, then, improves credit in partial equilibrium. However, inflation hurts households that do not have access to credit as they bear the full cost. In general equilibrium, the unemployment effects are ambiguous. To capture potential changes in inflation during this time period, we repeat the experiment above while letting R vary. In the baseline calibration, R is disciplined by the real user cost of holding M2.[31] Despite the large movements in inflation over this time period, the real user cost of holding M2 changed little. The average real user cost form 1978 to 1986 was 0.058% compared to 0.055% in the baseline calibration. The results from the previous experiment change little. Matching a similar decrease in unsecured credit, productivity falls by 4.6%, while unemployment increases by 35%.[32]

2008-2011 Table 2.3 reports the results of a similar experiment to that in Table 2.2 to mimic the US before and after the Great Recession. The experimental question is to what extent can the fall in unsecured credit, viewed as the result of changes in monitoring, explain the short-term movements in unemployment. Between 2008 and 2011, unsecured credit as a percentage of consumption fell by 22.9%. As before, we generate this decline by first decreasing ω to 67.9% to mimic the fraction of households in the 2010 SCF that reporting having at least one credit card. We then decrease ρ by 0.8% to generate the 22.9% fall in credit to consumption.

[31] See the discussion of the real user cost in Section 5.1 above.

[32] We also repeated the experiment above interpreting assets as Lucas trees in fixed supply and allow the price, R, to vary. The results are similar and are available upon request.

The model predicts the fall in unsecured credit between 2008 and 2011 led to a decline in aggregate productivity of 1.4%. This generates an increase in equilibrium unemployment from 5.13% to 5.53%, or 7.8% of the total 74% increase in unemployment over this time period. In the steady-state, the reduction in household's ability to finance purchases with unsecured credit explains about 10% of the total increase in unemployment over this time period.[33]

The Response of Unemployment to Exogenous Productivity Shocks: The Role of Unsecured Credit In this section, we analyze the role of the unsecured credit market in amplifying exogenous shocks in the model. Specifically, we return to a question posed by Petrosky-Nadeau (2014), and others, which asks to what extent do credit frictions amplify the response of unemployment to exogenous productivity shocks. We analyze this question in the context of the steady-state. Consider a decrease in the exogenous component of productivity, $\bar{z}$. In partial equilibrium, holding unsecured credit fixed, there is an increase in equilibrium unemployment. The value of a filled job decreases, firms post less vacancies causing unemployment to rise. This is just the standard mechanics of MP. In general equilibrium, however, an increase in unemployment also lowers households access to unsecured credit. This causes a further drop in measured productivity, z and a larger increase in unemployment. Figures 2.8 and 2.9 highlight this channel.

[33]The effects are robust to alternative calibration strategies presented in tables B.1 and B.2 in the Appendix. Varying ν within the range $(0.035, 0.065)$ affects the substitution between liquid assets and credit, but does not alter the magnitude of the effect on productivity and unemployment. Considering a smaller target for the retail markup of 30% also delivers similar results, though the effect on unemployment and productivity are slightly dampened.

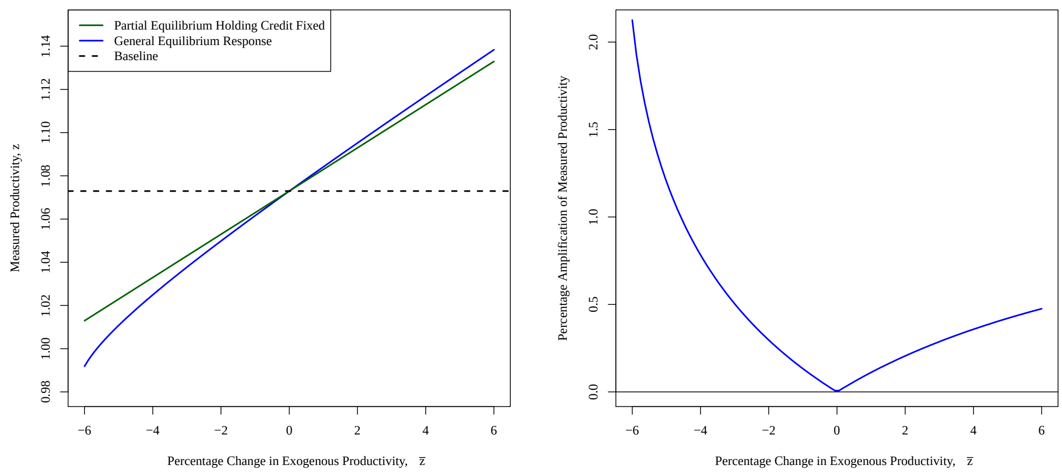

Figure 2.8: The credit amplification channel on measured productivity

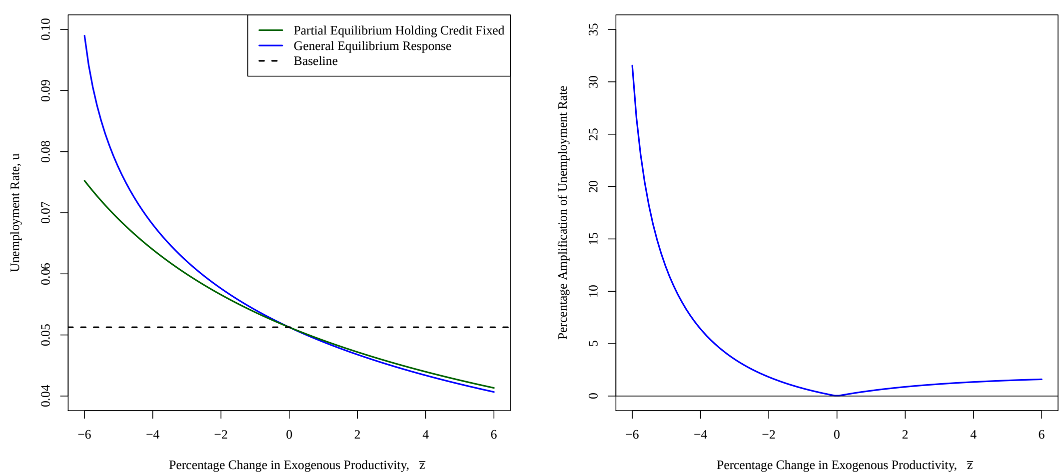

Figure 2.9: The role of credit in the amplification of unemployment to productivity shocks

The first graph of each figure shows the response in equilibrium steady-state measured productivity and unemployment, respectively, for an exogenous change in $\bar{z}$. The green line represents the partial equilibrium effect holding credit constant while the blue line represents the full general equilibrium movement. The difference between the two lines is caused by the endogenous response of credit. The graph on the right of each figure highlights the magnitude of this amplification. The y-axis represents the absolute distance between the green and blue lines in percentage terms.

First, the existence of unsecured credit can have a significant amplification effect on measured productivity and unemployment. For instance, consider a negative 4% shock to exogenous productivity, $\bar{z}$. Holding unsecured credit constant, measured productivity, z, falls by 3.7% and unemployment increases from 5.1% to 6.4%. As debt limits adjust, productivity decreases an additional 0.8% and unemployment increases further to 6.9%. Therefore the additional amplification due to unsecured credit is 7.8%. Secondly, the effects are highly asymmetric. Negative productivity shocks lead to a significantly larger amplification caused by unsecured credit. For instance, when exogenous productivity decreases by 6 percent, unsecured credit amplifies the effect on unemployment by around 30%. A positive 6% shock to exogenous productivity is only amplified by 1.6%.

2.6 Conclusion

This paper studies the relationship between aggregate unemployment and household unsecured credit. We develop a theory that incorporates a retail goods market with search frictions and limited commitment. Households purchase goods using both unsecured credit and liquid assets. The theory provides a channel through which credit can affect firm revenue and entry, thereby impacting steady-state unemployment. We show the possibility of multiple steady-state equilibria in which there exists a negative relationship between unemployment and unsecured debt. The key mechanism delivering the result is the complementarity between the endogenous borrowing limit and firms' entry decisions.

The model is also amenable to quantitative analysis. As an example, we illustrate the labor market effects of the long-term expansion of unsecured credit in the US between 1978 and 2008 as a result of an improvement in monitoring. The model is able to account for 71% of the low frequency movements in unemployment over this time period. The results suggest that unsecured credit is an important contributor to firm revenue and, as a result, has important implications on labor market outcomes.

	Baseline	Decrease ω		Decrease ρ		Decrease both		% change
		Level	% change	Level	% change	Level	% change	2008-1978
Credit & Goods Market								
Credit to Con., $\alpha(n)\omega\bar{d}/C$	0.23	0.20	-13.3	0.08	-66.0	0.07	-70.6	-70.6
M2 to Cons., $(1-\omega)R\tilde{a}/C$	0.74	1.01	36.3	0.68	-8.10	0.93	25.5	18.3
Agg. productivity, z	1.07	1.06	-0.81	1.03	-4.28	1.02	-4.45	
Liquidity shocks, $\alpha(n)$	0.23	0.23	-0.11	0.23	-0.83	0.23	-0.89	
Labor Market								
Unemployment rate (%)	5.13	5.34	4.18	6.69	30.6	6.82	33.0	46.0
Job Finding Rate, p	0.36	0.34	-4.23	0.27	-24.7	0.26	-26.2	16.1

Table 2.2: Unemployment and Credit, 1978-2008

	Baseline	Decrease ω		Decrease ρ		Decrease both		% change
		Level	% change	Level	% change	Level	% change	2008-2011
Credit & Goods Market								
Credit to Con., $\alpha(n)\omega\bar{d}/C$	0.23	0.21	-9.58	0.20	-15.1	0.18	-22.9	-22.9
M2 to Cons., $(1-\omega)R\tilde{a}/C$	0.74	0.94	26.1	0.73	-1.28	0.92	24.4	11.0
Agg. productivity, z	1.07	1.07	-0.58	1.06	-0.95	1.06	-1.44	
Liquidity shocks, $\alpha(n)$	0.23	0.23	-0.08	0.23	-0.13	0.23	-0.21	
Labor Market								
Unemployment rate (%)	5.13	5.28	2.95	5.38	4.95	5.53	7.80	74.1
Job Finding Rate, p	0.36	0.35	-3.31	0.34	-4.97	0.33	-7.63	-46.6

Table 2.3: Unemployment and Credit, 2008-2011

Chapter 3

Dynamic Indeterminacy and Welfare in Credit Economies

3.1 Introduction

The inability of individuals to commit to honor their future obligations is a key friction of decentralized economies that jeopardizes the Arrow-Debreu apparatus based on promises to deliver goods at different dates and in different states. Economies with limited commitment have been studied predominantly in monetary theory. Stark examples are pure currency economies where anonymity and lack of commitment make credit infeasible. Arguably, pure currency economies have become less relevant due to advances in record-keeping technologies that facilitate the use of credit. Yet, monitoring technologies do not purge economies from the limited commitment problem—they do not make individuals entirely trustworthy. Hence, this paper investigates the full set of equilibria and constrained-efficient allocations of a pure credit economy taking seriously the limited commitment friction.[1]

There are two recent contributions, one normative and one positive, that shed some light on economies with limited commitment. On the normative side, Alvarez and Jermann (2000), AJ thereafter, establish a Second Welfare Theorem for a pure exchange, one-good economy where agents are subject to endowment shocks and have limited commitment—a special case of the environment in Kehoe and Levine (1993). They prove that constrained-efficient allocations can be implemented by competitive trades subject to "not-too-tight" solvency constraints. These constraints specify that in every period agents can issue the maximum amount of debt that is incentive-compatible with no default, thereby allowing as much risk sharing as possible. From a positive perspective Gu *et al.* (2013b), GMMW thereafter, study a pure credit economy subject to the same "not-too-tight" solvency constraints and show the possibility of endogenous credit cycles.[2] The conditions for such cycles, however, are much more stringent than the ones in pure currency economies.[3]

The objective of this paper is to revisit these two key insights—the implementation of constrained-efficient allocations and the existence of endogenous credit cycles—in the context of a pure credit economy with limited commitment. Our

[1] In the words of Wicksell (1936), "a thorough analysis of this purely imaginary case seems to me to be worth while, for it provides a precise antithesis to the equally imaginary case of a pure cash system, in which credit plays no part whatever."

[2] In a related paper Bloise *et al.* (2013) prove indeterminacy of competitive equilibrium in sequential economies under "not-too-tight" solvency constraints. While they do not focus on endogenous credit cycles they show that for any value of social welfare in between autarchy and constrained optimality, there exists an equilibrium attaining that value.

[3] For instance, Lagos and Wright (2003) find that monetary economies can generate endogenous cycles under monotone trading mechanisms, such as buyers-take-all bargaining. Under the same trading mechanisms, GMMW do not find any cycle.

main contributions are twofold. On the positive side, we give a complete characterization of the (perfect Bayesian) equilibrium set of a pure credit economy. On the normative side, we characterize constrained-efficient allocations of economies with pairwise meetings and competitive economies with large meetings.

The pure credit economy we consider features random matching—in pairwise meetings or in large groups—and incorporates intertemporal gains from trade that can be exploited with one-period debt contracts. In the absence of public record keeping, the environment corresponds to the New-Monetarist framework of Lagos and Wright (2005) so that one can easily compare equilibrium allocations in credit and monetary economies. In the presence of a public record-keeping technology the environment is mathematically equivalent to the one in GMMW.[4]

We start with a simple mechanism where the borrower in each bilateral match sets the terms of the loan contract unilaterally, which allows us to analyze the economy as a standard infinitely-repeated game with imperfect monitoring. If we impose the AJ "not-too-tight" solvency constraints exogenously—which amounts to restricting strategies and beliefs such that any form of default is punished with permanent autarky—then there is a unique active steady-state equilibrium and no equilibrium with endogenous cycles. When we look for all perfect Bayesian equilibria, we find a continuum of steady-state equilibria, a continuum of periodic equilibria of any periodicity, and much more. Each equilibrium can be reduced to a sequence of debt limits, where the debt limit in a period specifies the amount that agents can be trusted to repay. Moreover, there is a wide variety of outcomes: in some credit cycle equilibria debt limits are binding in all periods, in other equilibria

[4] As we discuss later in details, there are differences regarding the timing of production that are inconsequential.

they are never binding, or they bind periodically. These results are robust to the choice of the mechanism to determine the terms of the loan contract—Nash or proportional bargaining, or even competitive pricing if agents meet in large groups.

Figure 3.1 plots the set of 2-period cycles in the space of debt limits for a representative example under competitive pricing (example 3A in GMMW). The horizontal axis gives the debt limit (expressed in terms of the good used for repayment) in even periods and the vertical axis specifies the debt limit in odd periods. The blue area represents the continuum of 2-period cycle equilibria. Under "not-too-tight" solvency constraints there is a unique active steady state, marked by a green star, and two strict 2-period cycles marked by red stars.

The multiplicity of credit equilibria captures the basic notion that trust is a self-fulfilling phenomenon. To that extent, and following (Mailath and Samuelson, 2006, p. 9), "we consider multiple equilibria a virtue." But this multiplicity does not imply that everything goes. Fundamentals, including preferences and market structure, do matter for an outcome to be consistent with an equilibrium. We show that the set of credit-cycle equilibria expands (in Figure 3.1 the blue area expands outwards) as trading frictions are reduced, agents are more patient, and borrowers have more bargaining power (in the version of the model with bargaining).

We also show that for a given trading mechanism the set of equilibrium outcomes of a pure monetary economy (with fiat money but no record keeping) is a strict subset of the outcomes of a pure credit economy (with record keeping but no fiat money). Hence all dynamic allocations of monetary economies (e.g., cycles, chaos) also exist in credit environments. The reverse is not true. There are equilibrium outcomes of the pure credit economy that cannot be sustained

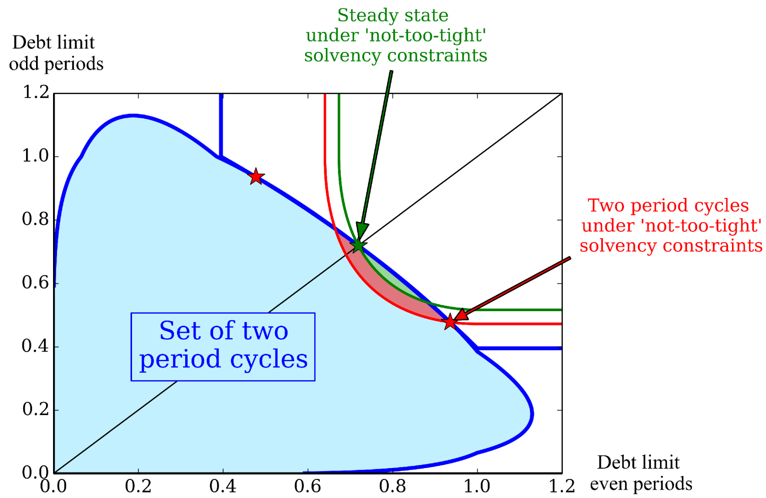

Figure 3.1: A representative example: Set of 2-period cycles under competitive pricing. Green star: steady state under ”not-too-tight” solvency constraints. Red star: cycle under ”not-too-tight” constraints. The red and green curves are society’s indifference curves.

as equilibrium outcomes of the pure currency economy. For instance, there are equilibria where credit and output shut down periodically—which is ruled out by backward-induction in monetary economies.

In order to understand why the equilibrium set for credit economies—the blue area in Figure 3.1—is so vastly larger than the one found in GMMW—the green and red stars—it is worth recalling that the AJ “not-too-tight” solvency constraints were meant to provide a way to decentralize constrained-efficient allocations in one particular economy with limited commitment. Such constraints are not warranted for positive analysis. We will argue later that they are also restrictive for normative analysis in our environment. We avoid arbitrary restrictions on the set of equilibrium outcomes by working with simple strategies that punish both default and excessive lending (i.e., lending in excess of the amount that is deemed trustworthy along the equilibrium path). We show that such simple strategies implement the full set of outcomes of perfect Bayesian equilibria (subject to mild restrictions).

In terms of normative analysis we determine the constrained-efficient allocations under different assumptions: we consider both economies where agents meet in pairs and economies with large-group meetings and, following the formulation in GMMW, we vary preferences to affect an agent's temptation to renege on his debt. If agents are matched bilaterally then the constrained-efficient allocation is implemented with take-it-or-leave-it offers by buyers and "not-too-tight" solvency constraints, which generalizes the AJ welfare theorem to economies with pairwise meetings. Under competitive pricing the "not-too-tight" solvency constraints are suboptimal when the temptation to renege is low. In this case the constrained-efficient allocation is non-stationary and it features slack participation constraints (i.e., solvency constraints that are "too-tight", according to the terminology of AJ) for all periods except the initial one. Slack participation constraints are socially optimal due to a pecuniary externality according to which a decrease in debt limits generates lower contemporaneous output, which reduces prices and increases gains from trade for buyers/borrowers, thereby relaxing borrowing constraints in earlier periods. Moreover, there exists a continuum of credit cycles that yield a higher social welfare than those with "not-too-tight" solvency constraints. We illustrate this point in Figure 3.1 by representing with red and green areas the set of 2-period cycles that dominate the equilibria obtained under "not-too-tight" solvency constraints.

3.1.1 Related literature

We adopt an environment similar to the pure currency economy of Lagos and Wright (2005) and Rocheteau and Wright (2005), but we replace currency with a public record-keeping technology, as in Sanches and Williamson (2010, Section 4). The first part of the paper, on the characterization of the equilibrium set (Sections 3.3 and 3.4), extends the analysis of Sanches-Williamson which focuses on steady states and of GMMW which focuses on cycles. In both cases, the equilibrium notion imposes the "not-too-tight" solvency constraints of AJ. Instead, we present our model as a repeated game with imperfect monitoring with few restrictions on strategies and beliefs (the same restrictions typically imposed on equilibria of pure currency economies).[5] In addition, we consider both stationary and non-stationary equilibria (including endogenous cycles and sunspots), various trading mechanisms (ultimatum games, axiomatic bargaining solutions, competitive pricing), and we conduct a normative analysis to determine constrained-efficient allocations. Our methods to characterize equilibrium outcomes (Sections 3.3 and 3.5) are related but different from the ones used by Abreu (1988) and Abreu *et al.* (1990) as our stage game has an extensive form and only buyers/borrowers are (imperfectly) monitored.

Kocherlakota (1998) shows that the set of implementable outcomes of monetary economies is a subset of the implementable outcomes of pure credit economies.[6]

[5]Repeated games where agents are matched bilaterally and at random and change trading partners over time are studied in Kandori (1992) and Ellison (1994). A thorough review of the literature is provided by Mailath and Samuelson (2006).

[6]Hellwig and Lorenzoni (2009) study an environment similar to Alvarez and Jermann (2000) and show that the set of equilibrium allocations with self-enforcing private debt is equivalent to the allocations that are sustained with money.

We find a similar result, but in contrast to Kocherlakota (1998), we take the trading mechanism as given and we do not restrict outcomes to stationary ones. Moreover, we establish in Proposition 13 that the condition to implement the first best allocation in the pure credit economy with record keeping is identical to the one in the pure currency economy with no record keeping (Hu *et al.*, 2009).

Our paper is part of the literature on limited commitment in macroeconomics. Seminal contributions on risk sharing in endowment economies where agents lack commitment include Kehoe and Levine (1993), Kocherlakota (1996), and AJ. Kocherlakota (1996) adopts a mechanism design approach in a two-agent economy with a single good. Our Section 3.5.1 on constrained-efficient allocations under pairwise meetings is related with some key differences: we study a two-good production economy where a continuum of agents search for new partners every period and we select the allocation that maximizes a social welfare criterion under quasi-linear preferences. Gu et al. (2013a, Section 7) has a similar environment but solves for the contract curve. In our Section 3.5.2 we study constrained-efficient allocations under large meetings and competitive pricing, as in Kehoe and Levine (1993) or AJ. Kehoe and Levine (1993, Section 7) conjectured that punishments based on partial exclusion might allow the implementation of socially desirable allocations.[7] This conjecture is verified in our economy with the caveat that the extent of exclusion has to vary over time. Our normative results are also related to the Second Welfare Theorem in AJ according to which constrained-efficient

[7] Azariadis and Kass (2013) relaxed the assumption of permanent autarky and assumed that agents are only temporarily excluded from credit markets. Gu *et al.* (2013a) and GMMW allow for partial monitoring, which is formally equivalent to partial exclusion, except that the parameter governing the monitoring intensity, π, is time-invariant. Kocherlakota and Wallace (1998) consider the case of an imperfect record-keeping technology where the public record of individual transactions is updated after a probabilistic lag.

allocations can be implemented with "not-too-tight" solvency constraints. We provide a necessary and sufficient condition under which this theorem applies to our environment.

3.2 Description of the game

Time is discrete and starts with period 0. Each date has two stages. The first stage will be referred to as DM (decentralized market) while the second stage will be referred to as CM (centralized market). There is a single, perishable good at each stage and the CM good will be taken as the numéraire. There is a continuum of agents of measure two divided evenly into a subset of buyers, $\mathbb{B}$, and a subset of sellers, $\mathbb{S}$.[8] The labels "buyer" and "seller" refer to agents' roles in the DM: only the sellers can produce the DM good (and hence will be lenders) and only the buyers desire DM goods (and hence will be borrowers). In the DM, a fraction $\alpha \in (0, 1]$ of buyers meet with sellers in pairs. (We consider a version of the model with large meetings later.) The CM will be the place where agents settle debts.

Preferences are additively separable over dates and stages. The DM utility of a seller who produces $y \in \mathbb{R}_+$ is $-v(y)$, while that of a buyer who consumes y is $u(y)$, where $v(0) = u(0) = 0$, v and u are strictly increasing and differentiable with v convex and u strictly concave, and $u'(0) = +\infty > v'(0) = 0$. Moreover, there exists $\tilde{y} > 0$ such that $v(\tilde{y}) = u(\tilde{y})$. We denote by $y^* = \arg\max [u(y) - v(y)] > 0$ the quantity that maximizes the match surplus. The utility of consuming $z \in \mathbb{R}$

[8]The assumption of ex-ante heterogeneity among agents is borrowed from Rocheteau and Wright (2005). Alternatively, one could assume that an agent's role in the DM is determined at random in every period without affecting any of our results.

units of the numéraire good is z, where $z < 0$ is interpreted as production.[9] Agents' common discount factor across periods is $\beta = 1/(1+r) \in (0,1)$.

With no loss in generality we restrict our attention to intra-period loans issued in the DM and repaid in the subsequent CM.[10] The terms of the loan contracts are determined according to a simple protocol whereby buyers make take-it-or-leave-it offers to sellers. We describe alternative mechanisms later in the paper. Agents cannot commit to future actions. Therefore, the repayment of loans in the CM has to be self-enforcing.

There is a technology allowing loan contracts in the DM and repayments in the CM to be publicly recorded. The entry in the public record for each loan is a triple, (ℓ, x, i), composed of the size of the loan negotiated in the DM in terms of the numéraire good, $\ell \in \mathbb{R}_+$, the amount repaid in the CM, $x \in \mathbb{R}_+$, and the identity of the buyer, $i \in \mathbb{B}$. If no credit is issued in a pairwise meeting, or if i was unmatched, the entry in the public record is $(0,0,i)$. The record is updated at the end of each period t as follows:

$$\rho_{t+1}^{i} = \rho_t^{i} \circ (\ell_t, x_t, i), \tag{3.1}$$

[9] Kehoe and Levine (1993) and AJ consider pure exchange economies. One could reinterpret our economy as an endowment economy as follows. Suppose that sellers receive an endowment $\bar{y}$ in the DM and $\bar{z}$ in the CM. Buyers have no endowment in the DM but an endowment $\bar{z}$ in the CM. The DM utility of the seller is $w(c)$ where w is a concave function with $w'(\bar{y}) = 0$. Hence, the opportunity cost to the seller of giving up y units of consumption is $v(y) = w(\bar{y}) - w(\bar{y} - y)$.

[10] Under linear payoffs in the CM one-period debt contracts are optimal, i.e., agents have no incentives to smooth the repayment of debt across multiple periods. This assumption will facilitate the comparison with pure monetary economies of the type studied in Lagos and Wright (2005).

where $\rho_0^i = (\ell_0, x_0, i)$. The list of records for all buyers, $\rho_t = \langle \rho_t^i : i \in \mathbb{B} \rangle$, is public information to all agents.[11] Agents have private information about their trading histories that are not recorded; in particular, if $\rho_t^i = (0, 0, i)$, then agents other than i do not know whether i was matched but his offer got rejected (in that case, the offer made is not observed either) or was unmatched. However, as discussed later, this private information plays no role in our construction of equilibria.

3.3 Equilibria

For each buyer $i \in \mathbb{B}$, a strategy, s^i, consists of two functions $s_t^i = (s_{t,1}^i, s_{t,2}^i)$ at each period t conditional on being matched: $s_{t,1}^i$ maps his private trading history, h_t^i, and public records of other buyers, ρ_t^{-i}, to an offer to the seller, (y_t, ℓ_t); $s_{t,2}^i$ maps $((h_t^i, \rho_t^{-i}), (y_t, \ell_t))$, together with the seller's response, to his CM repayment, x_t. For each seller $j \in \mathbb{S}$, a strategy, s^j, consists of one function at each period t, conditional on being matched with buyer i: s_t^j maps the seller's private trading history, h_t^j, the buyer's identity and public records, $(i, \rho_t^i, \rho_t^{-i})$, and his current offer, (y_t, ℓ_t), to a response, *yes* or *no*. We restrict our attention to perfect Bayesian equilibria (see Osborne and Rubinstein, 1994, Definition 232.1) satisfying the following conditions:

(A1) **Public strategies.** In any DM meeting the strategies only depend on histories that are common knowledge in the match, including the buyer's public

[11] We could make alternative assumptions regarding what is recorded in a match. For instance, the technology could also record the output level, y, together with the promises made by the buyer, i.e., $\rho^i = (y, \ell, x, i)$. Not surprisingly, this would expand the set of equilibrium outcomes. Moreover, we could assume that the seller only observes the record of the buyer he is matched with, ρ^i, without affecting our results.

trading history, his offer and the seller's response in the current match, but not on private histories (nor the public records of other buyers).[12]

(A2) **Symmetry.** All buyers adopt the same strategy, s^b, and all sellers adopt the same strategy, s^s. Moreover, the buyer's offer strategy, $s^b_{t,1}$, is constant over all public trading histories of the buyer that are consistent with equilibrium behavior, in particular, equilibrium offers at date-t are independent of matching histories.

(A3) **Threshold rule for repayments.** For each buyer i and each date t following any history, there exists a number, d_t, such that d_t is weakly larger than the equilibrium loan amount at date t, and $s^b_{t,2}(\rho^i_t, (y_t, \ell_t), yes) = \ell_t$ if $\ell_t \leq d_t$ and if ρ^i_t is consistent with equilibrium behavior.

We call a perfect Bayesian equilibrium, (s^b, s^s), satisfying conditions (A1)-(A3) above a *credit equilibrium.* A few remarks are in order about these conditions. Our record-keeping technology does not record all actions taken by the agents. Agents have private information about the number of matches they had, quantities they consumed, or offers that were rejected. Because of this private information using perfect Bayesian equilibrium (PBE) as the solution concept is both standard and necessary. Alternatively, one may assume that all actions are observable, and PBE is reduced to subgame perfection. Although we prefer our environment, which is closer to the existing literature on monetary economics, our multiplicity result does not rely on the presence of private information. In fact, because of our focus on public strategies, (A1), any PBE we construct is also a subgame-perfect

[12]For a formal definition of public strategies see Definition 7.1.1 in Mailath and Samuelson (2006).

equilibrium (SPE) if all actions were observable.[13] However, agents' belief about how other agents will respond to deviations do matter but they are pinned down by equilibrium strategies.

Conditions (A1) and (A2) imply that, for any credit equilibrium, its outcomes are characterized by $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$, the sequence of equilibrium offers made by buyers. Moreover, (A3) implies that $x_t = \ell_t$ for each t, and hence the sequence $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$ also determines the equilibrium allocation. Without (A1), equilibrium offers may depend on the buyer's past matching histories.[14] Condition (A3) is not vacuous either. It restricts sellers to believe that buyers will repay their debt when observing a deviating offer with obligations smaller than those in equilibrium.[15] This restriction will rule out inefficiently large trades. As we will see later, taken together the restrictions (A1)-(A3) will allow us to obtain a simple representation of credit equilibria with solvency constraints added to the bargaining problem.

Let $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$ be a sequence of equilibrium offers. Along the equilibrium path the lifetime expected discounted utility of a buyer at the beginning of period t is

$$V_t^b = \sum_{s=0}^{\infty} \beta^s \alpha [u(y_{t+s}) - \ell_{t+s}]. \tag{3.2}$$

[13]In such an equilibrium sellers' beliefs about buyers' private information are irrelevant for their decisions to accept or reject offers. Hence, actions that correspond to agents' private trading histories would not matter even if they were publicly observable.

[14]Obviously, when $\alpha = 1$, the matching-history-independence element in (A1) is vacuous. However, when $\alpha < 1$, it would be difficult to fully characterize all equilibrium outcomes without (A1) but it certainly adds many more equilibria.

[15]Without this restriction one could sustain equilibria in which $y_t > y^*$ for some t; to do so, one can adopt a strategy that triggers a permanent autarky for the buyer if his offer ℓ_t is smaller than the equilibrium one.

In each period $t+s$ the buyer is matched with a seller with probability α in which case the buyer asks for y_{t+s} units of DM output in exchange for a repayment of ℓ_{t+s} units of the numéraire in the following CM and the seller agrees. In any equilibrium $-\ell_t + \beta V^b_{t+1} \geq 0$, which simply says that a buyer must be better off repaying his debt and going along with the equilibrium rather than defaulting on his debt and offering no-trade in all future matches, $(y_{t+s}, \ell_{t+s}) = (0, 0)$ for all $s > 0$. By a similar reasoning the lifetime expected utility of a seller along the equilibrium path is

$$V^s_t = \sum_{s=0}^{\infty} \beta^s \alpha[-v(y_{t+s}) - \ell_{t+s}]. \tag{3.3}$$

The seller's participation constraint in the DM requires $-v(y_t) + \ell_t \geq 0$ since a seller can reject a trade without fear of retribution. (He is not monitored.) Given that buyers set the terms of trade unilaterally, and the output level is not part of the record ρ^i, this participation constraint holds at equality. Our first proposition builds on these observations to characterize outcomes of credit equilibria.

Proposition 5. *A sequence, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, is a credit equilibrium outcome if and only if, for each $t = 0, 1, ...,$*

$$\ell_t \leq \sum_{s=1}^{\infty} \beta^s \alpha[u(y_{t+s}) - \ell_{t+s}] \tag{3.4}$$

$$\ell_t = x_t = v(y_t) \leq v(y^*). \tag{3.5}$$

As mentioned earlier, a sequence of equilibrium offers, $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$, also determines the sequence of allocations, $\{(y_t, x_t)\}_{t=0}^{+\infty}$, with $x_t = \ell_t$ for each t, and hence, Proposition 5 also gives a characterization of allocations that can be sustained in a credit equilibrium. Condition (3.4), which follows directly from (3.2) and the incentive constraint $-\ell_t + \beta V^b_{t+1} \geq 0$, is analogous to the participation constraint (IR) in Kehoe and Levine (1993), and the participation constraint in Proposition 2.1 in Kocherlakota (1996). However, while Kehoe and Levine (1993) assume the IR constraint from the outset as a primitive condition, (3.4) is derived as an equilibrium condition in our framework. The condition (3.5) is the outcome of the buyer take-it-or-leave-if offer and pairwise Pareto efficiency (which follows from the threshold rule A3).[16]

Proposition 5 shows that the conditions (3.4)-(3.5) are not only necessary but also sufficient for an equilibrium by constructing a simple equilibrium strategy profile. This strategy profile relies on punishments—the "penal code" in the terminology of Abreu (1988)—for both default and excessive lending.[17] Specifically, buyers can be in two states at the beginning of period t, $\chi_{i,t} \in \{G, A\}$, where G means "good standing" and A means "autarky", and each buyer's initial state is $\chi_{i,0} = G$. The law of motion of a buyer i's state following a loan and repayment $(\tilde{\ell}, \tilde{x})$ are given by:

$$\chi_{i,t+1}\left(\tilde{\ell}, \tilde{x}, \chi_{i,t}\right) = \begin{cases} A & \text{if } \tilde{x} < \min(\tilde{\ell}, \ell_t) \text{ or } \chi_{i,t} = A \\ G & \text{otherwise} \end{cases}, \tag{3.6}$$

[16]To derive these conditions formally one has to use the assumption that y_t is not publicly recorded—only the loan contract is—and the threshold property in (A3). See proof of Proposition 5.

[17]There are different approaches for finding equilibria of repeated games. Abreu *et al.* (1990) introduce the idea of self-generating set of equilibrium payoffs while Abreu (1988) introduces the notion of simple strategies. See Mailath and Samuelson (2006, Section 2.5) for a review of these approaches. We use a related but different approach from the one of Abreu (1988) as our stage game has an extensive form and only a subset of the agents (the buyers) are monitored.

where $(\tilde{\ell}, \tilde{x})$ might differ from the loan and repayment along the equilibrium path, $\ell_t = x_t$. In order to remain in good standing, or state G, the buyer must repay his loan, $\tilde{x} \geq \tilde{\ell}$, if the size of the loan is no greater than the equilibrium loan size, $\tilde{\ell} \leq \ell_t$, and he must repay the equilibrium loan size, $\tilde{x} \geq \ell_t$, otherwise.[18] The autarky state, A, is absorbing: once a buyer becomes untrustworthy, he stays untrustworthy forever.[19] Sellers cannot be punished in future periods for accepting a loan larger than ℓ_t since their identity is not recorded. However, they are punished in the current period because buyers are allowed to partially default on loans larger than ℓ_t while keeping their good standing with future lenders.

The strategies, (s^b, s^s), depend on the buyer's state as follows. The seller's strategy, s^s_t, consists of accepting all offers, $(\tilde{y}, \tilde{\ell})$, such that $v(\tilde{y}) \leq \min\{\tilde{\ell}, \ell_t\}$ provided that the buyer's state is $\chi_{i,t} = G$. The buyer repays $s^b_{t,2} = \min\{\ell_t, \tilde{\ell}\}$ if he is in state G, and he does not repay anything otherwise, $s^b_{t,2} = 0$. These strategies are depicted in Figure 3.2 where (y_t, ℓ_t) is the offer made by a buyer in state G along the equilibrium path and $(\tilde{y}, \tilde{\ell})$ is any offer. By the one-stage-

[18]Note that the buyer can remain in state G even if he does not pay his debt in full, and hence default is with respect to the common belief that buyers repay up to the size of the equilibrium loan. Also, notice that there are alternative strategy profiles that deliver the same equilibrium outcome. For instance, an alternative automaton is such that the transition to state A only occurs if $\tilde{x} < \tilde{\ell} \leq \ell_t$. If a loan such that $\tilde{\ell} > \ell_t$ is accepted, then the buyer can default without fear of retribution.

[19]While no player has an incentive to deviate unilaterally along a subgame perfect equilibrium, at some heuristic level it seems that two players may want to "renegotiate" the punishment and coordinate on some preferred outcome in the Pareto sense. In the context of our credit equilibrium, the seller might want to forgive, and trust, a buyer who defaulted in the past instead of punishing him with no trade. This idea was formalized by Farrell and Maskin (1989), among others, with the refinement concept of weakly renegotiation-proof (WRP) equilibrium, defined as one where any two continuation payoffs are not Pareto-rankable. From our viewpoint, WRP eliminates the possibility of interesting coordination failures and the possibility of belief-driven credit cycles.

deviation principle it is then straightforward to show that any $\{(y_t, \ell_t)\}_{t=0}^{\infty}$ that satisfies (3.4)-(3.5) is an outcome for the strategy profile (s^b, s^s).

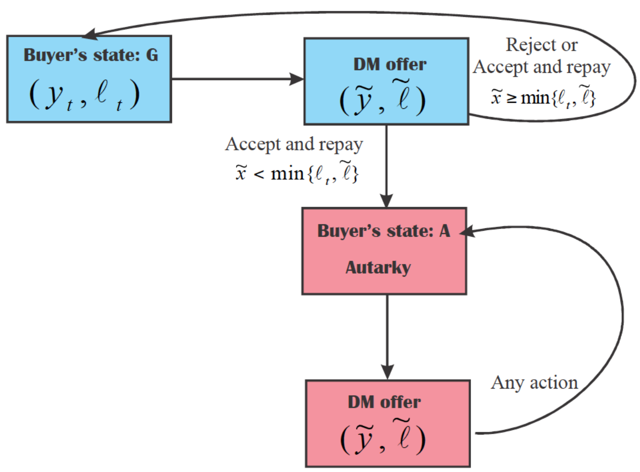

Figure 3.2: Automaton representation of the buyer's strategy

In the following we propose an alternative formulation of a credit equilibrium in terms of solvency constraints imposed on the bargaining problems in the DM. As in AJ in the context of an economy with competitive trades, a solvency constraint specifies an upper bound—called a *debt limit*—on the quantity of debt an agent can issue, $\ell \leq d_t$. According to this formulation, the buyer in a DM match sets the terms of the loan contract so as to maximize his surplus, $u(y) - \ell$, subject to the seller's participation constraint and the solvency (or borrowing) constraint, $\ell \leq d_t$, i.e.,

$$\max_{y,\ell} \{u(y) - \ell\} \quad \text{s.t.} \quad -v(y) + \ell \geq 0 \quad \text{and} \quad \ell \leq d_t. \tag{3.7}$$

The solution to (3.7) is $\ell_t = v(y_t)$ where

$$y_t = z(d_t) \equiv \min\{y^*, v^{-1}(d_t)\}. \tag{3.8}$$

The solvency constraint is reminiscent to the feasibility constraint in monetary models (e.g., Lagos and Wright, 2005) according to which buyers in bilateral matches cannot spend more than their real balances.

We say that a sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, is consistent with a credit equilibrium outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, if (y_t, ℓ_t) is a solution to the bargaining problem, (3.7), given d_t for all $t \in \mathbb{N}_0$, and the buyer's CM strategy consists of repaying his debt up to d_t provided that his past public histories (up to period $t-1$) are consistent with equilibrium behavior, that is, the sequence $\{d_t\}$ satisfies condition (A3).

It is easy to check from the proof of Proposition 5 that any credit equilibrium outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, is consistent with the sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, such that $d_t = \ell_t$ for all $t \in \mathbb{N}_0$. But the same equilibrium outcome may be implementable by multiple debt limits if (3.9) is slack and $y_t = y^*$. The following corollary summarizes these results and reduces a credit equilibrium to a sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, that satisfies a sequence of participation constraints.

Corollary 3. ***(Equilibrium representation with debt limits)*** *A sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, is consistent with a credit equilibrium outcome if and only if*

$$d_t \leq \sum_{s=1}^{\infty} \beta^s \alpha[u(y_{t+s}) - v(y_{t+s})] \tag{3.9}$$

$$v(y_t) = \min\{d_t, v(y^*)\}. \tag{3.10}$$

Corollary 3 gives a complete characterization of equilibrium outcomes using debt limits. Indeed, by (3.10), y_t is determined by d_t, and hence (3.9) can be viewed as an inequality that involves $\{d_t\}_{t=0}^{\infty}$ as the only endogenous variables. Without the danger of confusion, we also call a sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, a credit equilibrium if it satisfies (3.9) and (3.10).

While AJ introduces the solvency constraint as a primitive condition, we derive debt limits endogenously as part of equilibrium strategies. AJ focuses on solvency constraints that are "not-too-tight", meaning that d_t is the largest debt limit that solves the buyer's CM participation constraint, (3.9), at equality, thereby preventing default while allowing as much trade as possible. A "too-tight" solvency constraint is such that (3.9) is slack. In contrast to AJ and GMMW, we do not impose buyers' participation constraint to bind, i.e., the solvency constraint to be "not-too-tight", as such restriction would reduce the equilibrium set dramatically and might eliminate equilibria with good welfare properties. The next Corollary provides a sufficient condition for a credit equilibrium in recursive form.

Corollary 4. ***(Recursive sufficient condition)*** *Any bounded sequence,* $\{d_t\}_{t=0}^{+\infty}$*, that satisfies*

$$d_t \leq \beta \left\{ \alpha \left[u(y_{t+1}) - v(y_{t+1}) \right] + d_{t+1} \right\}, \tag{3.11}$$

where $v(y_t) = \min\{d_t, v(y^*)\}$*, is a credit equilibrium.*

The left side of (3.11) is the cost of repaying the current debt limit while the right side of (3.11) is the benefit which has two components: the expected match surplus of a buyer who has access to credit and his continuation value given by the debt limit next-period. In Figure 3.3 we represent the pairs, (d_t, d_{t+1}), that satisfy

(3.11) at equality by a red curve. We plot a truncated sequence of debt limits, (d_{T-2}, d_{T-1}, d_T), that solves (3.11), i.e., (d_{T-2}, d_{T-1}) and (d_{T-1}, d_T) are located to the left of the red curve. Under "not-too-tight" solvency constraints $\{d_t\}$ solves (3.11) where the weak inequality is replaced with an equality and hence any pair, (d_{t-1}, d_t), is on the red curve. The sequence of inequalities, (3.11), are sufficient conditions for a credit equilibrium, but they are not necessary. We will provide examples of credit equilibria that do not satisfy (3.11) in Section 3.3.

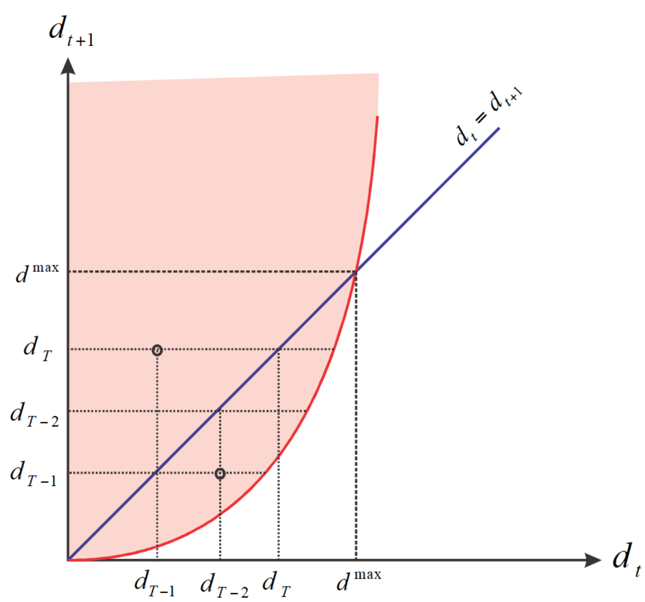

Figure 3.3: Recursive representation

3.3.1 Steady-state equilibria

We first characterize steady states where debt limits and DM allocations are constant over time, $(d_t, y_t, \ell_t) = (d, y, \ell)$ for all t. Under such restriction the incentive-compatibility condition, (3.9), or, equivalently, (3.11), can be simplified to read:

$$rd \leq \alpha \left\{ u\left[z(d)\right] - v\left[z(d)\right] \right\}, \tag{3.12}$$

where z given by (3.8) indicates the DM level of output as a function of d. The left side of (3.12) is the flow cost of repaying debt while the right side is the flow benefit from maintaining access to credit. This benefit is equal to the probability of a trading opportunity, α, times the match surplus, $u(y) - v(y)$, where $y = z(d)$. Let $d^{\max}$ denote the highest value of the debt limit that satisfies (3.12), i.e., $d^{\max}$ is the unique positive root to $rd^{\max} = \alpha\{u[z(d^{\max})] - v[z(d^{\max})]\}$. It is determined graphically in Figure 3.4 at the intersection of the left side of (3.12) that is linear and the right side of (3.12) that is concave. For all $d < d^{\max}$, the gain from defaulting is less than the cost associated with permanent autarky. The next Proposition shows that any debt limit between $d = 0$ and $d = d^{\max}$ is part of an equilibrium.

Proposition 6. ***(Steady-State Equilibria)*** *There exists a continuum of steady-state, credit equilibria indexed by $d \in [0, d^{\max}]$ with $d^{\max} > 0$.*

The two extreme debt limits, $\{0, d^{\max}\}$, correspond to the two steady-state equilibria under the AJ "not-too-tight" solvency constraints where the gain from defaulting is exactly equal to the cost of permanent autarky. Proposition 6 establishes that any debt limit in between these two extreme values is also part of an equilibrium. The intuition is as follows. For any debt d between 0 and $d^{\max}$ the gain from defaulting is strictly smaller than the cost associated with permanent autarky. So the buyer has incentives to repay such a loan. What about a slightly larger loan? The borrower cannot convince his current lender that he could repay more than d as his incentive to repay depends on how his future lenders (who he has not met yet) will "punish" him would he decide to renege on his current obligations, and these punishments are taken as given by the borrower and his lender. As a result, if a buyer offers $\ell > d$ then he only repays d, which is the repayment that keeps him trustworthy to the other sellers he may meet in future periods.

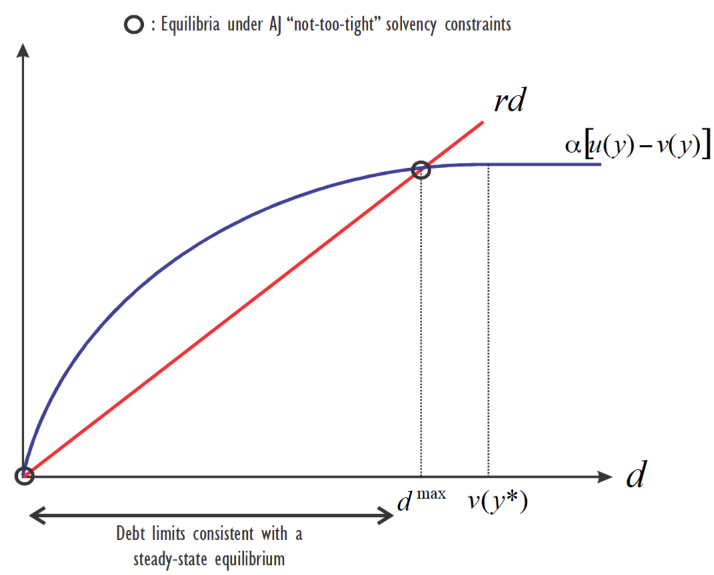

Figure 3.4: Set of debt limits at steady-state, credit equilibria

3.3.2 Periodic equilibria

Here we consider deterministic credit cycles where the extent to which buyers are trustworthy to repay their debts changes over time. We start with 2-period cycles, $\{d_0, d_1\}$, where d_0 is the debt limit in even periods and d_1 is the debt limit in odd periods. The incentive-compatibility condition, (3.9), becomes:

$$rd_t \leq \frac{\alpha \left[u(y_{(t+1)\mathrm{mod}2}) - \upsilon(y_{(t+1)\mathrm{mod}2})\right] + \beta\alpha \left[u(y_t) - \upsilon(y_t)\right]}{1+\beta}, \quad t \in \{0,1\}, \tag{3.13}$$

where we used that $\ell_t = \upsilon(y_t) = \min\{d_t, \upsilon(y^*)\}$ from (3.10). The term on the numerator on the right side of (3.13) is the buyer's expected discounted utility over the 2-period cycle starting in $t+1$. Obviously, the steady-state equilibria described in Proposition 6 are special cases of 2-period cycles; indeed, for any $d \in [0, d^{\max}]$, (d, d) satisfy (3.13). We define, for each $d_0 \in [0, d^{\max}]$,

$$\gamma(d_0) \equiv \max\{d_1 : (d_0, d_1) \text{ satisfies (3.13) with } t = 1\}, \tag{3.14}$$

the highest debt limit in odd periods consistent with a debt limit equal to d_0 in even periods. A 2-period-cycle equilibrium, or simply a *2-period cycle*, is a pair (d_0, d_1) that satisfies $d_0 \leq \gamma(d_1)$ and $d_1 \leq \gamma(d_0)$.

Lemma 4. *The function $\gamma(d)$ is positive, non-decreasing, and concave. Moreover, $d^{\min} \equiv \gamma(0) > 0$, $\gamma(d) > d$ for all $d \in (0, d^{\max})$, and $\gamma(d^{\max}) = d^{\max}$. If $\upsilon(y^*) < d^{\max}$, then $\gamma(d) = d^{\max}$ for all $d \in [\upsilon(y^*), d^{\max}]$.*

The function γ is represented in the two panels of Figure 3.5. It is non-decreasing because if the debt limit in even periods increases, then the punishment from defaulting gets larger and, as a consequence, higher debt limits can be sustained in odd periods. So there are complementarities between agents' trustworthiness in odd periods and agents' trustworthiness in even periods. The function $\gamma(d)$ is always positive because even if credit shuts down in even periods, it can be sustained in odd periods by the threat of autarky in both odd and even periods. For a given d_0 we define the set of debt limits in odd periods that are consistent with a 2-period cycle by

$$\Omega(d_0) \equiv \{d_1 : d_0 \leq \gamma(d_1), d_1 \leq \gamma(d_0)\}. \tag{3.15}$$

In Figure 3.5 the set of credit cycles is the area between γ and its mirror image with respect to the 45^o line.

Proposition 7. ***(2-Period Credit Cycles)*** *For all $d_0 \in [0, d^{\max})$ the set of 2-period cycles with initial debt limit, d_0, denoted $\Omega(d_0)$, is a nondegenerate interval.*

GMMW restrict attention to equilibria with "not-too-tight" solvency constraints, i.e., $d_0 = \gamma(d_1)$ and $d_1 = \gamma(d_0)$. Given the monotonicity and concavity of $\gamma(d)$ such equilibria do not occur outside of the 45^o line, i.e., there are no (strict) credit cycles with "not-too-tight" solvency constraints. Indeed, if $d_0 \in (0, d^{\max})$ then the maximum debt limit in odd periods is $d_1 = \gamma(d_0) > d_0$. But given d_1 the maximum debt limit in even periods is $d_0' = \gamma(d_1) > d_1 > d_0$. Following this argument we obtain an increasing sequence, $\{d_0, \gamma(d_0), \gamma(\gamma(d_0)), \dots\}$, that converges to $d^{\max}$. In contrast, by relaxing the restriction of "not-too-tight" solvency constraints we find a continuum of (strict) two-period cycle equilibria. Moreover, the set of steady-state equilibria is of measure 0 in the set of all 2-period equilibria. Indeed, for any d_0 in the interval $(0, d^{\max})$ there are a continuum of 2-period cycles where d_0 is the debt limit in even periods.

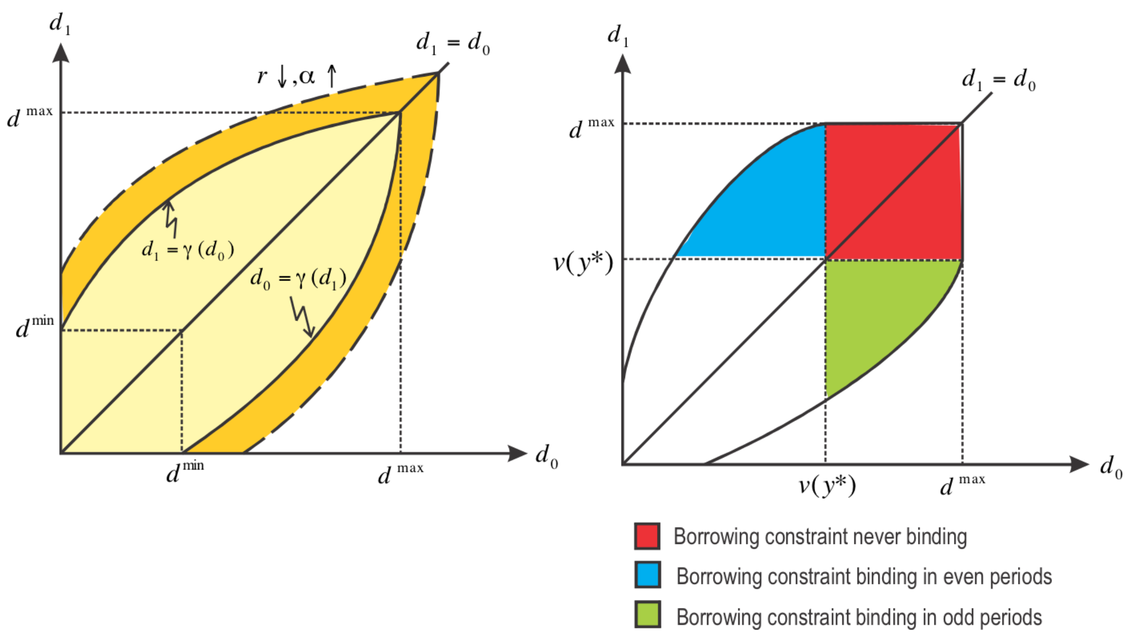

Figure 3.5: Set of two-period credit cycles

The set of credit equilibria described in Proposition 7 contains equilibria where credit dries up periodically. In the left panel of Figure 3.5 such equilibria correspond to the case where $d_0 = 0 < d_1 < \gamma(0) = d^{\min}$, i.e., even-period IOUs are believed to be worthless while odd-period IOUs are repaid. If a seller extends a loan in an even period, the buyer defaults, in accordance with equilibrium beliefs, but remains trustworthy in subsequent odd periods. Such outcomes are ruled out by backward induction in pure-currency economies. In contrast a credit economy has IOUs issued at different dates (and by different agents), and hence agents can form different beliefs regarding the terminal value of these different securities.

The result according to which there are a continuum of equilibria does not imply that everything goes. Fundamentals, such as preferences and matching technology, do matter for the outcomes that can emerge. The following corollary investigates how changes in fundamentals affect the equilibrium set.

Corollary 5. ***(Comparative statics)*** *As r decreases or α increases the set of 2-period cycles expands.*

If agents become more patient, i.e., r decreases, then γ shifts upward, as the discounted sum of future utility flows associated with a given allocation increases, and the set of 2-period cycle equilibria expands. The expansion of the equilibrium set is represented by the dark yellow area in the left panel of Figure 3.5. Similarly, if the frequency of matches, α, increases, then $d^{\max}$ increases as permanent autarky entails a larger opportunity cost, and the set of credit cycles expands.

Corollary 6. ***(Credit tightness over the cycle)***

If $r \geq \alpha\left[u(y^*) - \upsilon(y^*)\right]/\upsilon(y^*)$ then $\ell_t \leq d_t$ binds for both $t \in \{0, 1\}$ in any 2-period cycle.

If $r < \alpha\left[u(y^*) - \upsilon(y^*)\right]/\upsilon(y^*)$, then there are 2-period cycles such that $\ell_t \leq d_t$ is slack for all $t \in \{0, 1\}$, and there are 2-period cycles where $\ell_t \leq d_t$ binds only periodically.

Corollary 6 shows that if agents are sufficiently impatient, as in the left panel of Figure 3.5, then the debt limit binds and output is inefficiently low in every period for all credit cycles. However, if agents are patient, then there are equilibria where the debt limit binds periodically. Such equilibria are represented by the blue and green areas in the right panel of Figure 3.5. There are also equilibria where the debt limit fluctuates over time but never binds. These fluctuations, however, are payoff-irrelevant since the allocation is constant and the first best is implemented, $y_0 = y_1 = y^*$. These equilibria are represented by the red square, $[\upsilon(y^*), d^{\max}]^2$, in the right panel of Figure 3.5.

One can generalize the above arguments to T-period cycles, $\{d_j\}_{j=0}^{T-1}$. The debt limits must solve the following inequalities:

$$d_t \leq \frac{\alpha \sum_{j=1}^{T} \beta^j \left\{ u\left[y_{(t+j)\mathrm{mod}T}\right] - \upsilon\left[y_{(t+j)\mathrm{mod}T}\right]\right\}}{1-\beta^T}, \quad t = 0, ..., T-1 \qquad (3.16)$$

The numerator on the right side of (3.16) is the expected discounted sum of utility flows over the T-period cycle. Following the same reasoning as above:

Proposition 8. ***(T-Period Credit Cycles)*** *For any $T \geq 2$ and for all $d_0 \in [0, d^{\max})$, the set of T-period credit cycles with initial debt limit, d_0, denoted $\Omega_T(d_0)$, is a bounded, convex, and closed set in $\mathbb{R}^{T-1}$ with positive Lebesgue measure.*

Our environment can lead to cycles of any periodicity, and for a given length of the cycle there are a continuum of equilibria. As an illustration, in Figure 3.6 we represent the set of 3-period cycles for a given d_2. The outer edge of this set, which has positive measure in $\mathbb{R}^2$, is represented by a thick black curve. One can also see from the right panel that there is a non-empty set of 3-period cycles (the pink area) where credit shuts down periodically, once ($d_2 = 0$) or twice (e.g.,

$d_1 = d_2 = 0$) every three periods. Also, for our parametrization the first best is implementable, i.e., there are equilibria in the purple area with $d_t \geq d^* = 1$ for all $t \in \{0, 1, 2\}$.

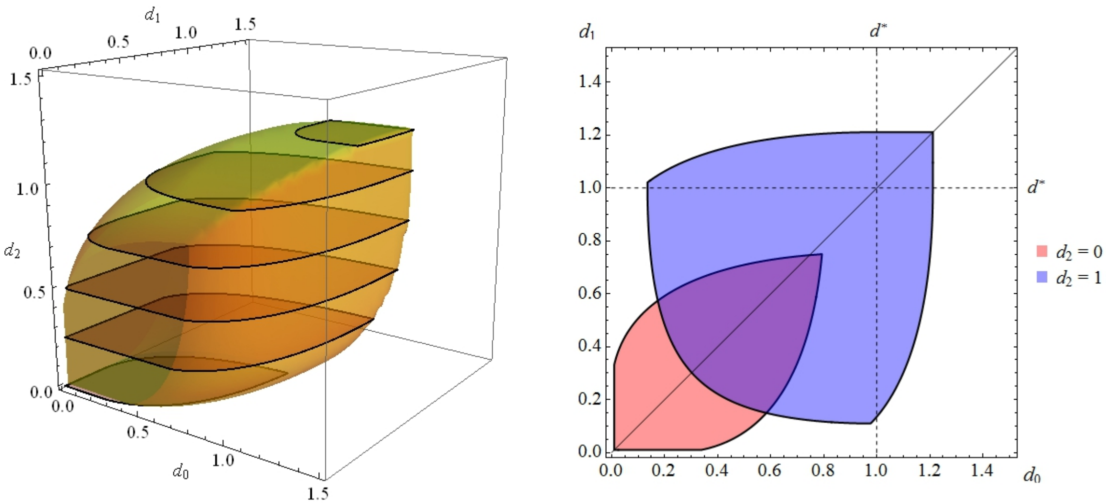

Figure 3.6: Set of three-period credit cycles: $u(y) = 2\sqrt{y}, v(y) = y, \beta = 0.9, \alpha = 0.25$

3.3.3 Monetary vs credit economies

We now consider the same environment as before but we shut down the record-keeping technology: individual trading histories are private information in a match. Without public memory credit is no longer incentive-feasible as a buyer would find it optimal to renege on his debt. Suppose that all buyers are endowed with $M = 1$ units of fiat money at time $t = 0$. Money is a perfectly divisible, storable, and intrinsically useless object, and its supply is constant over time.

The environment is now identical to the one in Lagos and Wright (2003, 2005).[20] We show in the following that any allocation, $\{(x_t, y_t)\}_{t=0}^{+\infty}$, of a pure monetary economy, where x_t is CM output and y_t is DM output, is also an allocation of a pure credit economy.

The CM price of money in terms of the numéraire good is denoted ϕ_t. The buyer's choice of money holdings in period t is the solution to the following problem:

$$\max_{m \geq 0} \left\{-\phi_t m + \beta\alpha \left[u(y_{t+1}) - \upsilon(y_{t+1})\right] + \beta\phi_{t+1} m\right\}, \tag{3.17}$$

where, from buyers' take-it-or-leave-it offers in the DM, sellers are indifferent between trading and not trading, $\upsilon(y_t) = \phi_t m$. From (3.17) it costs $\phi_t m$ to the buyer in the CM of period t to accumulate m units of money. In the following DM the buyer can purchase $y_{t+1} = \upsilon^{-1}(\phi_{t+1} m)$ if he happens to be matched with probability α. Otherwise the buyer can resell his units of money at the price ϕ_{t+1} in the CM of period $t+1$. From the first-order condition of (3.17), $\{\phi_t\}_{t=0}^{+\infty}$ solves the following first-order difference equation,

$$\phi_t = \beta\phi_{t+1} \left[1 + \alpha \frac{u'(y_{t+1}) - \upsilon'(y_{t+1})}{\upsilon'(y_{t+1})}\right]. \tag{3.18}$$

According to (3.18) the value of fiat money in period t is equal to the discounted value of money in period $t+1$ augmented with a liquidity term that captures the expected marginal surplus from holding an additional unit of money in a

[20] This version of the environment with ex-ante heterogeneity between buyers and sellers is due to Rocheteau and Wright (2005).

pairwise meeting in the DM. A monetary equilibrium is a bounded sequence, $\{(y_t, x_t, \phi_t)\}_{t=0}^{+\infty}$, that solves (3.18), with $v(y_t) = x_t = \min\{\phi_t, v(y^*)\}$

Proposition 9. ***(Monetary vs Credit Equilibria)*** *Let* $\{(y_t, x_t, \phi_t)\}_{t=0}^{+\infty}$ *be a monetary equilibrium of the economy with no record-keeping. Then,* $\{(y_t, x_t, \ell_t)\}_{t=0}^{+\infty}$ *where* $\ell_t = \min\{\phi_t, v(y^*)\}$ *is a credit equilibrium of the economy with record-keeping.*

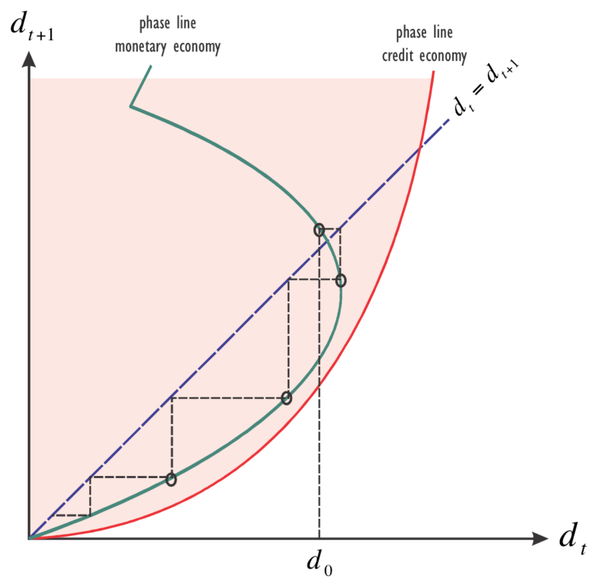

Figure 3.7: Monetary vs credit outcomes

Proposition 9 establishes that the set of (dynamic) equilibrium allocations in pure credit economies encompasses the set of equilibrium allocations of pure monetary economies taking as given the trading mechanism. This result is related to those in Kocherlakota (1998), but with a key difference: while Kocherlakota (1998) shows that the set of *all* implementable outcomes (allowing for arbitrary trading mechanisms) using money is contained in the set of all implementable outcomes with memory, we compare the equilibrium outcomes for the two economies under a *particular* trading mechanism. Later on we discuss the robustness to other trading mechanisms.

We illustrate this result in Figure 3.7 where the green, backward-bending line represents the first-order difference equation for a monetary equilibrium, (3.18), while the red area is the first-order difference inequality for a credit equilibrium, (3.11). Starting from some initial condition, d_0, we represent by a dashed line a sequence $\{d_t\}$ that satisfies the conditions for a monetary equilibrium. This sequence also satisfies the conditions for a credit equilibrium, i.e., all pairs (d_t, d_{t+1}) are located in the red area. Therefore, if the equilibrium set of a pure monetary economy contains cycles and chaotic dynamics, the same must be true for the equilibrium set of the same economy with no money but record-keeping.

The inclusion result in Proposition 9 breaks down if one imposes the "not-too-tight" solvency constraints since the phase line for credit economies differs from the phase line for monetary economies. The reason for this discrepancy is as follows. Under the "not-too-tight" solvency constraints the payment capacity of buyers, d_t, in the pure credit economy is the discounted sum of all future match surpluses,

$$d_t = \beta \left\{ \alpha \left[u(y_{t+1}) - v(y_{t+1}) \right] + d_{t+1} \right\}.$$

In pure monetary economy the payment capacity of buyers, ϕ_t, is the discounted sum of all future marginal surpluses multiplied by the value of money,

$$\phi_t = \beta \left\{ \alpha \frac{\partial \left[u \circ v^{-1}(\phi_{t+1}) - \phi_{t+1} \right]}{\partial \phi_{t+1}} \phi_{t+1} + \phi_{t+1} \right\}.$$

From the concavity of the match surplus, if $\phi_t = d_t$, then $\phi_{t+1} > d_{t+1}$.

The reverse of Proposition 9 does not hold; There are equilibria of pure credit economies that are not equilibria of pure monetary economies. As we saw above there are credit equilibria where trades shut down periodically, and such equilibria cannot be captured by Figure 3.7. (Recall that the recursive condition in Corollary 4 is sufficient but not necessary.) As another example, one can construct equilibria where the debt limit, d_t, increases in a monotonic fashion over time as buyers become more and more trustworthy. Such equilibria would not be sustainable in monetary economies.

3.3.4 Sunspot equilibria

There is a view that deterministic cycles might not provide a realistic description of actual business cycle fluctuations. Hence we show in the following that one can also construct sunspot equilibria where the DM allocation, $\{(y_\chi, \ell_\chi)\}$, depends on the realization of a sunspot state, $\chi \in \mathbb{X}$, at the beginning of the DM. Suppose that $\mathbb{X}$ is finite and the process driving the sunspot state is i.i.d. with distribution π. We assume that π has a full support, i.e., $\pi(\chi) > 0$ for all $\chi \in \mathbb{X}$. The value of a buyer along the equilibrium path solves

$$V_\chi^b = \alpha\left[u(y_\chi) - \upsilon(y_\chi)\right] + \beta \bar{V}^b \tag{3.19}$$

$$\bar{V}^b = \int V_{\chi'}^b d\pi(\chi'), \tag{3.20}$$

for all $\chi \in \mathbb{X}$. As before, the lifetime utility of a buyer is the expected discounted sum of the surpluses coming from DM trades. It follows that a sunspot credit equilibrium is a vector, $\langle d_\chi; \chi \in \mathbb{X}\rangle$, that satisfies $d_\chi \leq \beta \bar{V}^b$. Hence, $\{d_\chi\}$ satisfies

$$rd_\chi \leq \alpha \int \{u[z(d_{\chi'})] - \upsilon[z(d_{\chi'})]\} d\pi(\chi') \quad \forall \chi \in \mathbb{X} \tag{3.21}$$

Proposition 10. ***(Sunspot equilibria)*** *Suppose that $\mathbb{X}$ has at least two elements and let π be a distribution over $\mathbb{X}$ with a full support. For a given $(\chi, d_\chi) \in \mathbb{X} \times (0, d^{\max})$, the set of sunspot credit equilibria with debt limit d_χ in state χ, denoted by $\Omega_{\mathbb{X},\pi}(\chi, d_\chi)$, has a positive Lebesgue measure in $\mathbb{R}^{|\mathbb{X}|-1}$.*

3.4 Alternative trading mechanisms

In the following we show that our results regarding the equilibrium set of pure credit economies are robust to trading mechanisms other than take-it-or-leave-it offers by buyers. We also extend our model in order to parametrize buyers' temptation to renege on their debt. This extension adds a new parameter that plays a key role for the normative results in Section 3.5.

Suppose from now on that a buyer who promises to deliver ℓ units of goods in the next CM incurs the linear disutility of producing at the time he is matched

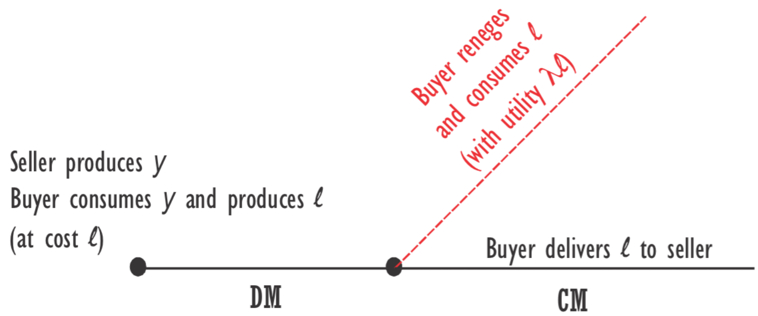

Figure 3.8: Timing of the extended model with temptation to renege

in the DM. This new timing is illustrated in Figure 3.8.[21] The effort exerted by the buyer in the DM, ℓ, is perfectly observable to the seller. At the time of delivery, at the beginning of the CM, the disutility of production has been sunk and the buyer has the option to renege on his promise to deliver the good. The buyer's utility from consuming his own output is $\lambda\ell$ with $\lambda \leq 1$. A buyer has no incentive to produce more good than the amount he promises to repay to the seller since the net utility gain from producing x units of the good for oneself is $(\lambda - 1)x \leq 0$. Although the physical environment is different, mathematically speaking, the model of the previous section can be regarded as a special case with $\lambda = 1$. As before we will focus on symmetric perfect Bayesian equilibria that satisfy (A1)-(A3).[22]

[21] The description of the buyer's incentive problem is taken from Gu *et al.* (2013a) and GMMW.

[22] GMMW also introduce an imperfect record-keeping technology as follows. At the end of the CM of period t the repayments are recorded for a subset of buyers, $\mathbb{B}_t^r \subset \mathbb{B}$, chosen at random among all buyers. The set, $\mathbb{B}_t^r$, of monitored buyers is of measure π, and the draws from $\mathbb{B}$ are independent across periods. So in every period, while his promise is always recorded, a buyer has a probability π of having his repayment decision being recorded. Any equilibrium of our model with $\pi < 1$ is also an equilibrium with $\pi = 1$. Hence, setting $\pi = 1$ is with no loss in generality.

Let $\{(d_t, y_t, \ell_t)\}_{t=0}^{\infty}$ be the sequence of equilibrium debt limits and trades. A necessary condition for the repayment of d_t to be incentive feasible is $\beta V_{t+1}^b \geq \lambda d_t$, where the left side is a buyer's continuation value from delivering the promised output and the right side of the inequality is the utility of a buyer if he keeps the output for himself, in which case he enjoys a utility flow λd_t, and goes to autarky. Following the same reasoning as before, a credit equilibrium is reduced to a sequence, $\{d_t\}_{t=0}^{\infty}$, that satisfies

$$\lambda d_t \leq \beta V_{t+1}^b = \alpha \sum_{s=1}^{+\infty} \beta^s \left[u(y_{t+s}) - \ell_{t+s} \right], \qquad t \in \mathbb{N}_0, \tag{3.22}$$

where the relationship between y_t, ℓ_t, and d_t will depend on the assumed trading mechanism.

3.4.1 Bargaining

It is standard in the literature on markets with pairwise meetings to determine the outcome of a meeting by an axiomatic bargaining solution. In this section we consider two well-known solutions: (i) the Kalai (1977) proportional bargaining solution and (ii) the generalized Nash solution. We adopt the representation of the equilibrium with solvency constraints, $\ell_t \leq d_t$, in order to obtain a convex bargaining set.[23] For a given sequence of debt limits, $\{d_t\}_{t=0}^{+\infty}$, the buyer repays $\min\{\ell_t, d_t\}$ if his date-t obligation from his DM trade is ℓ_t. Due to the linearity

[23] Even though the bargaining solution is axiomatic we could consider a simple game where upon being matched the buyer and the seller receive a proposal that they can either accept or reject. The focus here, however, is not on strategic foundations for axiomatic bargaining solutions.

of the CM value functions, the buyer's surplus from a DM trade, (y_t, ℓ_t) with $\ell_t \leq d_t$, is $u(y_t) - \ell_t$ and the seller's surplus is $-\upsilon(y_t) + \ell_t$.

Kalai proportional bargaining We amend the take-it-or-leave-it offer game by restricting the set of buyers' feasible offers: a buyer can only make offers such that the fraction of the match surplus he receives is no greater than a given $\theta \in [0, 1]$, i.e.,

$$u(y) - \ell \leq \theta[u(y) - \upsilon(y)]. \tag{3.23}$$

Thus, the buyer's offer in the DM, assuming he is in state G, solves

$$(y_t, \ell_t) = \arg\max_{y,\ell} [u(y) - \ell] \text{ s.t. (3.23) and } \ell \leq d_t. \tag{3.24}$$

According to (3.24) the buyer maximizes his utility of consumption net of the cost of repaying his debt subject to the feasibility constraint, (3.23), and the repayment constraint, $\ell \leq d_t$. The solution to (3.24) is

$$y_t = z(d_t) \equiv \min\{y^*, \eta^{-1}(d_t)\} \text{ and } \ell_t = \eta[z(d_t)]. \tag{3.25}$$

where $\eta(y) = (1 - \theta)u(y) + \theta\upsilon(y)$. In equilibrium, the buyer offers (y_t, ℓ_t) given by (3.25) and the seller accepts it. The seller rejects any offer from a buyer with state A.

Proposition 11. ***(Credit equilibrium under proportional bargaining)*** *A sequence, $\{d_t\}$, is a credit equilibrium under proportional bargaining if and only if*

$$\lambda d_t \leq \alpha\theta \sum_{i=1}^{+\infty} \beta^i \left[u(y_{t+i}) - v(y_{t+i}) \right], \quad \forall t \in \mathbb{N}_0, \tag{3.26}$$

where (y_t, ℓ_t) is given by (3.25).

Proposition 11 describes the set of all debt limits, $\{d_t\}_{t=0}^{+\infty}$, and associated allocations, $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$, that are generated by credit equilibria under bargaining weight θ. The right side of (3.26) takes into account that buyers only a receive a fraction θ of the match surplus. Note that Corollary 3 is a special case of Proposition 11 by taking $\theta = \lambda = 1$.

We can generalize Proposition 6 by showing that the set of steady-state equilibria is the interval $[0, d^{\max}]$, where $d^{\max}$ is the largest nonnegative root to $r\lambda d = \alpha\theta\{u[z(d)] - v[z(d)]\}$, and $d^{\max} > 0$ if and only if $\lambda r < \alpha\theta/(1-\theta)$. If buyers do not have all the bargaining power, then an active steady-state credit equilibrium exists only if buyers are sufficiently patient. The lower the value of θ the lower the rate of time preference that is required for credit to emerge. Indeed, if θ decreases buyers receive a lower share in current and future match surpluses and, for a given d, the amount of DM consumption they can purchase is lower. Both effects reduce the gains from participating in the DM and hence reduce the maximum sustainable debt limit. It can also be checked that a higher λ reduces $d^{\max}$. As a result any (equilibrium) allocation under $\lambda = 1$ is also an (equilibrium) allocation under $\lambda < 1$. We now move to equilibria with endogenous fluctuations.

Proposition 12. ***(2-Period Credit Cycles under proportional bargaining)*** *If $\lambda r < \alpha\theta/(1-\theta)$, then there exists a continuum of strict, 2-period, credit cycle equilibria. Moreover, if $r < \sqrt{1+\alpha\theta/[\lambda(1-\theta)]} - 1$, then there exist equilibria where credit shuts down periodically.*

Proposition 12 establishes a condition for the existence of a continuum of credit-cycle equilibria under proportional bargaining. The set of equilibria can also be represented by Figure 3.5, and, under proportional bargaining, the outer envelope shifts outward as θ increases. Moreover, if agents are sufficiently patient then there are equilibria where credit shuts down periodically, i.e., $\gamma(0) > 0$. In contrast, if we impose the "not-too-tight" solvency constraints, then there are no periodic equilibrium under proportional bargaining, irrespective of the buyer's bargaining share.[24]

Generalized Nash bargaining Under generalized Nash bargaining the terms of the loan contract are

$$(y_t, \ell_t) = \arg\max \left[u(y) - \ell\right]^{\theta} \left[\ell - \upsilon(y)\right]^{1-\theta} \quad \text{s.t.} \quad \ell \leq d_t.$$

The solution is given by (3.25) where

$$\eta(y) = \Theta(y)\upsilon(y) + \left[1 - \Theta(y)\right] u(y) \text{ and } \Theta(y) = \theta u'(y) / \left[\theta u'(y) + (1-\theta)\upsilon'(y)\right]. \tag{3.27}$$

[24] Propositions 8 and 9 regarding the existence of N-period credit cycles and the relationship between monetary and credit equilibria can be generalized to proportional bargaining in a similar fashion.

A sequence, $\{d_t\}_{t=0}^{+\infty}$, is a credit equilibrium under generalized Nash bargaining if and only if

$$\lambda d_t \leq \alpha \sum_{i=1}^{+\infty} \beta^i \left[u(y_{t+i}) - \eta(y_{t+i}) \right], \quad \forall t \in \mathbb{N}_0, \tag{3.28}$$

where y_t is the solution to (3.25).

We denote $\hat{y} = \arg\max \{u(y) - \eta(y)\}$ the output level that maximizes the buyer's surplus. Unlike the proportional solution $\hat{y} < y^*$ for all $\theta < 1$. As a result the buyer's surplus, $u(y) - \eta(y)$, in the right side of the participation constraint, (3.28), is non-monotonic with the debt limit provided that $\theta < 1$.[25] It follows that the function $\gamma(d)$ is hump-shaped, reaching a maximum at $d = \hat{d} \equiv \eta(\hat{y})$ and it is constant for $d > \eta(y^*)$. In Figure 3.9 we represent the function γ and the set of pairs, (d_0, d_1), consistent with a 2-period credit cycle equilibrium. One can see that the results are qualitatively unchanged except for the fact that the credit limits at a periodic equilibrium can be greater than the highest debt limit at a stationary equilibrium.[26] This result will have important normative implications.

The two red stars in the left panel of Figure 3.9 are the strict two-period cycles under 11not-too-tight" solvency constraints that GMMW focuses on. Such cycles are located at the intersection of γ and its mirror image with respect to the line $d_1 = d_0$. It should be clear that the non-monotonicity of the trading mechanism is necessary to obtain such cycles. It can also be checked that cycles under "not-too-tight" solvency constraints do not exist when $\lambda = 1$ (see GMMW).

[25]This non-monotonicity property of the Nash bargaining solution and its implications for monetary equilibria is discussed at length in Aruoba *et al.* (54).

[26]In the Appendix we prove that any 2-period cycle under proportional bargaining is also a 2-period cycle under Nash bargaining.

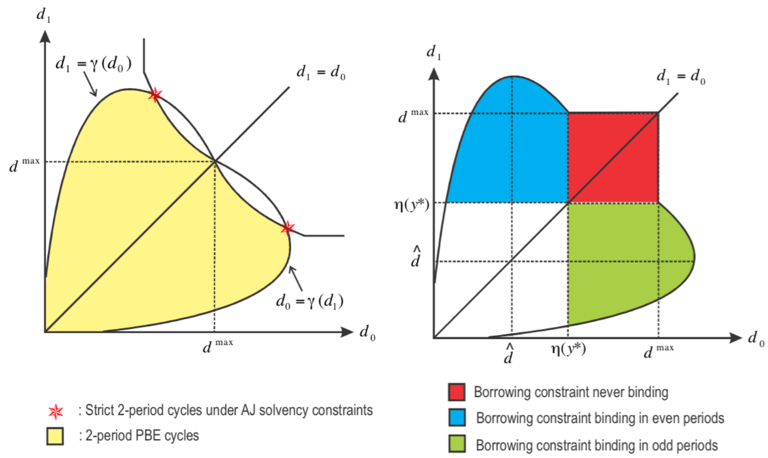

Figure 3.9: 2-period cycles under Nash bargaining or competitive pricing

In the top panels of Figure 3.10 we plot the numerical examples in Gu and Wright (2011) under generalized Nash bargaining for the following functional forms and parameter values: $u(y) = [(x+b)^{1-a} - b^{1-a}]/(1-a)$ with $a = 2$ and $b = 0.082$, $v(y) = Ay$, $\beta = 0.6$, $\alpha = 1$, $\theta = 0.01$, and $\lambda = 3/40$. In the top left panel, $A = 1.1$, the two 2-period cycles under "not-too-tight" solvency constraints are such that borrowing constraints bind periodically. In the top right panel, $A = 1.5$, the borrowing constraint binds in all periods. For both examples there exists a continuum of PBE 2-period cycles, a fraction of which feature borrowing constraints that bind periodically and a fraction of which have borrowing constraints that bind in all periods.

3.4.2 Competitive pricing

Here we follow Kehoe and Levine (1993) and AJ and assume that the terms of the loan contract in the DM are determined by competitive pricing. We reinterpret matching shocks as preference and productivity shocks, i.e., only α buyers want to consume and only α sellers can produce. As in the previous sections, buyers' repayment strategy follows a threshold rule: for a given sequence of debt limits, $\{d_t\}_{t=0}^{+\infty}$, the buyer repays $\min\{\ell_t, d_t\}$ if his date-t obligation from his DM trade is ℓ_t.[27] Moreover, the overall amount of debt issued by a buyer in the DM of period t, ℓ_t, is known to all agents. Hence, if p_t denotes the price of DM output in terms of the numéraire, a buyer's demand is subject to the borrowing constraint, $p_t y \leq d_t$. For a given $\{d_t\}_{t=0}^{+\infty}$ the market-clearing price is given by $p_t = v'(y_t)$, where

$$y_t = z(d_t) \equiv \min\{y^*, \eta^{-1}(d_t)\} \text{ and } \ell_t = \eta[z(d_t)], \tag{3.29}$$

with $\eta(y) = v'(y)y$.[28] The buyer's surplus is $u(y) - py = u(y) - v'(y)y$. For a given p, the buyer's surplus is non-decreasing in his borrowing capacity, d_t. However, once one takes into account the fact that $p = v'(y)$ then the buyer's surplus is non-monotone in his capacity to borrow, d_t. Provided that v is strictly convex, the buyer's surplus reaches a maximum for $y = \hat{y} < y^*$. A sequence, $\{d_t\}_{t=0}^{+\infty}$, is a credit equilibrium under competitive pricing if and only if (3.28) holds for all

[27] If a buyer repays $x_t \neq \ell_t$ in the CM, then each unit of IOU issued by that buyer has a payoff equal to x_t/ℓ_t units of numéraire to its owner.

[28] The buyer's problem is $\max_y \{u(y) - p_t y\}$ s.t. $p_t y \leq d_t$. The solution is $y_t = \min\left\{u'^{-1}(p_t), d_t/p_t\right\}$. Using that there is the same measure, α, of buyers and sellers participating in the market, market clearing implies $p_t = v'(y_t)$. As a result $y_t = y^*$ if $y^* v'(y^*) \geq d_t$ and $y_t v'(y_t) = d_t$ otherwise. For a detailed description of this problem in the context of a pure monetary economy, see Rocheteau and Wright (2005, Section 4).

$t \in \mathbb{N}_0$, where y_t is given by (3.29). A steady state is a d such that

$$r\lambda d \leq \alpha \left\{ u\left[z(d)\right] - v'\left[z(d)\right] z(d) \right\}. \tag{3.30}$$

Under some weak assumptions on v (for example, $\eta(y) = v'(y)y$ is convex), $d^{\max} > 0$, i.e., there exists a continuum of steady-state equilibria. This also implies that there exist a continuum of strict, 2-period, credit cycle equilibria.[29] This result can be contrasted with the ones in GMMW (Corollary 1-3) where conditions on parameter values are needed to generate a finite number (typically, two) of cycles. The right panel of Figure 3.9 illustrates these differences. Under "not-too-tight" solvency constraints credit cycles are determined at the intersection between $\gamma(d)$ and its mirror image with respect to the 45° line. These cycles are marked by a red star. If we allow for slack buyers' participation constraints, cycles are at the intersection of the area underneath $\gamma(d)$ and its mirror image with respect to the 45° line—the blue area in the figure. Finally, Proposition 9 on the equivalence result between monetary equilibria and credit equilibria holds for Walrasian pricing as well. (See the Supplementary Appendix C.0.8 for a formal proof).

We now review the numerical examples in GMMW in the case where the DM market is assumed to be competitive. The functional forms are $u(y) = y$, $v(y) = y^{1+\gamma}/(1+\gamma)$, and there are no idiosyncratic shocks, $\alpha = 1$. The first example in the bottom left panel of Figure 3.10 is obtained with the following

[29] Under competitive pricing, the function γ (analogous to (3.14)) may not be monotone or concave, but the logic for Proposition 7 does not depend on those properties. See also the supplementary appendix S2 for a formal proof of the existence of 2-period cycles.

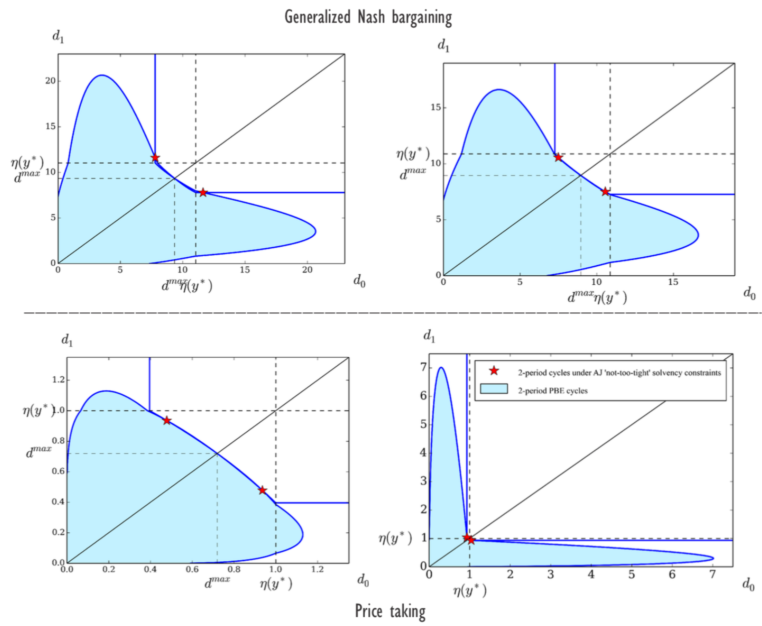

Figure 3.10: The blue area is the set of all PBE credit cycles. The red dots are credit cycles under AJ "not-too-tight" solvency constraints. The top panels are obtained under generalized Nash bargaining while the bottom panels are obtained under price taking.

parameter values: $\gamma = 2.1$, $\beta = 0.4$, $\lambda = 1/6$. GMMW identify two (strict) two-period cycles under "not-too-tight" solvency constraints , $(d_0, d_1) = (0.477, 0.936)$ and its converse, marked by red dots in the figure. The second example in the bottom right panel is obtained with the following parameter values: $\gamma = 0.5$, $\beta = 0.9$, $\lambda = 1/10$. The credit cycles under "not-too-tight" solvency constraints, $(d_0, d_1) = (0.933, 1.037)$ and its converse, are such that period allocations fluctuate between being debt-constrained and unconstrained. We find a much bigger set of PBE credit cycles represented by the blue colored region. There are a continuum of cycles such that the allocations fluctuate between being debt-constrained and unconstrained and a continuum of cycles such that agents are debt-constrained in all periods. In the second example, the credit cycle under "not-too-tight" solvency constraints is such that $(y_0, y_1) = (0.96, 1.00)$ while the most volatile PBE is $(y_0, y_1) = (0.96, 0.00)$.

3.5 Normative analysis

We now turn to the normative implications of our model. We will characterize constrained-efficient allocations under two alternative market structures: pairwise meetings and large-group meetings. We will show that the optimal mechanism for the economy with pairwise meetings is the one studied in Section 3.3 where buyers have all the bargaining power, and "not-too-tight" solvency constraints are socially optimal. Under large-group meetings the optimality of "not-too-tight" solvency constraints depends on λ that parameterizes buyers' temptation to renege on their obligations.

3.5.1 Optimal mechanism with pairwise meetings

We study the problem of a planner who chooses the allocation, $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$, in order to maximize the discounted sum of all match surpluses subject to incentive-feasibility conditions:[30]

$$\max_{\{(y_t,\ell_t)\}} \sum_{t=0}^{+\infty} \beta^t \alpha \left[u(y_t) - \upsilon(y_t)\right] \tag{3.31}$$

$$\text{s.t.} \quad \lambda \ell_t \leq \sum_{s=1}^{+\infty} \beta^s \alpha \left[u(y_{t+s}) - \ell_{t+s}\right] \tag{3.32}$$

$$\upsilon(y_t) \leq \ell_t \leq u(y_t). \tag{3.33}$$

The inequality, (3.32), is the participation constraint guaranteeing that buyers prefer to repay their debt rather than going to permanent autarky. The conditions in (3.33) make sure that both buyers and sellers receive a positive surplus from their DM trades. Coalition-proofness in pairwise meetings requires that $y_t \leq y^*$, which is satisfied endogenously (and hence ignored thereafter). We call a solution to (3.31)-(3.33) a constrained-efficient allocation (c.e.a.). In the following we use $y^{\max}$ to denote the highest, stationary level of output consistent with both the seller's and buyer's participation constraints. It is the positive solution to $\lambda r \upsilon(y^{\max}) = \alpha \left[u(y^{\max}) - \upsilon(y^{\max})\right]$.

Proposition 13. ***(c.e.a. under pairwise meetings)***

1. *If $y^* \leq y^{\max}$, then any c.e.a. is such that $y_t = y^*$ and $\ell_t \in [\upsilon(y^*), \bar{\ell}]$ for all $t \in \mathbb{N}_0$, where $\bar{\ell} = \alpha \left[u(y^*) - \upsilon(y^*)\right] / \lambda r$.*

[30] Kocherlakota (1996) and Gu *et al.* (2013a, Section 7) study a Pareto problem to determine a contract curve linking the expected discounted utilities of buyers and sellers. In contrast the planner's objective in our model is a social welfare function that aggregates the buyers' and sellers' utilities. One can interpret this social welfare function as the ex ante expected utility of a representative agent in a version of the model where the role of an agent in the DM is determined at random in each period.

2. *If $y^* > y^{\max}$, then the c.e.a. is such that $y_t = y^{\max}$ and $\ell_t = \upsilon(y_t)$ for all $t \in \mathbb{N}_0$.*

If agents are sufficiently patient (r low) and if the temptation to renege is not too large (λ low), then the first-best allocation is implementable.[31] In contrast, if $\lambda r > \alpha\left[u(y^*)/\upsilon(y^*) - 1\right]$, then the c.e.a. is $y_t = y^{\max} < y^*$, which corresponds to the highest steady state. The c.e.a. can be implemented by having buyers set the terms of the loan contract unilaterally, in which case $\ell_t = \upsilon(y_t)$ for all t.[32] By giving all the bargaining power to buyers the planner relaxes participation constraints in the CM, which allows for higher levels of output. Moreover, the solvency constraint in the buyer's bargaining problem must be "not-too-tight," in accordance with AJ's Second Welfare Theorem. We summarize this implementation result in the following Corollary.

Corollary 7. ***(Second Welfare Theorem for economies with pairwise meetings)*** *The c.e.a. is implemented with take-it-or-leave-it offers by buyers under "not-too-tight" solvency constraints.*

3.5.2 Optimal mechanism with large-group meetings

Suppose next that agents meet in a centralized location in the DM. If we do not allow for defections with coalitions, then the planner's problem is subject to

[31] Hu *et al.* (2009) derive the same condition for pure monetary economies in the case where $\lambda = 1$. A difference, however, is that the game where buyers make take-it-or-leave-it offers is not the optimal mechanism in monetary economies.

[32] Notice, however, that the c.e.a. is not uniquely implemented by the optimal mechanism. There are a continuum of equilibria under "not-too-tight" solvency constraints converging to the autarky steady state. The c.e.a. is the only bounded sequence that does not converge to the autarky equilibrium. For related results in the context of the AJ model see Bloise *et al.* (2013).

the same incentive constraints as before, (3.32) and (3.33), and Proposition 13 holds. However, the restriction according to which no coalition of agents can defect from the proposed allocation is binding when $\upsilon'' > 0$.[33] In order to prevent such defections we impose the core requirement in the DM or, equivalently, the competitive equilibrium outcome.[34] Hence, from (3.29) the terms of the loan contract are given by $\ell = \eta(y) = \upsilon'(y)y$.

The planner's problem, which is analogous to (3.31)-(3.33), is easier to solve when written recursively with the buyer's "promised utility," ω_t, as a new state variable.[35] Society's welfare, denoted $V(\omega)$, solves the following Bellman equation,

$$V(\omega) = \max_{y,\omega'} \{\alpha [u(y) - \upsilon(y)] + \beta V(\omega')\} \tag{3.34}$$

$$\text{s.t.} \quad -\eta(y) + \beta \frac{\omega'}{\lambda} \geq 0 \tag{3.35}$$

$$\omega' \geq (1+r) \{\omega - \alpha [u(y) - \eta(y)]\} \tag{3.36}$$

$$y \in [0, y^*], \ \omega' \in [0, \bar{\omega}], \tag{3.37}$$

where $\bar{\omega} = \max_{y\in[0,y^*]} [u(y) - \eta(y)] / (1-\beta)$ is an upper bound for the lifetime expected utility of a buyer. Equation (3.35) is the buyer's participation constraint in the CM that replaces (3.32) taking into account the competitive pricing mech-

[33] Indeed, a buyer and two sellers can form a deviating coalition in which each seller produces $y_t/2$ at a total cost of $2\upsilon(y_t/2) < \upsilon(y_t)$ and the buyer compensates the sellers by offering them a positive surplus, $\ell_t/2 - \upsilon(y_t/2) > 0$.

[34] See Hu *et al.* (2009) and Wallace (2013) for a related assumption in the context of monetary economies. The equivalence result between the core and competitive equilibrium allocations for economies with a continuum of agents was first shown by Aumann (1964). See supplementary appendix $S3$ for a proof of this equivalence result in the context of our model.

[35] Our recursive formulation is similar to the self-generation technique in Abreu *et al.* (1990), which characterizes the set of payoffs generated by Perfect Public Equilibria. We go beyond their characterization by providing the set of allocations and debt limits.

anism, $\ell = \eta(y) = v'(y)y$. The novelty is the promise-keeping constraint, (3.36), according to which the lifetime expected utility promised to the buyer along the equilibrium path, ω, is implemented by generating an expected surplus in the current period equal to $\alpha\left[u(y) - \eta(y)\right]$ and by promising $\beta\omega'$ for the future. In the Supplementary Appendix S4 we show that there is a unique V solution to (3.34)-(3.37) in the space of continuous, bounded and concave functions, and this solution is non-increasing. As a result, the maximum value for society's welfare is $V(0) = \max_{\omega\in[0,\bar{\omega}]} V(\omega)$, as the initial promised utility to the buyer is a choice variable.

We define two critical values for DM output:

$$\hat{y} = \arg\max_{y\in[0,y^*]} [u(y) - \eta(y)] \tag{3.38}$$

$$y^{\max} = \max\{y > 0 : \alpha[u(y) - \eta(y)] \geq r\lambda\eta(y)\}. \tag{3.39}$$

The quantity $\hat{y}$ is the output level that maximizes the buyer's surplus in the DM. The quantity $y^{\max}$ is the highest, stationary level of output that is consistent with the buyer's participation constraint in the CM. We assume that both $\hat{y}$ and $y^{\max}$ are well-defined and, for all $0 \leq y \leq y^{\max}$, $\alpha[u(y) - \eta(y)] \geq r\lambda\eta(y)$.

Proposition 14. ***(c.e.a. under centralized meetings)*** *Assume η is a convex function.*

1. *If $y^* \leq y^{\max}$, then the c.e.a. is such that $y_t = y^*$ for all $t \in \mathbb{N}_0$.*

2. *If $y^{\max} \leq \hat{y} \leq y^*$, then the c.e.a. is such that $y_t = y^{\max}$ for all $t \in \mathbb{N}_0$.*

3. *If $\hat{y} < y^{\max} < y^*$ then there are two cases:*

(a) *If* $\lambda \geq \alpha\left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$, *then the c.e.a. is such that* $y_t = y^{\max}$ *for all* $t \in \mathbb{N}_0$.

(b) *If* $\lambda < \alpha\left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$, *then the c.e.a. is such that* $y_0 \in (y^{\max}, y^*)$ *and* $y_t = y_1 \in (\hat{y}, y^{\max})$ *for all* $t \geq 1$, *where* (y_0, y_1) *is the unique solution to*

$$\max_{y_0, y_1} \left\{ u(y_0) - v(y_0) + \frac{u(y_1) - v(y_1)}{r} \right\} \tag{3.40}$$

$$s.t. \quad \eta(y_0) = \frac{\alpha[u(y_1) - \eta(y_1)]}{\lambda r}. \tag{3.41}$$

In accordance with "Folk theorems" for repeated games, provided that agents are sufficiently patient, $r \leq \alpha[u(y^*) - \eta(y^*)]/\lambda\eta(y^*)$, the first-best allocation is an equilibrium outcome. If $y^{\max} < y^*$ then the first best violates the buyers' participation constraint. In this case, the characterization of the c.e.a. depends on the ordering of $y^{\max}$ and $\hat{y}$. As shown in the left panel of Figure 3.11, if $y^{\max} \leq \hat{y}$ then a buyer's welfare and society's welfare are both increasing with y over $(0, y^{\max})$ and the highest steady state maximizes social welfare.

We now turn to the case where $\hat{y} < y^{\max} < y^*$. For all $y \in (\hat{y}, y^*)$ the buyer's surplus, $u(y) - \eta(y)$, and society's surplus, $u(y) - v(y)$, covary negatively with y, as shown in the right panel of Figure 3.11. This negative relationship gives rise to a trade-off between social efficiency and incentives for debt repayment. As a result of this trade-off the highest steady state, $y^{\max}$, might no longer be the PBE outcome that maximizes social welfare.

It is shown in the proof of Proposition 14 that it is always socially optimal to keep future output constant, $y_t = y_1$ for all $t \geq 1$. Moreover, if the first best

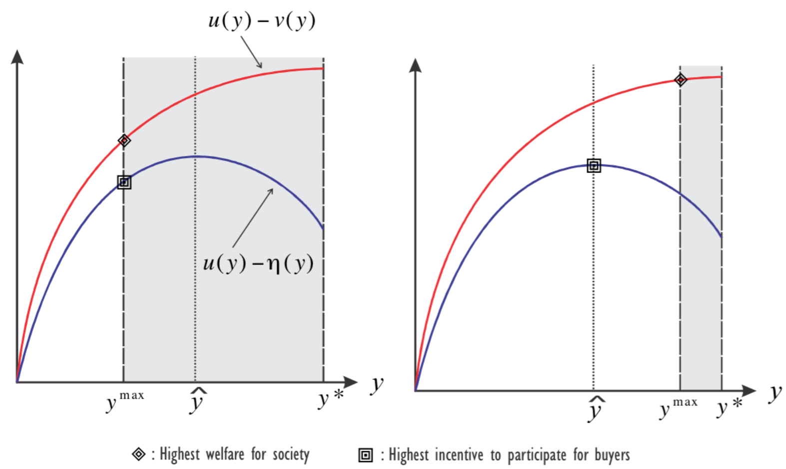

Figure 3.11: Left panel: No trade-off between efficiency and incentives over $[0, y^{\max}]$; Right panel: A trade-off between efficiency and incentives over $[\hat{y}, y^{*}]$.

cannot be achieved, then the buyer's participation constraint at $t = 0$ must be binding since otherwise y_0 could be raised without affecting any future incentive constraints. As a result of these two properties the buyer's participation constraint at $t = 0$ is given by (3.41). If $y_1 > \hat{y}$, (3.41) gives a trade-off between current and future output. The magnitude of this trade-off in the neighborhood of the highest steady state is:

$$\left.\frac{dy_1}{dy_0}\right|_{y^{\max}} = \frac{\lambda r \eta'(y^{\max})}{\alpha[u'(y^{\max}) - \eta'(y^{\max})]} < 0.$$

When $\lambda \geq \alpha\left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$ exploiting this trade-off is harmful since one would have to implement a large drop in future output below $y^{\max}$ in order to raise current output by a small amount above $y^{\max}$ while maintaining the buyer's incentive to repay his debt.

In contrast, when λ is small, it is optimal to exploit the trade-off between current and future output arising from (3.41). The optimal allocation is such that y_0 is larger than $y^{\max}$ while y_1 is lower than $y^{\max}$. Even though, in future periods, society would be better-off at the highest steady state, $u(y_1) - v(y_1) < u(y^{\max}) - v(y^{\max})$, buyers enjoy a higher surplus, $u(y_1) - \eta(y_1) > u(y^{\max}) - \eta(y^{\max})$, which relaxes their incentive constraint for repayment at $t = 0$. As a result, output and society's welfare in the initial period are higher than the highest steady-state levels, $u(y_0) - v(y_0) > u(y^{\max}) - v(y^{\max})$.[36]

In Figure 3.12 we illustrate the determination of (y_0, y_1). The red curve labelled IR corresponds to (3.41). It slops downward because of the trade-off between current and future output described above. By definition the IR curve intersects the 45$^{\text{o}}$-line at $y^{\max}$. The blue curve labelled FOC corresponds to the first-order condition of the problem (3.40)-(3.41). Given the strict concavity of the surplus function it is optimal to smooth consumption by increasing y_0 when y_1 increases. When λ is low the FOC curve is located above the IR curve at $y_1 = y^{\max}$. Hence, the optimal solution, denoted (y_0^{**}, y_1^{**}), is such that $y_0^{**} > y^{\max}$ and $y_1^{**} < y^{\max}$. The next Corollary reviews the role of "not-too-tight" solvency constraints to implement a constrained-efficient allocation.

Corollary 8. ***(Second Welfare Theorem under large-group meetings)*** *Assume that η is a convex function.*

[36] Kehoe and Levine (1993) provide an example where partial exclusion leads to a welfare-improving outcome. See their Example 2 on p. 875. In the Supplementary Appendix S5 we conduct a similar analysis for different trading mechanisms, η. We show that the results obtained under Nash bargaining are qualitatively similar to the ones obtained in this section under competitive pricing.

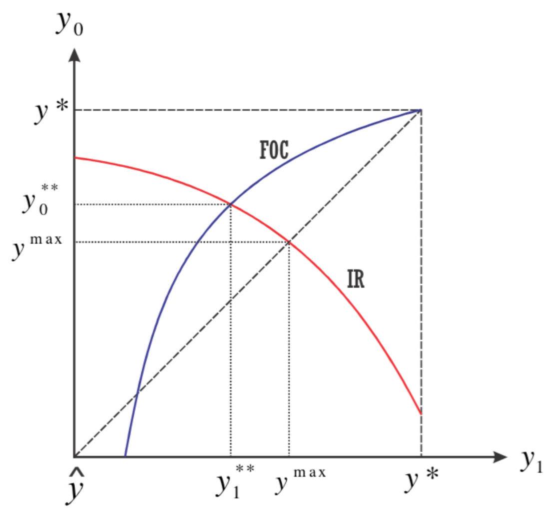

Figure 3.12: Determination of the constrained-efficient allocation, (y_0, y_1)

1. *If either* $y^{\max} \leq \hat{y} \leq y^*$ *or* $\hat{y} < y^{\max} < y^*$ *and* $\lambda \geq \alpha \left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$, *then the c.e.a. is implemented with "not-too-tight" solvency constraints.*

2. *If* $\hat{y} < y^{\max} < y^*$ *and* $\lambda < \alpha \left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$, *then the c.e.a. is implemented with slack repayment constraints (i.e., "too-tight" solvency constraints) in all future periods,* $t \geq 1$.

The failure of the AJ Welfare Theorem in the second part of Corollary 8 is surprising as one would conjecture that higher debt limits allow society to generate larger gains from trade. This reasoning is valid in a static sense. If d_t increases, the sum of all surpluses in period t, $\alpha \left[u(y_t) - v(y_t)\right]$, increases. However, there is a general equilibrium effect according to which more IOUs are competing for DM goods, which raises the price of DM goods, $p_t = v'(y_t)$. If the economy is close enough to the first best, this pecuniary externality lowers the buyers' welfare (even though society as a whole is better off) and worsens their incentive to repay their debt in earlier periods.

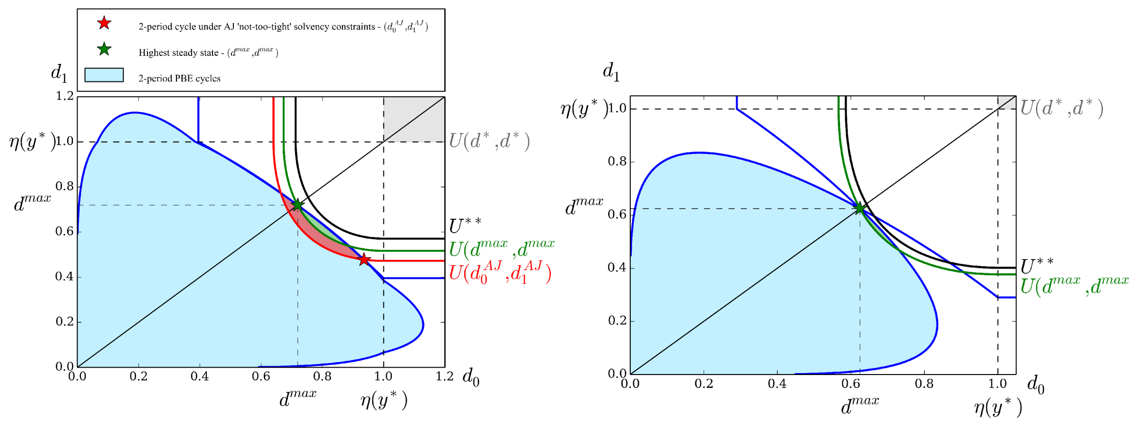

Figure 3.13: The blue area is set of all 2-period cycles. The red star is the 2-period cycle in GMMW and the green star is the highest steady state. Left panel: $\lambda = 1/6$; Right panel: $\lambda = 1/4$.

The results in Proposition 14 are robust if we restrict the equilibrium set to 2-period cycles. To see this we adopt the numerical example from the left panel of Figure 3.10, $\gamma = 2.1$, $\beta = 0.4$, $\lambda = 1/6$. For these parameter values $\lambda < \alpha \left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$. Society's welfare over a 2-period cycle is measured by $u\left[y(d_0)\right] - v\left[y(d_0)\right] + \beta \left\{u\left[y(d_1)\right] - v\left[y(d_1)\right]\right\}$. In the left panel of Figure 3.13 we highlight in red and green the set of 2-period cycles, (d_0, d_1), that dominate the equilibria under "not-too-tight" solvency constraints. There exist a continuum of such cycles that feature slack participation constraints. Hence, the imposition of "not-too-tight" solvency constraints eliminates good equilibria. Moreover, we represent society's welfare at the c.e.a. with a black indifference curve. This curve lies outside of the set of 2-period cycles (the blue area), which confirms

Part 3(b) of Proposition 14, i.e., the c.e.a. is not a 2-period cycle. The right panel of Figure 3.13 reduces λ from $\lambda = 1/6$ to $\lambda = 1/4$. The condition in Part 3(b) of Proposition 14 holds so that the highest steady state is not constrained efficient. There is no credit cycle under the "not-too-tight" solvency constraints, but there are a continuum of cycles under "too-tight" constraints, a fraction of which dominate the highest steady state.

3.6 Conclusion

We have characterized the set of equilibrium outcomes of a pure credit economy and their welfare properties. The economy features intertemporal gains from trade that can be exploited with simple one-period loan contracts. Such contracts and their execution are publicly recorded. Agents interact either through random, pairwise meetings under various trading mechanisms, as in the New-Monetarist literature, or in competitive spot markets, as in AJ. In contrast with the existing literature we have shown that such economies exhibit a continuum of steady states and a continuum of endogenous cycles of any periodicity. Moreover, any equilibrium outcome of the pure monetary economy with no record-keeping but fiat money is an outcome of the pure credit economy, but the reverse is not true.

Finally, we have characterized the constrained-efficient allocations for economies with pairwise and large-group meetings. We generalized the AJ Second Welfare Theorem to economies with pairwise meetings by showing that constrained-efficient allocations are implemented with take-it-or-leave-it offers by buyers and "not-too-tight" solvency constraints. In contrast, under large group meetings the

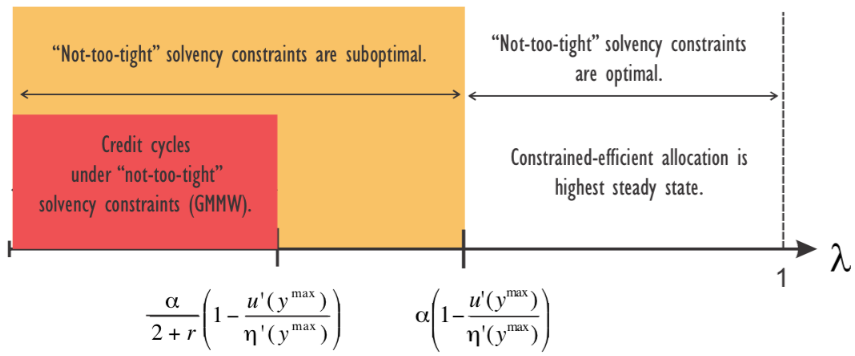

Figure 3.14: Optimality of "not-too-tight" solvency constraints and credit cycles under competitive pricing.

AJ Second Welfare Theorem fails when the temptation to renege (λ) is small. As shown in Figure 3.14, cycles under "not-too-tight" solvency constraints emerge for low values of λ (see GMMW), but for such values constrained efficiency requires slack participation constraints or, equivalently, "too-tight" solvency constraints. Hence, imposing the "not-too-tight" solvency constraint entails a loss in generality for both positive and normative analysis.

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Appendices

Appendix A

Consumer Credit, Unemployment, and Aggregate Labor Market Dynamics

A.0.1 Figures and Tables

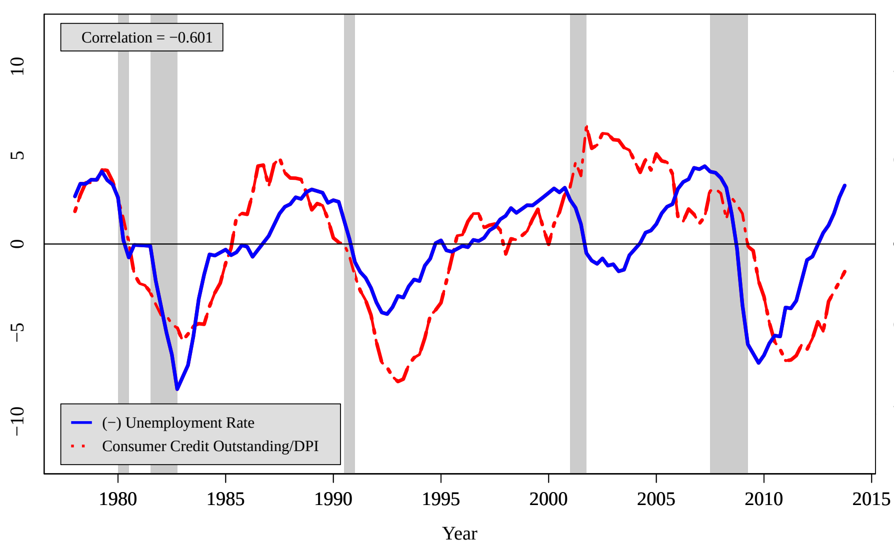

Figure A.1: Consumer credit outstanding to disposable personal income and the civilian unemployment rate. 1978 Q1-2013 Q4. Series are detrended with a Hodrick-Prescott filter with smoothing parameter $\lambda = 100,000$. Sources: Federal Reserve Board Flow of Funds Accounts. Table B.100 and Bureau of Labor Statistics. NBER recessions are shown in grey.

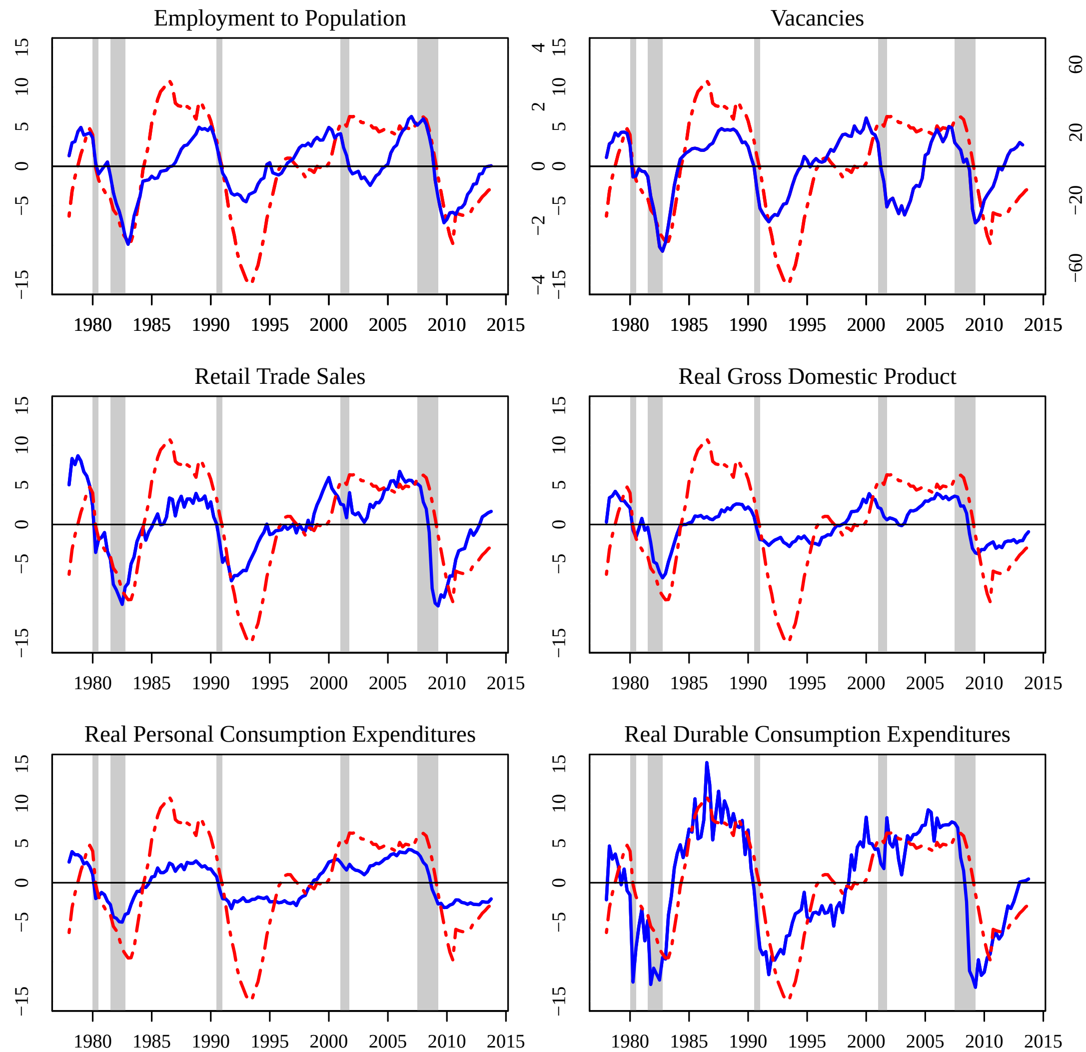

Figure A.2: Each time series is logged then de-trended with a Hodrick-Prescott filter with smoothing parameter $\lambda = 100,000$.

Table A.2: Unemployment and Consumer Credit, 2007-2009

	(1) 2007	(2) 2009	(3) Difference (2)-(1)
Consumer Debt ($)	4,780	5,441	661*
	(150)	(340)	
Credit Card Debt ($)	2,438	2,226	-212
	(113)	(113)	
Auto Debt ($)	2,513	3,086	573*
	(97)	(283)	
Credit Card {0,1}	0.67	0.62	-0.43***
	(0.01)	(0.01)	
CC Monthly Charges ($)	447	393	-54**
	(16.5)	(14.4)	
CC Debt Limit ($1,000)	15.74	14.50	-1.24*
	(0.52)	(0.48)	
Applied	0.63	0.43	-0.20***
	(0.01)	(0.01)	
Denied | Applied	0.21	0.27	0.06***
	(0.01)	(0.01)	
Income ($1000)	44.72	42.82	-1.90*
	(0.61)	(0.61)	
Weekly Hours	29.96	25.64	-4.32***
	(0.34)	(0.34)	
Observations	3,820	3,820	

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. Dollar values represent real 2007 dollars adjusted using the CPI.

Table A.1: Dynamics of Consumer Credit and Macroeconomic Aggregates, 1978-2013

	(1)	(2)	(3)
	Std(Variable)	Corr(Variable,CC)	Std(CC)/Std(Variable)
Consumer Credit	0.062	1.000	1.000
Unemployment Rate	0.012	-0.617	5.211
Employment/Population	0.010	0.646	6.462
Vacancies	0.184	0.511	0.336
Retail Trade Sales	0.043	0.643	1.423
Gross Domestic Product	0.025	0.721	2.529
Personal Consumption Expenditures	0.025	0.543	2.456
Durable Personal Consumption Expenditures	0.067	0.771	0.918

Note: All series have been logged and detrended using a Hodrick-Prescott Filter with a smoothing parameter $\lambda = 100,000$. Consumer credit corresponds to total consumer credit outstanding from the the Federal Reserve Board's Flow of Funds, Table B.100 (FRED Series: TOTALSL) . The unemployment rate is the civilian unemployment rate (FRED Series: UNRATE). The employment to population ratio is the fraction of the civilian noninstitutional population employed (FRED Series EMRATIO). Vacancies are correspond to the composite Help-Wanted Index calculated by Barnichon (2010). Retail trade sales are given by the OECD Main Economic Indicators (FRED Series: SLRTTO02USA189N). Gross domestic product, personal consumption expenditures, and durable personal consumption expenditures come from the National Income and Product Accounts (FRED Series GDPC1, PCEC, and PCDG, respectively).

Table A.3: Balancing Test, Pre-Treatment 2007

	(1) $EU = 0$	(2) $EU = 1$	(3) Difference
Individual Characteristics			
Male	0.34	0.33	-0.01
	(0.01)	(0.03)	
Black	0.20	0.23	0.03
	(0.01)	(0.03)	
Age	48.0	43.1	-4.88***
	(0.25)	(0.77)	
High School	0.41	0.39	-0.02
	(0.01)	(0.03)	
Some College	0.22	0.19	-0.03
	(0.01)	(0.02)	
College Degree	0.37	0.42	0.05*
	(0.01)	(0.03)	
Outcome Variables			
Consumer Debt ($)	4,711	5,525	814
	(148)	(784)	
Credit Card Debt ($)	2,457	3,304	946
	(102)	(756)	
Auto Debt ($)	2,540	2,221	-319
	(102)	(305)	
Credit Card {0,1}	0.67	0.65	-0.02
	(0.01)	(0.03)	
CC Monthly Charges {0,1}	0.53	0.58	0.05
	(0.01)	(0.03)	
CC Monthly Charges ($)	441	507	66.4
	(16.1)	(89.1)	
CC Debt Limit ($1,000)	16.20	10.75	-5.45***
	(0.56)	(1.08)	
Applied	0.64	0.56	-0.08**
	(0.01)	(0.01)	
Denied | Applied	0.21	0.20	-0.01
	(0.01)	(0.03)	
Income ($1000)	45.43	37.10	-8.33***
	(0.76)	(2.36)	
Labor Income ($1000)	28.43	27.85	-0.47
	(0.66)	(2.03)	
Weekly Hours	29.42	35.87	6.45***
	(0.37)	(1.09)	
Observations	3,515	305	

Note: See note in Table A.2.

Table A.9: Calibration Summary: Parameters and Stochastic Steady State Targets

Description	Value	Source/Target
Labor Market Parameters		
matching curvature, η_L	0.710	job filling rate, JOLTS
separation rate, δ	0.032	unemployment rate, BLS
vacancy posting costs, k	0.150	Silva and Toledo (2009)
labor bargaining weight, λ	0.500	normalization
utility from leisure, ℓ	0.250	Hall and Milgrom (2008)
unemployment income, b	0.700	average decline in credit upon job loss, estimated in Section 1.2.2, SCF
Goods/Credit Market Parameters		
matching curvature, η_G	1.27	debt to income, FRB Z.1 Flow of Funds
utility curvature, γ	0.390	MPC out of credit, Gross and Souleles (2002)
mean of labor productivity, μ_z	1.000	normalization
persistence of labor productivity, ρ_z	0.884	output per job, BLS
s.d. of labor productivity shock, σ_z	0.0075	output per job, BLS
mean of agg. financial conditions, μ_ν	0.512	FRB Z.1 Flow of Funds
persistence of agg. financial conditions, ρ_ν	0.967	FRB Z.1 Flow of Funds
s.d. of agg. financial conditions, σ_ν	0.0083	FRB Z.1 Flow of Funds

Table A.4: Impact of Unemployment on Changes in Consumer Debt, 2007-2009

	(1)	(2)	(3)
	ΔConsumer Debt ($)	ΔCredit Card Debt ($)	ΔAuto Debt ($)
EU×$\mathbb{I}\{t = 2009\}$	-2,809**	-2,504***	-173
	(941)	(771)	(550)
Observations	7640	7640	7640
R^2	0.01	0.03	0.03
Demographic Controls	Y	Y	Y

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.

Table A.5: Impact of Unemployment on Credit Card Use, 2007-2009

	(1)	(2)	(3)
	ΔCredit Card {0,1}	ΔCC Charges {0,1}	ΔCC Charges ($)
EU$\times\mathbb{I}\{t = 2009\}$	-0.06***	-0.15***	-250*
	(0.04)	(0.04)	(89.5)
Observations	7640	7640	7640
R^2	0.20	0.18	0.07
Demographic Controls	Y	Y	Y

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.

Table A.6: Impact of Unemployment on Credit Applications and Denials, 2007-2009

	(1)	(2)	(3)	
	ΔApplied	ΔDenied|Applied	ΔDenied$_{Credit}$|Applied	ΔDenied$_{Emp./Inc.}$|Applied
EU$\times\mathbb{I}\{t = 2009\}$	0.14***	0.05***	-0.07	0.07**
	(0.04)	(0.02)	(0.05)	(0.03)
Observations	7640	4112	4112	4112
R^2	0.10	0.08	0.07	0.02
Demo. Controls	Y	Y	Y	Y

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.

Table A.7: Impact of Unemployment on Income and Assets, 2007-2009

	(1)	(2)	(3)	
	ΔIncome	ΔUnemp. Benefits	ΔLiquid Assets	ΔNo Liquid{0,1}
EU×$\mathbb{I}\{t = 2009\}$	-20,000***	1,188***	-5,292*	0.15***
	(2,957)	(212)	(3,184)	(0.03)
Observations	7640	7640	7640	7640
R^2	0.20	0.18	0.07	0.07
Demographic Controls	Y	Y	Y	Y

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.

Table A.8: Functional Forms and Stochastic Processes

labor market matching	$m(s,o) = \frac{so}{(s^{\eta_L}+o^{\eta_L})^{1/\eta_L}}$
goods market matching	$\alpha(n) = \frac{n}{(n^{\eta_G}+1)^{1/\eta_G}}$
DM utility	$u(y) = \frac{y^{1-\gamma}}{1-\gamma}$
aggreagte productivity	$\hat{\hat{z}}_{t+1} = \rho_z \hat{z}_t + \epsilon_{z,t}$ where $\epsilon_{z,t} \sim N(0, \sigma_z^2)$
aggregate financial frictions	$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \epsilon_{\nu,t}$ where $\epsilon_{\nu,t} \sim N(0, \sigma_{]} nu^2)$

A.0.2 Derivation of Wage equation

From (1.16), we can write $V_t(1)$ and $V_t(0)$ combining (1.5) and (1.22) as

$$V_t(1) = \alpha(n_t)\mu[\upsilon(y_t^1) - y_t^1] + w_t + \beta\big[(1-\delta)V_{t+1}(1) + \delta V_{t+1}(0)\big] \quad \text{(A.1)}$$

$$V_t(0) = \alpha(n_t)\mu[\upsilon(y_t^0) - y_t^0] + (\ell + b) + \beta[p(\theta_t)V_{t+1}(1) + (1-p(\theta_t))V_{t+1}(0)] \quad \text{(A.2)}$$

Solving for $V_t(1)$ in (A.1) and subtracting $V_t(0)$ obtains the surplus of an employed worker

$$\begin{aligned} V_t(1) - V_t(0) = {} & \alpha(n_t)\mu\big\{[\upsilon(y_t^1) - y_t^1] - [\upsilon(y_t^0) - y_t^0]\big\} + w_t - (\ell + b) \\ & + \beta(1-\delta-p(\theta_t))[V_{t+1}(1) - V_{t+1}(0)] \end{aligned} \quad \text{(A.3)}$$

From (1.26) and the free entry condition $k = \beta f(\theta_t)J_{t+1}$, we can write (A.3) as

$$\begin{aligned} V_t(1) - V_t(0) = {} & \alpha(n_t)\mu\big\{[\upsilon(y_t^1) - y_t^1] - [\upsilon(y_t^0) - y_t^0]\big\} + w_t - (\ell + b) \\ & + (1-\delta-p(\theta_t))\frac{\lambda}{1-\lambda}\frac{k}{f(\theta_t)} \end{aligned} \quad \text{(A.4)}$$

Similarly, from (1.25), (1.26), and the free entry condition we can write the value of a filled job as

$$J_t = z_t - w_t + (1-\delta)\frac{k}{f(\theta_t)} \quad \text{(A.5)}$$

Combining (A.4) and (A.5) using (1.26) we obtain

$$(1-\lambda)[\alpha(n_t)\mu\big\{[\upsilon(y_t^1)-y_t^1]-[\upsilon(y_t^0)-y_t^0]\big\}+w_t-(\ell+b)]=\lambda[z_t-w_t+\lambda\theta_t k] \tag{A.6}$$

where we have used the result that $p(\theta_t)=\theta_t f(\theta_t)$. Rearranging (A.6) yields the wage equation (1.27)

$$w_t=\lambda[z_t(w_t)+\theta_t k]+(1-\lambda)(b+\ell-\alpha(n_t)\mu[S^1(w_t)-S^0])=\Gamma_t(w_t) \tag{A.7}$$

A.0.3 Proofs of Lemmas and Propositions

Proof of Lemma 2: Part (i): Taking the derivative with respect to θ in (1.31) we have

$$\frac{\partial S^f}{\partial\theta}=-\lambda k+\frac{(1-\lambda)\alpha(n)\nu[\upsilon'(y^1)-1]}{(1-\mu)\upsilon'(y^1)+\mu}\frac{\partial w}{\partial\theta} \tag{A.8}$$

Taking derivative of (1.29) with respect to θ yields

$$\frac{\partial w}{\partial\theta}=\frac{\lambda k}{1-(\lambda-\mu)\alpha(n)\frac{\nu[\upsilon'(y^1)-1]}{(1-\mu)\upsilon'(y^1)+\mu}} \tag{A.9}$$

Evaluated at the equilibrium wage, the denominator in (A.9) is positive, hence $\partial w/\partial\theta>0$. Let $\partial S^1/\partial w=nu[\upsilon'(y^1)-1]/[(1-\mu)\upsilon'(y^1)+\mu]$. Combining (A.8) and (A.9) we can write $\partial S^f/\partial\theta$ as

$$\frac{\partial S^f}{\partial\theta}=-\lambda k\frac{1-(1-\mu)\alpha(n)\partial S^1/\partial w}{1-(\lambda-\mu)\alpha(n)\partial S^1/\partial w} \tag{A.10}$$

The sign of (A.10) depends on the magnitude of $\partial S^1/\partial w$. Its maximum value is $\nu\mu/(1-\mu)$ when $y^1 = 0$. Since $(1-\mu)\alpha(n)\nu\mu/(1-\mu) \leq 1$, then $\partial S^f/\partial\theta \leq 0$ for any n.

Part (ii and iii): Taking the derivative with respect to n in (1.31) yields

$$\frac{\partial S^f}{\partial n} = (1-\lambda)\alpha'(n)[S^1 - S^0] + (1-\lambda)(1-\mu)\xi S^0 + (1-\lambda)\alpha(n)\frac{\partial S^1}{\partial w}\frac{\partial w}{\partial n} \quad \text{(A.11)}$$

where $\xi = \frac{\alpha'(n)n - \alpha(n)}{n^2} < 0$. If $S^0 = 0$ the result in part (ii) immediately follows. Taking the derivative of (1.29) with respect to n yields.

$$\frac{\partial w}{\partial n} = \frac{(\lambda-\mu)\alpha'(n)S^1 + \lambda(1-\mu)\xi S^0}{1-(\lambda-\mu)\alpha(n)\partial S^1/\partial w} \quad \text{(A.12)}$$

The sign of the denominator is positive at the equilibrium wage. However, the sign of the numerator is ambiguous since $\xi < 0$. Plugging (A.12) into (A.11) yields

$$\frac{\partial S^f}{\partial n} = (1-\lambda)\alpha'(n)[S^1 - S^0] + (1-\lambda)(1-\mu)\xi S^0 + \frac{(1-\lambda)\alpha(n)(\lambda-\mu)\alpha'(n)S^1\frac{\partial S^1}{\partial w} + (1-\lambda)\lambda(1-\mu)\xi\alpha(n)S^0\frac{\partial S^1}{\partial w}}{1-(\lambda-\mu)\alpha(n)\partial S^1/\partial w} \quad \text{(A.13)}$$

Proof of Lemma 3: From Lemma 2, we know $\partial S^f/\partial\theta \leq 0$ which implies $\partial S^f/\partial J \leq 0$. Let $\bar{S}_t$ be the solution to $S^f(1-u, 0)$. From (1.31), $\bar{S}_f > 0$ for any u. Since the right-hand side of (1.34) is strictly increasing and crosses through the origin, there exists a unique fixed point of the problem. Further since S^f is bounded below by $(1-\lambda)(\bar{z}-\ell) - \lambda\theta k$, $J^* > J^{DMP}$.

A.0.4 Continuous Time Derivation

To come. Available upon request.

A.0.5 Data Appendix

Survey of Consumer Finances, 2007-2009 'Denied' Variable Definitions

Denied because of credit related reasons includes households that were told they haven't established a credit history, credit rating service reports, credit records/history from another institution, bankruptcy, amounts of debt, size of other payments, or ability to repay loan too high, insufficient credit references, or other credit characteristics of the borrower.

Denied because of asset related reasons includes lack of assets, collateral, property to secure the loan or insufficient collateral or equity.

Denied because of income related reason includes lack of assets, collateral or property to secure the loan, time on current job, the type of job or work (i.e. steady or secure, a good job), lack of job or not working, amount of income or the source of income for retired households, and any other financial characteristics of the household.

Appendix B

Aggregate Unemployment and Household Unsecured Debt

B.0.6 Tables

	$\nu = .035$	$\nu = .065$	Markup = 20%	
	Decrease both	Decrease both	Decrease both	% change
	% change	% change	% change	2008-1980
Credit & Goods Market				
Credit to Con., $\alpha(n)\omega\bar{d}/C$	-70.6	-70.6	-70.6	-70.6
M2 to Cons., $(1-\omega)R\tilde{a}/C$	7.01	33.3	31.0	18.3
Agg. productivity, z	-4.63	-4.34	-3.58	
Liquidity shocks, $\alpha(n)$	-0.94	-0.85	-0.64	
Labor Market				
Unemployment rate (%)	34.6	31.3	23.7	40.0
Job Finding Rate, p	-27.1	-25.1	-20.2	16.1

Table B.1: Unemployment and Credit, 1970-2008, Sensitivity Analysist

	$\nu = .035$	$\nu = .065$	Markup = 20%	
	Decrease both	Decrease both	Decrease both	% change
	% change	% change	% change	2008-2011
Credit & Goods Market				
Credit to Con., $\alpha(n)\omega\bar{d}/C$	-22.9	-22.9	-22.9	-22.9
M2 to Cons., $(1-\omega)R\tilde{a}/C$	19.9	26.2	25.4	11.0
Agg. productivity, z	-1.51	-1.36	-1.14	
Liquidity shocks, $\alpha(n)$	-0.21	-0.20	-0.16	
Labor Market				
Unemployment rate (%)	8.21	7.33	6.02	74.1
Job Finding Rate, p	-8.00	-7.19	-5.99	-46.6

Table B.2: Unemployment and Credit, 2008-2011, Sensitivity Analysis

B.0.7 Proofs of Lemmas and Propositions

Proof of Proposition 1 First we derive Equation (2.32) that determines the debt limit. Let $\Lambda_e = [-a + \beta U_e(a)] - \left[-\tilde{a} + \beta \tilde{U}_e(\tilde{a})\right]$, where a and $\tilde{a}$ are the optimal choices of liquid assets of households with and without access to credit, respectively. From (2.14) and (2.16) it can be checked that

$$
\begin{aligned}
U_1(a) =& \alpha(n)\mu\left[\upsilon(y) - y\right] + (R-1)a + \Delta - T + (1-\delta)\left[w + \beta U_1(a)\right] \\
& + \delta\left[\ell + b + \beta U_0(a)\right] \\
\tilde{U}_1(\tilde{a}) =& \alpha(n)\mu\left[\upsilon(\tilde{y}) - \tilde{y}\right] + (R-1)\tilde{a} + \Delta - T + (1-\delta)\left[w + \beta \tilde{U}_1(\tilde{a})\right] \\
& + \delta\left[\ell + b + \beta \tilde{U}_0(\tilde{a})\right] \\
U_0(a) =& \alpha(n)\mu\left[\upsilon(y) - y\right] + (R-1)a + \Delta - T + p\left[w + \beta U_1(a)\right] \\
& + (1-p)\left[\ell + b + \beta U_0(a)\right] \\
\tilde{U}_0(\tilde{a}) =& \alpha(n)\mu\left[\upsilon(\tilde{y}) - \tilde{y}\right] + (R-1)\tilde{a} + \Delta - T + p\left[w + \beta \tilde{U}_1(\tilde{a})\right] \\
& + (1-p)\left[\ell + b + \beta \tilde{U}_0(\tilde{a})\right].
\end{aligned}
$$

After some calculation,

$$
\begin{aligned}
\Lambda_1 &= -(1-\beta R)(a-\tilde{a}) + \beta\alpha(n)\mu\left\{\left[\upsilon(y) - y\right] - \left[\upsilon(\tilde{y}) - \tilde{y}\right]\right\} + (1-\delta)\beta\Lambda_1 + \delta\beta\Lambda_0 \\
\Lambda_0 &= -(1-\beta R)(a-\tilde{a}) + \beta\alpha(n)\mu\left\{\left[\upsilon(y) - y\right] - \left[\upsilon(\tilde{y}) - \tilde{y}\right]\right\} + p\beta\Lambda_1 + (1-p)\beta\Lambda_0.
\end{aligned}
$$

It follows that $\Lambda_1 = \Lambda_0 = \Lambda$ where

$$
\Lambda = \frac{-(1+r-R)(a-\tilde{a}) + \alpha(n)\mu\left\{\left[\upsilon(y) - y\right] - \left[\upsilon(\tilde{y}) - \tilde{y}\right]\right\}}{r}. \tag{B.1}
$$

Thus, the cost of losing access to credit is the same for employed and unemployed households. From (2.29) and (B.1), $\bar{d}$ is a fixed point to (2.32). Next, we prove the claims in Proposition 1 by distinguishing the case where households with no access to credit hold liquid assets (case 1) from the case where they don't (case 2).

Case 1: $\tilde{a} > 0$. From (2.25) this requires $(1 + r - R)/R < \alpha(n)\nu\mu/(1-\mu)$, i.e., $R > \underline{R}$.

The left side of (2.32), $r\bar{d}$, is linear. Let us turn to the right side of (2.32), $\Gamma(\bar{d})$. If $\bar{d} \leq R\nu\tilde{a}$, then the debt limit is less than the payment capacity of households with no access to credit. It follows from (2.25) that households with access to credit will choose the same payment capacity as the one of households with no access to credit, i.e., $\bar{d} + R\nu\hat{a} = R\nu\tilde{a}$ and $y = \tilde{y}$, since they face the same marginal condition for the choice of liquid assets. Consequently, the right side of (2.32) is linear, $\Gamma(\bar{d}) \equiv \rho\,(1 + r - R)\,\bar{d}/R\nu$. Since $\Gamma(0) = 0$, $\bar{d} = 0$ is a solution to (2.32).

If $\bar{d} > R\nu\tilde{a}$, then the debt limit is greater than the payment capacity of households with no access to credit. Consequently, from (2.25), households with access to credit choose not to accumulate liquid assets, $a = 0$, and the derivative of Γ is

$$\Gamma'(\bar{d}) \equiv \rho\alpha(n)\mu\left[\frac{\upsilon'(y) - 1}{(1-\mu)\upsilon'(y) + \mu}\right] \geq 0.$$

In that case $\Gamma(\bar{d})$ is a concave function of $\bar{d}$. For all $\bar{d} \geq (1-\mu)\upsilon(y^*) + \mu y^*$, $y = y^*$ and $\Gamma'(\bar{d}) = 0$. A necessary and sufficient condition for a unique $\bar{d} > 0$ solution to (2.32) to exist is that the slope of the left side of (2.32) is less than the slope of the right side of (2.32) evaluated at $\bar{d} = 0$, i.e., $r < \Gamma'(0)$. This condition can be rewritten as $r < \rho\left(\frac{1+r-R}{R\nu}\right)$ or $R < \bar{R}$.

If $R = \bar{R}$, then $r = \Gamma'(0)$ and $\Gamma(\bar{d}) = r\bar{d}$ for all $\bar{d} \leq R\nu\tilde{a}$. Moreover, for all $\bar{d} > R\nu\tilde{a}$, $\Gamma(\bar{d})$ is located underneath $r\bar{d}$ so that there is no solution to $\Gamma(\bar{d}) = r\bar{d}$. This proves the claim that if $R = \bar{R}$, then any $\bar{d} \in [0, R\nu\tilde{a}]$ is a solution to (2.32).

Case 2: $\tilde{a} = 0$. This happens if $\frac{1+r-R}{R} \geq \frac{\alpha(n)\nu\mu}{1-\mu}$, i.e., $R \leq \underline{R}$. In this case (2.32) can be expressed as

$$r\bar{d} = \rho\alpha(n)\mu\left[\upsilon\left(y\right) - y\right].$$

It follows that $\bar{d} > 0$ iff $r < \frac{\rho\alpha(n)\mu}{1-\mu}$. Finally, notice that $\underline{R} < \bar{R}$ iff $r < \frac{\rho\alpha(n)\mu}{1-\mu}$.

Putting together cases 1 and 2, $\bar{d} > 0$ is unique iff $\underline{R} < \bar{R}$ and $R < \bar{R}$, as claimed in Proposition 1.

Proof of Proposition 2 We establish first that n, y and $\tilde{y}$ are monotone, continuous functions of θ. From (2.38), $n = 1 - u = m(1,\theta)/[m(1,\theta) + \delta]$ is a continuous and increasing function of θ. Therefore, from (2.36), $\tilde{y}$ is a continuous and non-decreasing function of θ. By virtue of the assumption, $r < \rho\alpha(n_0)\mu/(1-\mu)$, for all $\theta \geq \theta_0$ the condition, $r < \rho\alpha(n)\mu/(1-\mu)$, holds. Therefore, from Proposition 1, for all $\theta \geq \theta_0$ and $R < \bar{R} \equiv \rho(1+r)/(r\nu+\rho)$, there exists a unique $\bar{d} > 0$ solution to (2.32). Moreover, $\bar{d}$ is continuous and increasing (Corollary 2) with θ. From (2.35) for all $\theta \geq \theta_0$, y is continuous and increasing with θ. Let us turn to the mapping defining equilibrium market tightness:

$$\Psi(\theta) \equiv \frac{(r+\delta)\,k}{m\left(\frac{1}{\theta},1\right)} + \beta\lambda\theta k$$
$$-(1-\lambda)\left\{\frac{\alpha\,[n(\theta)]}{n(\theta)}(1-\mu)\left\{\omega\left[\upsilon\left(y(\theta)\right)-y(\theta)\right]+(1-\omega)\left[\upsilon\left(\tilde{y}(\theta)\right)-\tilde{y}(\theta)\right]\right\}+\Phi\right\},$$

where $\Phi = +\bar{z} - \ell - b$. From (2.39) an equilibrium value for θ solves $\Psi(\theta) = 0$. From the assumption that $(1-\lambda)(\bar{z}-\ell-b) > (r+\delta)\,k$, and using the definition of θ_0, it can be checked that

$$\Psi(\theta_0) =$$
$$-(1-\lambda)\left\{\frac{\alpha\,[n(\theta_0)]}{n(\theta_0)}(1-\mu)\left\{\omega\left[\upsilon\left(y(\theta_0)\right)-y(\theta_0)\right]+(1-\omega)\left[\upsilon\left(\tilde{y}(\theta_0)\right)-\tilde{y}(\theta_0)\right]\right\}\right\}$$
$$< 0.$$

Since $\Psi(\infty) = +\infty$, there exists a $\theta > \theta_0$ solution to $\Psi(\theta) = 0$. Moreover, since $r < \rho\alpha(n_0)\mu/(1-\mu) \leq \rho\alpha(n)\mu/(1-\mu)$, the corresponding debt limit is positive, $\bar{d} > 0$.

Appendix C

Dynamic Indeterminacy and Welfare in Credit Economies

C.0.8 Proofs of Lemmas and Propositions

Proof of Proposition 5 **(⇒)** Here we prove necessity. Suppose that $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$ is an equilibrium outcome in a credit equilibrium, (s^b, s^s).

(i) Here we show condition (3.4). Because the worst payoff to buyers at each period is 0 (autarky) while the equilibrium payoff at period t is $u(y_t) - x_t$, condition (3.4) is necessary for buyers to repay their promises at each period.

(ii) To show condition (3.5), we first show that $x_t = v(y_t)$ for all t. Note that (A3) implies that $x_t = \ell_t$ for all t. If $x_t < v(y_t)$, then the seller would not accept the offer. Suppose, by contradiction, that $x_t > v(y_t)$. Then, the buyer may deviate and offer (y', ℓ_t) with $v(y') \in (v(y_t), \ell_t)$. Because this deviation does not affect the buyer's public record and the buyer has the same incentive to repay his debt, it is dominant for the seller to accept it. It then is a profitable deviation because $y' > y_t$.

Next, to show that $y_t \leq y^*$ for all t, suppose, by contradiction, that $y_t > y^*$ and hence $u(y_t) \geq x_t \geq v(y_t) > v(y^*)$. Then there exists an alternative offer, $(y', \ell') = (y', x')$, such that $u(y') - x' > u(y_t) - x_t$ and $-v(y') + x' > -v(y_t) + x_t$ and $\ell' \leq \ell_t$. It is dominant for the seller to accept this alternative offer. The seller's payoff at the current period is 0 if he rejects. However, if he accepts, then by (A3), the threshold rule for repayment, the buyer will repay his promise $\ell' = x'$. Then, by accepting the offer the seller obtains $-v(y') + x' > 0$. Thus, (y', ℓ') is a profitable deviation for the buyer.

(⇐) Here we show sufficiency. Let $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$ be a sequence satisfying (3.4) and (3.5). Consider (s^b, s^s) given as follows. Buyers can be in two states, $\chi_{i,t} \in \{G, A\}$, and each buyer's initial state is $\chi_{i,0} = G$. The law of motion of the buyer i's state are given by:

$$\chi_{i,t+1}\left[(\ell', x', i), \chi_{i,t}\right] = \begin{cases} A & \text{if } x' < \min(x_t, \ell') \text{ or } \chi_{i,t} = A \\ G & \text{otherwise} \end{cases} . \tag{C.1}$$

The strategies are such that $s_{t,1}^b(\rho_t^i) = (y_t, \ell_t)$ if the state for ρ_t^i is G and $s_{t,1}^b(\rho_t^i) = (0, 0)$ otherwise; $s_{t,2}^b(\rho_t^i, (y', \ell'), yes) = \min\{\ell', \ell_t\}$ if the state for ρ_t^i is G and $s_{t,2}^b(\rho_t^i, (y', \ell'), yes) = 0$ otherwise; $s_t^s(\rho_t^i, (y', \ell')) = yes$ if the state for ρ_t^i is G and $v(y') \leq \min\{\ell', \ell_t\}$, and $s_t^s(\rho_t^i, (y', \ell')) = no$ otherwise. We show that (s^b, s^s) is a credit equilibrium.

Given s^b, s^s is optimal: the seller expects a buyer in state G to repay up to ℓ_t at period t and hence he accepts an offer, (y', ℓ'), if $v(y') \le \min\{\ell', \ell_t\}$; with buyers in state A he expects no repayment at all and hence rejects any offer. Next, we show that s^b is optimal given s^s. Consider a buyer with state A at the beginning of period t. Any offer to the seller is rejected and therefore it is optimal for the buyer to offer $(0, 0)$. Similarly, for such a buyer at the CM stage at period t with a promise ℓ', his state will remain in A, independent of his repayment decision and hence it is optimal to repay nothing.

Now consider a buyer with state G at the CM stage of period t, with a promise ℓ' made to the seller. The buyer has to pay $\min\{\ell_t, \ell'\}$ to maintain state G. By (3.4), paying this amount is better than becoming an A person, whose continuation value is 0. Finally, consider a buyer with state G at the beginning of period t. Note that under s^b, his continuation value from period $t+1$ onward is independent of his offer at period t. Moreover, for any offer (y, ℓ), the seller accepts the offer if and only if $v(y) \le \min\{\ell, \ell_t\}$. Thus, a buyer's problem is

$$\max_{(y,\ell)} u(y) - \min\{\ell, \ell_t\} \text{ s.t. } v(y) \le \min\{\ell, \ell_t\}.$$

Because $\ell_t = v(y_t) \le v(y^*)$, (y_t, ℓ_t) is a solution to the problem. □

Proof of Corollary 3 (⇐) Here we show sufficiency. Let $\{d_t\}_{t=0}^{\infty}$ be a sequence satisfying (3.9) and (3.10). Then, we can determine the outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, consistent with $\{d_t\}_{t=0}^{\infty}$ by the solution to the bargaining problem, (3.8), that is, $x_t = \ell_t = v(y_t) = \min\{v(y^*), d_t\}$ for each t. It remains to show that $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$ is the outcome of a credit equilibrium, (s^b, s^s), with buyers' repayment strategy consistent with $\{d_t\}_{t=0}^{\infty}$. As in the proof of Proposition 5, the strategy follows a simple finite automaton with two states, $\chi_{i,t} \in \{G, A\}$, and each buyer's initial state is $\chi_{i,0} = G$. The law of motion of the buyer i's state are given by:

$$\chi_{i,t+1}\left[(\ell', x', i), \chi_{i,t}\right] = \begin{cases} A & \text{if } x' < \min(d_t, \ell') \text{ or } \chi_{i,t} = A \\ G & \text{otherwise} \end{cases}. \tag{C.2}$$

This law of motion is the same as (C.1), where d_t replaces x_t. The strategies are analogous to those constructed in the proof of Proposition 5, but with d_t as the maximum amount of debt the buyer repays: at date t, the buyer offers (y_t, ℓ_t) in state G, the seller accepts the offer (y', ℓ') iff $v(y') \leq \ell' \leq d_t$ and the buyer's state is G, and the buyer repays $\min(\ell', d_t)$ in the CM in state G if ℓ' is the loan issued in DM. Following exactly the same logic as in the proof of Proposition 5, (3.9) and (3.10) ensure that (s^b, s^s) is a credit equilibrium.

(⇒) Here we show necessity. Let $\{d_t\}_{t=0}^{\infty}$ be a sequence consistent with a credit equilibrium outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$. By definition, $\{d_t\}_{t=0}^{\infty}$ satisfies (3.10). To show (3.9), consider a buyer at period-t CM with a loan size $\ell' = d_t$ (perhaps on an off-equilibrium path). For repayment of d_t to be optimal in state G, (3.9) must hold, i.e., the buyer prefers repaying d_t to permanent autarky.

Proof of Corollary 4 Rewrite the incentive-compatibility constraint (3.11) at time $t+1$ and multiply it by β to obtain:

$$\beta d_{t+1} \leq \beta^2 \left\{\alpha\left[u(y_{t+2}) - v(y_{t+2})\right] + d_{t+2}\right\}. \tag{C.3}$$

Combining (3.11) and (C.3) we get:

$$d_t \leq \beta \left\{ \alpha \left[u(y_{t+1}) - v(y_{t+1}) \right] \right\} + \beta^2 \left\{ \alpha \left[u(y_{t+2}) - v(y_{t+2}) \right] \right\} + \beta^2 d_{t+2}.$$

By successive iterations we generalize the inequality above as follows:

$$d_t \leq \sum_{s=1}^{T} \beta^s \left\{ \alpha \left[u(y_{t+s}) - v(y_{t+s}) \right] \right\} + \beta^{t+T} d_{t+T}. \tag{C.4}$$

By assumption, $\{d_t\}$ is bounded, $\lim_{T\to\infty} \beta^{t+T} d_{t+T} = 0$. Hence, by taking T to infinity, it follows from (C.4) that $\{d_t\}$ satisfies (3.9).

Proof of Proposition 6 Define the right side of (3.12) as a function

$$\Psi(d) = \alpha \left\{ u\left[z(d)\right] - v\left[z(d)\right] \right\}. \tag{C.5}$$

Ψ is continuous in d with $\Psi(0) = 0$ and $\Psi(d) = \alpha \left[u\left(y^*\right) - v\left(y^*\right) \right]$ for all $d \geq v(y^*)$. Moreover, it is differentiable with

$$\Psi'(d) = \alpha \left\{ \frac{u'[z(d)] - v'[z(d)]}{v'[z(d)]} \right\} \text{ if } d \in (0, v(y^*)], \text{ and } \Psi'(d) = 0 \text{ if } d > v(y^*).$$

This derivative is decreasing in d for all $d \in (0, v(y^*))$. Hence, Ψ is a concave function of d, and the set of values for d that satisfies (3.12) is an interval $[0, d^{\max}]$, where $d^{\max} \geq 0$ is the largest number that satisfies $\Psi(d^{\max}) = r d^{\max}$. Moreover, $d^{\max} > 0$ if and only if $\Psi'(0) > r$, which is always satisfied since $\Psi'(0) = \infty$ by assumption on preferences.

Proof of Lemma 4 Define the correspondence $\Gamma : \mathbb{R}_+ \supset \mathbb{R}_+$ as follows:

$$\Gamma(d) = \{x \in \mathbb{R}_+ : r\,(1+\beta)\,x \leq \alpha\,\{u\,[z(d)] - \upsilon\,[z(d)]\} + \beta\alpha\,\{u\,[z(x)] - \upsilon\,[z(x)]\}\,. \tag{C.6}$$

Then, $\gamma(d) = \max\Gamma(d)$. First we show that $\Gamma(d)$ is a closed interval and γ is well-defined. By definition, $x \in \Gamma(d)$ if and only if

$$r(1+\beta)x \leq \Psi(d) + \beta\Psi(x),$$

where $\Psi(d) = \alpha\,\{u\,[z(d)] - \upsilon\,[z(d)]\}$. Using a similar argument to that in Proposition 6, $\Gamma(d)$ is a closed interval with zero as the lower end point. Thus, γ is well-defined, and $\gamma(d)$ is the largest x that satisfies

$$r\,(1+\beta)\,x = \Psi(d) + \beta\Psi(x). \tag{C.7}$$

Moreover, if $d > d'$, then $\Gamma(d') \subseteq \Gamma(d)$, and hence γ is a non-decreasing function: Because $\Psi(d)$ is constant for all $d \geq \upsilon(y^*)$, γ is constant for all $d \geq \upsilon(y^*)$, but it is strictly increasing for $d < \upsilon(y^*)$. Now we show that

$$\gamma(0) > 0,\ \ \gamma(d^{\max}) = d^{\max},$$

where $d^{\max}$ is given in Proposition 6. First, as $\Psi(0) = 0$, and $\Psi(x)$ is a concave function, $\gamma(0) > 0$ if and only if $r(1+\beta) < \Psi'(0) = \infty$, which holds by Inada conditions. Moreover, as the two curves $r\,(1+\beta)\,x$ and $\beta\Psi(x)$ intersect at $\gamma(0) \equiv d^{\min} > 0$, by concavity of Ψ we have $\beta\Psi'(d^{\min}) < r(1+\beta)$. Second, by Proposition

6, $d^{\max} > 0$ and $rd^{\max} = \Psi(d^{\max})$. Therefore, $r\,(1+\beta)\,d^{\max} = \Psi(d^{\max}) + \beta\Psi(d^{\max})$ and hence $\gamma(d^{\max}) = d^{\max}$.

Finally, we show that γ is a concave function. Applying the implicit function theorem to (C.7), for all $0 < d < v(y^*)$,

$$\gamma'(d) = \frac{\Psi'(d)}{(1+\beta)\,r - \beta\Psi'[\gamma(d)]}.$$

Note that $(1+\beta)\,r - \beta\Psi'[\gamma(0)] = (1+\beta)\,r - \beta\Psi'(d^{\min}) > 0$ and hence $(1+\beta)\,r - \beta\Psi'[\gamma(d)] > 0$ for all d. By concavity of Ψ, $\gamma'(d)$ is decreasing in d. Hence, γ is a concave function.

Proof of Proposition 7 Notice that, by definition, any pair (d_0, d_1) that satisfies $d_0 \leq \gamma(d_1)$ and $d_1 \leq \gamma(d_0)$ also satisfies (3.13) with $y_0 = z(d_0)$ and $y_1 = z(d_1)$, and hence (d_0, d_1) is a 2-period credit cycle. By Lemma 4, γ is a concave function with $\gamma(0) > 0$ and $\gamma(d^{\max}) = d^{\max}$, and hence, $\gamma(d) > d$ for all $d \in [0, d^{\max})$, where $d^{\max}$ is given in Proposition 6. Thus, for each $d_0 \in [0, d^{\max})$, the interval $[d_0,\ \gamma(d_0)]$ is nondegenerate and $\gamma(d_0) < d^{\max}$. Hence, for each $d_1 \in [d_0,\ \gamma(d_0)]$, $d_0 \leq d_1 < \gamma(d_1)$, where we used that $\gamma(d) > d$ for all $d \leq \gamma(d_0) < d^{\max}$, so (d_0, d_1) is a 2-period credit cycle. This gives a full characterization of the set of 2-period cycles with $d_0 \leq d_1$, and the set of cycles with $d_1 \leq d_0$ is its mirror image with respect to the 45° line. Thus, for each $d_0 \in [0, d^{\max})$, the set $\Omega(d_0)$ is a nondegenerate interval.

Proof of Corollary 5 As shown earlier, a pair (d_0, d_1) is a 2-period cycle if and only if $d_0 \leq \gamma(d_1; \alpha, r)$ and $d_1 \leq \gamma(d_0; \alpha, r)$, where γ is given by Lemma 1.

Note that here we make the parameters (α, r) explicit. By Proposition 7, for all $d_0 \in [0, d^{\max})$, there exists a continuum of d_1 such that (d_0, d_1) is a 2-period cycle. Now, $d_1 \leq \gamma(d_0; \alpha, r)$ if and only if

$$\frac{r(2+r)}{1+r} d_1 \leq \Psi(d_0; \alpha) + \frac{1}{1+r} \Psi(d_1; \alpha), \tag{C.8}$$

where $\Psi(d; \alpha) = \alpha \{u[z(d)] - \upsilon[z(d)]\}$. By the proof of Proposition 7, for each $d_0 \in [0, d^{\max})$, (d_0, d_1) is a 2-period cycle with $d_0 \leq d_1$ if and only if d_1 satisfies (C.8). Let $\overline{\Omega}(d_0; \alpha, r)$ be the set of such d_1. Because of symmetry between d_0 and d_1 in a 2-period cycle, it suffices to show that $\overline{\Omega}(d_0; \alpha, r)$ expands as α increases and as r decreases. Because $\Psi(d; \alpha)$ is strictly increasing in α, it follows that for any $\alpha' > \alpha''$, $d_1 \in \overline{\Omega}(d_0; \alpha'', r)$ implies that $d_1 \in \overline{\Omega}(d_0; \alpha', r)$, but there exists $d_1 \in \overline{\Omega}(d_0; \alpha', r)$ that is not in $\overline{\Omega}(d_0; \alpha'', r)$, that is, $\overline{\Omega}(d_0; \alpha'', r) \subsetneqq \overline{\Omega}(d_0; \alpha', r)$. Similarly, because the left-side of (C.8) is increasing in r but the right-side is decreasing in r, for any $r' > r''$, $d_1 \in \overline{\Omega}(d_0; \alpha, r')$ implies that $d_1 \in \overline{\Omega}(d_0; \alpha, r'')$, but there exists $d_1 \in \overline{\Omega}(d_0; \alpha, r'')$ that is not in $\overline{\Omega}(d_0; \alpha, r')$, that is, $\overline{\Omega}(d_0; \alpha, r') \subsetneqq \overline{\Omega}(d_0; \alpha, r'')$.

Proof of Corollary 6 Note that $d^{\max} \leq \upsilon(y^*)$ if and only if $r \geq \alpha [u(y^*) - \upsilon(y^*)] / \upsilon(y^*)$. We prove the two cases separately.

Case 1: $d^{\max} \leq \upsilon(y^*)$. Consider a 2-period cycle, (d_0, d_1), with $d_0 \leq d_1$. By Proposition 7, $d_0 \leq d_1 \leq \gamma(d_0) \leq d^{\max} \leq \upsilon(y^*)$. Thus, by (3.8), the loan contract is given by $\ell_t = d_t$ for $t = 0, 1$. The case $d_0 > d_1$ is completely symmetric.

Case 2: $d^{\max} > \upsilon(y^*)$. Consider a 2-period cycle, (d_0, d_1), with $d_0 \leq d_1$. If $d_0 \leq d_1 \leq \upsilon(y^*)$, then using identical arguments as case (1) above, we can show that the borrowing constraints always bind. Because, as shown in Lemma 4,

$\gamma[v(y^*)] = d^{\max} > v(y^*)$, there exists a unique $\widehat{d}_0 < v(y^*)$ such that $\gamma(\widehat{d}_0) = v(y^*)$. Then, for any $d_0 \in (\widehat{d}_0, d^{\max}]$ and for any $d_1 \in [d_0, \gamma(d_0)]$, $d_1 > v(y^*)$, and hence, by (3.8), $\ell_1 = v(y^*) < d_1$. Thus, for any 2-period cycle, (d_0, d_1), with $\widehat{d}_0 < d_0 \leq v(y^*)$ and $d_1 \in [d_0, \gamma(d_0)]$, the borrowing constraint is slack in odd periods but binds in even periods. For any 2-period cycle, (d_0, d_1), with $v(y^*) < d_0$ and $d_1 \in [d_0, \gamma(d_0)]$, the borrowing constraints are slack in all periods. The case $d_0 > d_1$ is symmetric.

Proof of Proposition 8 By a similar argument to Proposition 7, a T-tuple, $(d_0, ..., d_{T-1}) \in \mathbb{R}^T_+$, is a T-period credit cycle if and only if $d_t \leq \gamma_T(d_{t+1}, .., d_{t+T-1})$, where $\gamma_T(d_0, ..., d_{T-2})$ is the largest x that satisfies

$$r\frac{\beta}{1-\beta^T}x = \sum_{t=0}^{T-2} \beta^t \Psi(d_t) + \beta^{T-1}\Psi(x), \tag{C.9}$$

and $\Psi(x) = \alpha \{u[z(x)] - v[z(x)]\}$. The function $\gamma_T(d_0, ..., d_{T-2})$ is a non-decreasing function in all its arguments. By similar arguments to Lemma 4, we can also show that $\gamma_T(d^{\max}, ..., d^{\max}) = d^{\max}$ and $\gamma_T(d, ..., d) > d$ for all $d \in [0, d^{\max})$. Moreover, γ_T is a concave function. For each $d_0 \in [0, d^{\max})$, define

$$\Omega_T(d_0) = \left\{(d_1, ..., d_{T-1}) \in \mathbb{R}^{T-1}_+ : d_t \leq \gamma_T(d_{t+1}, .., d_{t+T-1}) \text{ for all } t = 0, ..., T\right\}.$$

The set $\Omega_T(d_0)$ is closed and bounded. Moreover, $(d_0, ..., d_0) \in \Omega_T(d_0)$ where all inequalities in the definition above are strict inequalities. Hence, there exists an open ball with a positive radius centered at $(d_0, ..., d_0)$ that is contained in

$\Omega_T(d_0)$. Thus, $\Omega_T(d_0)$ has positive Lebesgue measure in $\mathbb{R}^{T-1}$. Finally, because γ_T is concave, $\Omega_T(d_0)$ is a convex set.

Proof of Proposition 9 Replace $d_t = \phi_t$ into the buyer's optimality condition in a monetary economy, (3.18), to get

$$d_t = \beta d_{t+1}\left[1 + \alpha \frac{u'(y_{t+1}) - \upsilon'(y_{t+1})}{\upsilon'(y_{t+1})}\right]. \tag{C.10}$$

The right side of (C.10), $[u'(y_{t+1}) - \upsilon'(y_{t+1})]/\upsilon'(y_{t+1})$, is the derivative of the function, $u[\upsilon^{-1}(d_{t+1})] - d_{t+1}$, with respect to d_{t+1}. From the strict concavity of the function and the fact that it is equal to 0 when evaluated at $d_{t+1} = 0$,

$$\frac{u'(y_{t+1}) - \upsilon'(y_{t+1})}{\upsilon'(y_{t+1})} d_{t+1} < u(y_{t+1}) - \upsilon(y_{t+1}). \tag{C.11}$$

From (C.10) and (C.11),

$$d_t < \beta\alpha\left[u(y_{t+1}) - \upsilon(y_{t+1})\right] + \beta d_{t+1}. \tag{C.12}$$

Iterating (C.12),

$$d_t < \sum_{j=1}^{J} \beta^j \alpha\left[u(y_{t+j}) - \upsilon(y_{t+j})\right] + \beta^J d_{t+J}. \tag{C.13}$$

Applying the transversality condition, $\lim_{J\to\infty} \beta^J d_{t+J} = 0$ to (C.13), we prove that the sequence, $\{d_t\}$, is a solution to (C.10) satisfies (3.9), and hence it is part of a credit equilibrium.

Proof of Proposition 10 Here we show that for any distribution over $\mathbb{X}$ with a full support, denoted by π, we have a continuum of sunspot equilibria indexed by $d \in (0, d^{\max})$. For any $d \in (0, d^{\max})$, we have

$$rd < \alpha\{u[z(d)] - v[z(d)]\}. \tag{C.14}$$

Fix an element $\chi_0 \in \mathbb{X}$ and let $\mathbb{X}_{-0} = \mathbb{X} - \{\chi_0\}$. Define the set

$$\Omega_{(\mathbb{X},\pi)}(d_{\chi_0}) = \Bigg\{\langle d_\chi; \chi \in \mathbb{X}_{-0}\rangle : rd_\chi \leq \pi(\chi_0)\alpha\{u[z(d_{\chi_0})] - v[z(d_{\chi_0})]\} \\ + \sum_{\chi\in\mathbb{X}_{-0}} \pi(\chi)\alpha\{u[z(d_\chi)] - v[z(d_\chi)]\} \text{ for all } \chi \in \mathbb{X}\Bigg\}.$$

By (C.14), the sequence $\langle d_\chi; \chi \in \mathbb{X}_{-0}\rangle$ with $d_\chi = d_{\chi_0}$ for all $\chi \in \mathbb{X}_{-0}$ is in $\Omega_{(\mathbb{X},\pi)}(d_{\chi_0})$ where all inequalities in the definition above are strict inequalities. Thus, the set $\Omega_{(\mathbb{X},\pi)}(d_{\chi_0})$ contains an open ball with a positive radius centered at $\langle d_\chi; \chi \in \mathbb{X}_{-0}\rangle$ with $d_\chi = d_{\chi_0}$ for all $\chi \in \mathbb{X}_{-0}$. Hence, it has a positive Lebesgue measure in $\mathbb{R}^{|\mathbb{X}_{-0}|}$ and almost all points in it satisfy $d_\chi \neq d_{\chi'}$ for all $\chi \neq \chi'$. Note that for any $\langle d_\chi; \chi \in \mathbb{X}_{-0}\rangle \in \Phi(d)$, $\langle d_\chi; \chi \in \mathbb{X}\rangle$ with $d_{\chi_0} = d$ is a sunspot credit equilibrium by (3.21).

Proof of Proposition 11 In the main text we have shown that (3.22) is necessary for the buyer to deliver their promised output in the DM, and, by (3.25), for a given sequence of debt limits, $\{d_t\}$, the equilibrium amount of loan is given by $\ell_t = \eta[z(d_t)]$ and hence $u(y_t) - \ell_t = \theta[u(y_t) - v(y_t)]$. Thus, (3.22) becomes (3.26). This proves the necessity. For sufficiency, let $\{(y_t, \ell_t, d_t)\}$ be a sequence satisfying (3.25) and (3.26). We use the same strategies as in the proof of Corollary 3, but we have to modify s_2^b in accordance with the new environment. As the buyer makes the production decision in the DM, the buyer strategy s_2^b becomes a delivery strategy that specifies the amount of the output that the buyer delivers to the seller in the CM, the difference being what the buyer consumes. Analogous to the strategies in Corollary 3, s_2^b satisfies the following threshold property. If ℓ' is the amount of output that the buyer promises to deliver in the period-t DM, then his actual delivery is $x' = \min\{\ell', d_t\}$. As in the proof of Corollary 3, (3.26) ensures that this delivery strategy is optimal.

Proof of Proposition 12 From (3.26), a pair (d_0, d_1) is a 2-period credit cycle equilibrium outcome if and only if $d_0 \leq \gamma(d_1)$ and $d_1 \leq \gamma(d_0)$, where $\gamma(d)$ is the largest x that satisfies

$$r\lambda\,(1+\beta)\,x = \Psi(d) + \beta\Psi(x), \tag{C.15}$$

and

$$\Psi(d) = \alpha\theta\left\{u\left[z(d)\right] - \upsilon\left[z(d)\right]\right\}.$$

The left side of the equation in (C.15), $r\lambda\,(1+\beta)\,x$, is linear and increasing while the right side, $\beta\alpha\theta\left\{u\left[z(x)\right] - \upsilon\left[z(x)\right]\right\}$, is non-decreasing and concave. Given that the first term on the right side is non-negative, $\gamma(d)$ is well-defined. Note that $\gamma(d^{\max}) = d^{\max}$, where $d^{\max}$ is defined as the highest solution to $r\lambda d = \Psi(d)$, and, by similar arguments to Lemma 4, we can show γ is concave.

Note that $d^{\max} > 0$ if and only if $\Psi'(0) > r\lambda$. Now,

$$\Psi'(d) = \alpha\theta \left[\frac{u'[z(d)] - v'[z(d)]}{(1-\theta)u'[z(d)] + \theta v'[z(d)]} \right]$$

and hence $\Psi'(0) = \alpha\theta/(1-\theta)$, which shows that $d^{\max} > 0$ if and only if $r\lambda < \alpha\theta/(1-\theta)$.

When $d^{\max} > 0$, i.e., when $r\lambda < \alpha\theta/(1-\theta)$, for each $0 < d_0 < d^{\max}$, $d_0 < \gamma(d_0)$, and, for each $d_1 \in (d_0, \gamma(d_0)]$, $d_0 < \gamma(d_0) \leq \gamma(d_1)$. So any such (d_0, d_1) is a 2-period cycle and there are continuum of them.

To show the existence of 2-period cycles with periodic credit shutdowns, we need to show that $\gamma(0) > 0$. From (C.15), $\gamma(0) > 0$ if and only if $r\lambda(1+\beta) < \beta\Psi'(0) = \beta\alpha\theta/(1-\theta)$, which corresponds to the condition $r < \sqrt{1 + \alpha\theta/[\lambda(1-\theta)]} - 1$. Given that $\gamma(0) > 0$, any $(d_0, d_1) \in \{0\} \times (0, \gamma(0))$ is a credit equilibrium where credit shuts down in even periods.

Proof of Proposition 13 (1) Suppose that $y^* \leq y^{\max}$. Then, the outcome $\{(y_t, \ell_t)\}_{t=0}^{\infty}$ with $y_t = y^*$ and $\ell_t = v(y_t)$ for all t is implementable.

(2) Suppose that $y^* > y^{\max}$. We show that the optimal sequence that has $y_t = y^{\max}$ and $\ell_t = v(y_t)$ for all t. Suppose, by contradiction, that there is another sequence $\{y'_t, \ell'_t\}_{t=0}^{\infty}$ satisfying (3.32) and (3.33) with a strictly higher welfare. It then follows that $y^* \geq y'_t > y^{\max}$ for some t. Let t_0 be the first t such that $u(y'_t) - v(y'_t) > u(y^{\max}) - v(y^{\max})$. Now we show that for some $t_1 > t_0$, $y'_{t_1} > y'_{t_0}$. Suppose, by contradiction, that $y'_t \leq y'_{t_0}$ for all $t > t_0$. We have the following

inequality,

$$v(y'_{t_0}) \leq \ell'_{t_0} \leq \lambda^{-1} \sum_{s=1}^{+\infty} \beta^s \alpha \left[u(y'_{t_0+s}) - \ell'_{t_0+s}\right] \leq \lambda^{-1} \sum_{s=1}^{+\infty} \beta^s \alpha \left[u(y'_{t_0}) - v(y'_{t_0})\right],$$

where the first inequality follows from the seller's participation constraint, (3.33), at $t = t_0$, the second follows from the buyer's participation constraint, (3.32), and the third follows from $u(y'_{t_0+s}) - \ell'_{t_0+s} \leq u(y'_{t_0+s}) - v(y'_{t_0+s}) \leq u(y'_{t_0}) - v(y'_{t_0})$ since $u - v$ is increasing for $y < y^*$ and $\ell'_{t_0+s} \leq v(y'_{t_0+s})$ for all s. Because $y^{\max}$ is the maximal value of y'_{t_0} that equalizes the left side and the right side of this series of inequalities, it follows that $y'_{t_0} \leq y^{\max}$, a contradiction. So $y^* \geq y'_{t_1} > y'_{t_0}$ for some t_1 (and we choose $t_1 > t_0$ to be the first index for this to happen). By induction, we can then find a subsequence $\{y'_{t_i}\}$ that is strictly increasing and is bounded from above. So there exists a limit $\widetilde{y} = \lim_{i\to\infty} y'_{t_i} > y^{\max}$. Hence, by monotonicity, we have for all i,

$$rv(y'_{t_i}) \leq r\ell_{t_i} \leq \frac{\alpha[u(\widetilde{y}) - v(\widetilde{y})]}{\lambda},$$

and, by taking i to infinity, we have

$$rv(\widetilde{y}) \leq \frac{\alpha[u(\widetilde{y}) - v(\widetilde{y})]}{\lambda}.$$

However, as explained above, this implies that $\widetilde{y} \leq y^{\max}$, and this leads to a contradiction.

Proof of Proposition 14 The program that selects the best PBE is

$$\max_{\{y_t\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \alpha \left[u(y_t) - v(y_t)\right] \tag{C.16}$$

$$\text{s.t.} \qquad \lambda \eta(y_t) \leq \alpha \sum_{s=1}^{+\infty} \beta^s \left[u(y_{t+s}) - \eta(y_{t+s})\right] \tag{C.17}$$

$$y_t \leq y^* \text{ for all } t = 0, 1, 2, ... \tag{C.18}$$

(1) Suppose that $y^* \leq y^{\max}$. In this case, the outcome $\{y_t\}_{t=0}^{\infty}$ with $y_t = y^*$ for all t is implementable and hence is the c.e.a.

(2) Suppose that $y^* > y^{\max}$ but $y^{\max} \leq \hat{y}$. We show that the outcome $\{y_t\}_{t=0}^{\infty}$ with $y_t = y^{\max}$ for all t is the optimum. Suppose, by contradiction, that there is another outcome $\{y_t'\}_{t=0}^{\infty}$ satisfying (C.17) and (C.18) with a strictly higher welfare. First we show that $y_t' \leq \hat{y}$ for all t. Suppose, by contradiction, that there is a t such that $y_t' > \hat{y}$. Then, because $\hat{y} \geq y^{\max}$,

$$\lambda \eta(y_t') > \lambda \eta(\hat{y}) \geq \sum_{s=1}^{\infty} \beta^s \alpha [u(\hat{y}) - \eta(\hat{y})] \geq \sum_{s=1}^{\infty} \beta^s \alpha [u(y_{t+s}') - \eta(y_{t+s}')],$$

a contradiction to (C.17). Given that this alternative outcome can only lie in the range $[0, \hat{y}]$ and hence the trade surplus is increasing in the output, the rest of the arguments are exactly the same as those in the proof of Proposition 13.

(3) Suppose that $\hat{y} < y^{\max} < y^*$. In the supplementary Appendix S4 we show that the constrained-efficient allocation, $\{x_t, y_t\}$, can be determined recursively as follows:

$$V(\omega) = \max_{y,\omega'} \{\alpha [u(y) - v(y)] + \beta V(\omega')\} \tag{C.19}$$

$$\text{s.t.} \quad -\eta(y) + \beta \frac{\omega'}{\lambda} \geq 0 \tag{C.20}$$

$$\beta\omega' \geq \{\omega - \alpha [u(y) - \eta(y)]\} \tag{C.21}$$

$$y \in [0, y^*], \ \omega' \in [0, \bar{\omega}], \tag{C.22}$$

with $\omega_0 = 0$, $\omega_{t+1} = \omega'(\omega_t)$, $y_t = y(\omega_t)$, and $x_t = \eta(y_t)$. Moreover, the value function V is unique, and it is nonincreasing and concave.

The Lagrangian associated with the above Bellman equation is

$$\begin{aligned} \mathscr{L} &= \alpha [u(y) - v(y)] + \beta V(\omega') + \xi \left(\beta \frac{\omega'}{\lambda} - \eta(y)\right) \\ &\quad + \nu \{\alpha [u(y) - \eta(y)] + \beta\omega' - \omega\}, \end{aligned} \tag{C.23}$$

where the Lagrange multipliers, ξ and ν, are non-negative. In general V may not be differentiable everywhere. However, because V is concave, the following first-order conditions are still necessary and sufficient for (y, ω') to be optimal (Clark (1976), Theorems 1 and 2):

$$\alpha [u'(y) - v'(y)] - \xi\eta'(y) + \nu\alpha [u'(y) - \eta'(y)] = 0 \tag{C.24}$$

$$\beta V'_+(\omega') + \beta \frac{\xi}{\lambda} + \beta\nu \leq 0 \leq \beta V'_-(\omega') + \beta \frac{\xi}{\lambda} + \beta\nu, \tag{C.25}$$

where $V'_+(\omega') = \lim_{\omega \downarrow \omega'} V'(\omega)$ and $V'_-(\omega') = \lim_{\omega \uparrow \omega'} V'(\omega)$. Both $V'_+(\omega')$ and $V'_-(\omega')$ exist because of concavity. The envelope condition, provided that $V'(\omega)$ exists, is

$$V'(\omega) = -\nu. \tag{C.26}$$

We define two critical values for the buyer's promised utility:

$$\omega^{\max} = \frac{\alpha\left[u(y^{\max}) - \eta(y^{\max})\right]}{1-\beta} \text{ and } \bar{\omega} = \frac{\alpha[u(\hat{y}) - \eta(\hat{y})]}{1-\beta}.$$

The first threshold is the buyer's life-time expected utility at the highest steady state, while the second is the maximum life-time expected utility achieved by the buyers across all steady states. Note that by the definition of $y^{\max}$, $\eta(y^{\max}) = \beta\omega^{\max}/\lambda$.

(a) $\lambda \geq \alpha\left[1 - u'(y^{\max})/\eta'(y^{\max})\right].$

The following claim provides conditions under which the constrained-efficient allocation corresponds to the highest steady state. In order to establish this claim, we shows that, for $\omega = 0$ and $\omega = \omega^{\max}$, the optimal solution to the maximization problem in (C.19)-(C.22) is $(\omega^{\max}, y^{\max})$.

Claim 1. *If $\hat{y} < y^{\max} < y^*$ and $\lambda \geq \alpha\left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$, then the unique solution to (C.19)-(C.22) is*

$$V(\omega) = \frac{\alpha\left[u(y^{\max}) - v(y^{\max})\right]}{1-\beta} \text{ if } \omega \in [0, \omega^{\max}], \tag{C.27}$$

$$= \frac{\alpha\left\{u[g(\omega)] - v[g(\omega)]\right\}}{1-\beta} \text{ if } \omega \in (\omega^{\max}, \bar{\omega}], \tag{C.28}$$

where $g(\omega)$ *is the unique solution to* $\alpha[u(y)-\eta(y)] = (1-\beta)\omega$ *for all* $\omega \in (\omega^{\max}, \bar{\omega}]$.

The function V given by (C.27)-(C.28) is flat in the interval $[0, \omega^{\max}]$ and is strictly concave for all $\omega \in (\omega^{\max}, \bar{\omega})$, and hence is concave overall. To show the strict concavity, we compute $V''(\omega)$ for all $\omega \in (\omega^{\max}, \bar{\omega})$. By the Implicit Function Theorem, we have

$$g'(\omega) = \frac{1-\beta}{\alpha\{u'[g(\omega)] - \eta'[g(\omega)]\}} < 0,$$

and hence

$$V'(\omega) = \frac{u'[g(\omega)] - v'[g(\omega)]}{u'[g(\omega)] - \eta'[g(\omega)]} \tag{C.29}$$

for all $\omega \in (\omega^{\max}, \bar{\omega})$. Thus,

$$\begin{aligned} V''(\omega) &= \frac{\{u''[g(\omega)] - v''[g(\omega)]\}\{u'[g(\omega)] - \eta'[g(\omega)]\}}{\{u'[g(\omega)] - \eta'[g(\omega)]\}^2} g'(\omega) \\ &+ \frac{-\{u'[g(\omega)] - v'[g(\omega)]\}\{u''[g(\omega)] - \eta''[g(\omega)]\}}{\{u'[g(\omega)] - \eta'[g(\omega)]\}^2} g'(\omega) < 0. \end{aligned}$$

Note that, for all $\omega \in (\omega^{\max}, \bar{\omega})$, $u'[g(\omega)] - \eta'[g(\omega)] < 0$ as $g(\omega) > \hat{y}$ and that $u'[g(\omega)] - v'[g(\omega)] > 0$ as $g(\omega) \leq y^{\max} < y^*$.

To prove that V satisfies (C.27) and (C.28), we consider two cases.

(i) Suppose that $\omega \in [0, \omega^{\max}]$. The solution to the maximization problem in (C.19)-(C.22) is given by $(\omega', y) = (\omega^{\max}, y^{\max})$. This solution is feasible because $(\omega^{\max}, y^{\max})$ satisfies (C.21) for all $\omega \leq \omega^{\max}$ and it satisfies (C.20) at equality. Next, we show that it satisfies (C.24)-(C.25) with $\nu = 0$ and

$$\xi = \frac{\alpha[u'(y^{\max}) - v'(y^{\max})]}{\eta'(y^{\max})} > 0.$$

The condition (C.24) holds by the definition of ξ. To establish (C.25), first note that $V'_{-}(\omega^{\max}) = 0$ and

$$V'_{+}(\omega^{\max}) \equiv \lim_{\omega \downarrow \omega^{\max}} V'(\omega) = \frac{u'(y^{\max}) - v'(y^{\max})}{u'(y^{\max}) - \eta'(y^{\max})}.$$

Thus, $V'_{-}(\omega^{\max}) + \xi/\lambda > 0$ and the first inequality in (C.25) holds if and only if

$$\begin{aligned} & V'_{+}(\omega^{\max}) + \frac{\xi}{\lambda} \\ = \; & \frac{1}{\lambda} \frac{[u'(y^{\max}) - v'(y^{\max})]}{\eta'(y^{\max})} \left\{ \alpha + \lambda \frac{\eta'(y^{\max})}{u'(y^{\max}) - \eta'(y^{\max})} \right\} \leq 0, \end{aligned}$$

and, because $\hat{y} < y^{\max} < y^{*}$ and hence $u'(y^{\max}) - v'(y^{\max}) > 0$ and $u'(y^{\max}) - \eta'(y^{\max}) < 0$, the last inequality holds if and only if

$$\alpha \left[\frac{u'(y^{\max})}{\eta'(y^{\max})} - 1 \right] \geq -\lambda,$$

that is, $\lambda \geq \alpha \left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$. This implies V satisfies (C.27).

(ii) Suppose that $\omega \in (\omega^{\max}, \bar{\omega})$. Here we show that $(\omega', y) = (\omega, g(\omega))$ is the solution to the maximization problem in (C.19)-(C.22). This solution is feasible: (C.21) holds by construction; because $\omega' = \omega = \alpha[u(y) - \eta(y)]/(1-\beta)$ and because $y = g(\omega) \leq y^{\max}$,

$$\lambda \eta(y) \leq \beta \alpha [u(y) - \eta(y)]/(1-\beta),$$

(C.20) holds. Next, we show that the FOC's (C.24) and (C.25) are satisfied by $(\omega', y) = (\omega, g(\omega))$ with $\xi = 0$ and

$$\nu = -\frac{u'[g(\omega)] - v'[g(\omega)]}{u'[g(\omega)] - \eta'[g(\omega)]} > 0.$$

The FOC for y, (C.24), holds by the definition of ν. The FOC for ω', (C.25), holds if and only if

$$\nu + V'(\omega) = 0,$$

which holds by (C.29). Thus, if $\omega_0 = \omega \in (\omega^{\max}, \bar{\omega})$, then the optimal sequence is $(\omega_t, y_t) = (\omega, g(\omega))$ for all t. Hence, $V(\omega)$ is satisfies (C.28) for all $\omega \in (\omega^{\max}, \bar{\omega})$. Finally, V satisfies (C.28) at $\omega = \bar{\omega}$ by continuity.

(b) $\lambda < \alpha \left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$.

We will show that $V(\omega)$ has the same closed-form solution as derived in claim 1 when $\omega > \omega^{\max}$. Given this observation, we will establish that if $\omega = 0$ then $\omega' > \omega^{\max}$ and y can be solved in closed form.

Claim 2. *Suppose that $\hat{y} < y^{\max} < y^*$ and $\lambda < \alpha \left[1 - u'(y^{\max})/\eta'(y^{\max})\right]$. Then, there exists a unique (y_0, y_1) with $\hat{y} < y_1 < y^{\max} < y_0 < y^*$ that solves (3.40)-(3.41), and the unique V that solves (C.19)-(C.22) satisfies*

$$\begin{aligned} V(\omega) &= \alpha[u(y_0) - v(y_0)] + \frac{\beta}{1-\beta}\alpha[u(y_1) - v(y_1)] \quad \text{if } \omega = 0, && \text{(C.30)} \\ &= \frac{\alpha}{1-\beta}\{u[g(\omega)] - v[g(\omega)]\} \quad \text{if } \omega \in [\omega^{\max}, \bar{\omega}], && \text{(C.31)} \end{aligned}$$

where $g(\omega)$ is given in Part 1.

The fact that V satisfies (C.31) follows the proof of the second case in the claim in the proof of Part 1 and the Contraction Mapping Theorem. Note that by (C.31), $V'(\omega)$ is given by (C.29) for $\omega > \omega^{\max}$ and hence the proof there applies exactly.

Here we show (C.30). First we rewrite the problem in (C.19)-(C.22) at $\omega = 0$ as follows:

$$\max_{y,\omega'} \{\alpha\left[u(y) - v(y)\right] + \beta V(\omega')\} \tag{C.32}$$

$$\text{s.t.} \quad -\eta(y) + \beta\frac{\omega'}{\lambda} \geq 0 \tag{C.33}$$

$$y \in [0, y^*],\ \omega' \in [0, \bar{\omega}]\,. \tag{C.34}$$

Note that (C.21) is trivially satisfied when $\omega = 0$. Now, conjecturing that $\omega' \geq \omega^{\max}$, we can replace $V(\omega')$ by the expression given by (C.31), y by y_0 and $g(\omega')$ by y_1, and transform the above problem to

$$\max_{(y_0,y_1)\in[0,y^*]\times[\hat{y},y^{\max}]} \left\{\alpha\left[u(y_0) - v(y_0)\right] + \alpha\frac{u(y_1) - v(y_1)}{r}\right\} \tag{C.35}$$

$$\text{s.t.} \qquad -\eta(y_0) + \alpha\frac{u(y_1) - \eta(y_1)}{\lambda r} \geq 0, \tag{C.36}$$

which is exactly the same as (3.40)-(3.41). By the Kuhn-Tucker conditions, a pair (y_0, y_1) solves the above problem if it satisfies the following FOC and feasibility condition:

$$\frac{u'(y_0) - v'(y_0)}{\eta'(y_0)} = -\frac{\lambda}{\alpha}\left[\frac{u'(y_1) - v'(y_1)}{u'(y_1) - \eta'(y_1)}\right] \tag{C.37}$$

$$\alpha[u(y_1) - \eta(y_1)] = r\lambda\eta(y_0). \tag{C.38}$$

In order to show that the solution (y_0, y_1) is also a solution to the problem in (C.19)-(C.22) at $\omega = 0$ we only need to verify our conjecture,

$$\omega_1 = \frac{1}{1-\beta}\alpha[u(y_1) - \eta(y_1)] > \omega^{\max},$$

because the necessary conditions, (C.37)-(C.38), are also sufficient by the concavity of V over its entire domain.

Now we show that there exists a unique pair (y_0, y_1) with $\hat{y} < y_1 < y^{\max} < y_0 < y^*$ that satisfies (**??**)-(**??**). For each $y_1 \in (\hat{y}, y^{\max}]$, define

$$h(y_1) = \eta^{-1}\left[\frac{\alpha}{r\lambda}[u(y_1) - \eta(y_1)]\right].$$

as the unique solution of y_0 to (**??**) for a given y_1. Note that $h(y^{\max}) = y^{\max}$. For any $y_1 \in (\hat{y}, y^{\max}]$,

$$h'(y_1) = \frac{\alpha}{r\lambda}\frac{[u'(y_1) - \eta'(y_1)]}{\eta'[h(y_1)]} < 0.$$

Substituting y_0 by its expression given by $h(y_1)$ in the left side of (C.37), we rewrite (C.37) as $H(y_1) = 0$ where

$$H(y_1) = \frac{u'[h(y_1)] - v'[h(y_1)]}{\eta'[h(y_1)]} + \frac{\lambda}{\alpha}\left[\frac{u'(y_1) - v'(y_1)}{u'(y_1) - \eta'(y_1)}\right].$$

The function $H(y_1)$ is continuous and strictly increasing in $(\hat{y}, y^{\max}]$ with

$$\lim_{y_1 \downarrow \hat{y}} H(y_1) = -\infty,$$

and, at $y_1 = y^{\max}$, we have

$$\begin{aligned} H(y^{\max}) &= \frac{u'(y^{\max}) - v'(y^{\max})}{\eta'(y^{\max})} + \frac{\lambda}{\alpha}\left[\frac{u'(y^{\max}) - v'(y^{\max})}{u'(y^{\max}) - \eta'(y^{\max})}\right] \\ &= [u'(y^{\max}) - v'(y^{\max})]\left\{\frac{1}{\eta'(y^{\max})} + \frac{\lambda}{\alpha}\left[\frac{1}{u'(y^{\max}) - \eta'(y^{\max})}\right]\right\} > 0 \end{aligned}$$

because $\lambda < \alpha\,[1 - u'(y^{\max})/\eta'(y^{\max})]$. Thus, by Intermediate Value Theorem, there exists a unique $y_1 \in (\hat{y}, y^{\max})$ such that $H(y_1) = 0$ and hence (C.37) holds for $(h(y_1), y_1)$, and $h(y_1) > y^{\max}$ as h is strictly decreasing with $h(y^{\max}) = y^{\max}$. This proves that there exists a unique pair (y_0, y_1) with $\hat{y} < y_1 < y^{\max} < y_0 < y^*$ that satisfies (C.37) and (C.38).

Finally, because $\hat{y} < y_1 < y^{\max} < y_0 < y^*$ and because $(\omega', y) = (\omega_1, y_0)$ with $\omega_1 = \alpha[u(y_1) - \eta(y_1)]/(1 - \beta)$ is the solution to the maximization problem in (C.19)-(C.22) for $\omega = 0$, V satisfies (C.30). $\square$

SUPPLEMENTARY APPENDICES

S1. Equivalence between monetary and credit equilibria

Here we extend the equivalence result, Proposition 9, to other trading mechanisms. We first consider bargaining in the pairwise meetings and then consider Walrasian pricing for large group meetings. We adopt the environment introduced in Section 4 without record-keeping. The monetary trades follow a similar pattern to that in Section 3.3: buyers who cannot commit to deliver goods in the CM use money to buy DM goods from sellers in the DM. They produce CM goods in the first stage of each period in order to sell them for money in the CM. Notice that the timing of producing CM goods (whether it takes place in the first or second stage of each period) is irrelevant for buyers' behavior because it is only incentive-feasible to sell these goods in the CM for money. Sellers use money obtained from DM sales to buy CM goods. Because $\lambda \leq 1$, buyers never produce CM goods for self-consumption. As a result, the parameter λ plays no role in monetary equilibria. So with no loss of generality we set $\lambda = 1$.

Bargaining Under a general bargaining solution represented by the function $\eta(y)$, the sequence for the values of money, $\{\phi_t\}$, solves

$$\max_{m \geq 0} \{\phi_t m + \beta\alpha [u(y_{t+1}) - \eta(y_{t+1})]\}$$

where $\phi_{t+1} m = \eta(y_{t+1})$ for all t. Replace $d_t = \phi_t$ for all t in the above problem and take the FOC, we obtain

$$d_t = \beta d_{t+1} \left\{ \alpha \left[\frac{u'(y_{t+1})}{\eta'(y_{t+1})} - 1 \right] + 1 \right\}, \tag{C.39}$$

where $\eta(y_t) = d_t$ for all t. In the credit economy, the debt limits, $\{d_t\}$, solves

$$d_t \leq \beta \{\alpha [u(y_{t+1}) - \eta(y_{t+1})] + d_{t+1}\}. \tag{C.40}$$

Because η is concave, $u \circ \eta^{-1}(d_t) - d_t$ is concave in terms of the value of money. The right side of (C.39), $[u'(y_{t+1}) - \eta'(y_{t+1})] / \eta'(y_{t+1})$, is the derivative of the function, $u[\eta^{-1}(d_{t+1})] - d_{t+1}$, with respect to d_{t+1}. From the strict concavity of the function and the fact that it is equal to 0 when evaluated at $d_{t+1} = 0$,

$$\frac{u'(y_{t+1}) - \eta'(y_{t+1})}{\eta'(y_{t+1})} d_{t+1} < u(y_{t+1}) - \eta(y_{t+1}). \tag{C.41}$$

From (C.39) and (C.41),

$$d_t < \beta\alpha \left[u(y_{t+1}) - \eta(y_{t+1})\right] + \beta d_{t+1}. \tag{C.42}$$

Iterating (C.42),

$$d_t < \sum_{j=1}^{J} \beta^j \alpha \left[u(y_{t+j}) - \eta(y_{t+j})\right] + \beta^J d_{t+J}. \tag{C.43}$$

Applying the transversality condition, $\lim_{J\to\infty} \beta^J d_{t+J} = 0$ to (C.43), we prove that the sequence, $\{d_t\}$, solution to (C.39) satisfies (C.40), and hence it is part of a credit equilibrium.

This concavity of η is satisfied for the proportional bargaining solution and for the general Nash bargaining solution under the functional forms for u and v that guarantee the concavity of the buyer's surplus.

Walrasian pricing Suppose the DM is competitive and p_t denotes the price of DM goods in terms of CM goods. In a monetary economy the buyer chooses money holdings as the solution to:

$$\max_{m, y_{t+1} \geq 0} \left\{ -\phi_t m + \beta\alpha \left[u(y_{t+1}) - p_{t+1} y_{t+1} \right] + \beta\phi_{t+1} m \right\}, \tag{C.44}$$

where, $\phi_{t+1} m \geq p_{t+1} y_{t+1}$. The first-order condition for (C.44) is

$$\phi_t = \beta\phi_{t+1} \left\{ \alpha \left[\frac{u'(y_{t+1})}{p_{t+1}} - 1 \right] + 1 \right\}.$$

From the seller's maximization problem, $p_{t+1} = v'(y_{t+1})$ so that $\{\phi_t\}$ solves

$$\phi_t = \beta\phi_{t+1} \left\{ \alpha \left[\frac{u'(y_{t+1})}{v'(y_{t+1})} - 1 \right] + 1 \right\}. \tag{C.45}$$

It should be noticed that it is the same first-order difference equation as the one obtained under buyers' take-it-or-leave-it offers. Notice, using $\phi_{t+1} = v'(y_{t+1}) y_{t+1}$ by market-clearing (i.e., $m = 1$), that

$$\phi_{t+1} \left[\frac{u'(y_{t+1})}{v'(y_{t+1})} - 1 \right] = u'(y_{t+1}) y_{t+1} - v'(y_{t+1}) y_{t+1} < u(y_{t+1}) - v'(y_{t+1}) y_{t+1},$$

from the concavity of u. Recall that a sufficient condition for the sequence of debt limits to be a credit equilibrium is

$$d_t \leq \beta \left\{ \alpha \left[u(y_{t+1}) - v'(y_{t+1}) y_{t+1} \right] + d_{t+1} \right\}.$$

This proves that the phase of the monetary equilibrium is located to the left of the phase line of the credit equilibrium. Hence, by the same reasoning as before, any outcome of the monetary economy is an outcome of the credit economy.

S2. Existence of 2-period cycles under alternative mechanisms

Walrasian pricing Under Walrasian pricing, $\eta(y) = v'(y)y$. Here we show existence of a continuum of 2-period cycles when $\eta(y)$ is convex. Recall that $z(d) = \min\{\eta^{-1}(d), y^*\}$. Let $d^{\max}$ be the unique positive solution to

$$r\lambda d = \alpha \left\{u\left[z(d)\right] - \eta\left[z(d)\right]\right\}. \tag{C.46}$$

Lemma 5. *Suppose that $\eta(y)$ is convex. For each $d_0 \in [0, d^{\max})$, there is a nondegenerate interval, $\Omega(d_0)$, such that for any $d_1 \in \Omega(d_0)$, (d_0, d_1) is a (strict) 2-period cycle.*

Proof. Because $\eta(y)$ is convex, there is a unique positive number, denoted $y^{\max}$, such that

$$r\lambda\eta(y^{\max}) = \alpha[u(y^{\max}) - \eta(y^{\max})].$$

It can be verified that that $d^{\max}$ is given by

$$d^{\max} = \{.\eta(y^{\max}) \text{ if } y^* \geq y^{\max} \frac{\alpha\left\{u(y^*) - \eta(y^*)\right\}}{r\lambda} \text{ otherwise.}$$

Note that any $d \in [0, d^{\max}]$ corresponds to a steady-state equilibrium. Let us turn to 2-period cycles. A pair, (d_0, d_1), is a 2-period cycle if for $t = 0, 1$,

$$r\lambda d_t \leq \frac{\alpha\left\{u\left[z(d_{t+1})\right] - \eta\left[z(d_{t+1})\right]\right\} + \beta\alpha\left\{u\left[z(d_t)\right] - \eta\left[z(d_t)\right]\right\}}{1+\beta}. \tag{C.47}$$

Hence,

$$\Omega(d_0) = \{d_1 \geq 0 : (d_0, d_1) \text{ satisfies (C.47)}\}.$$

For all $d \in [0, d^{\max})$, because $r\lambda d < \alpha \{u[z(d)] - \eta[z(d)]\}$, $(d_0, d_1) = (d, d)$ satisfies (C.47) with a strict inequality. Hence, by continuity, there is a nonempty open set contained in $\Omega(d)$. Moreover, because η is concave, the set $\Omega(d)$ is convex and hence is a nondegenerate interval. □

Nash bargaining For all $y \leq y^*$, $u(y) - \eta(y) \geq \theta[u(y) - \upsilon(y)]$ and hence $\eta(y) \leq (1-\theta)u(y) + \theta\upsilon(y)$. Under proportional bargaining a 2-period cycle solves

$$r\lambda [(1-\theta)u(y_t) + \theta\upsilon(y_t)] \leq \frac{\{\alpha\theta [u(y_{t+1}) - \upsilon(y_{t+1})] + \beta\alpha\theta [u(y_t) - \upsilon(y_t)]\}}{1+\beta}.$$

It implies

$$r\lambda\eta(y_t) \leq \frac{\{\alpha [u(y_{t+1}) - \eta(y_{t+1})] + \beta\alpha [u(y_t) - \eta(y_t)]\}}{1+\beta}.$$

Hence (y_t, y_{t+1}), and the associated $(d_t, d_{t+1}) = (\eta(y_t), \eta(y_{t+1}))$, is a credit cycle under generalized Nash bargaining.

S3. Core and competitive equilibrium

Recall that an allocation, $\mathscr{L} = \{(y(i), x(i)), (y(j), x(j)) : i \in \mathbb{B}, j \in \mathbb{S}\}$, where $(y(i), x(i))$ denotes buyer i's DM and CM consumptions and $(y(j), x(j))$ denotes seller j's DM and CM consumptions, is in the core if there is no blocking (finite) coalition, $\mathscr{I} \subset \mathbb{B} \cup \mathbb{S}$, such that each agent in $\mathscr{I}$ enjoys at least the same utility as his allocation in $\mathscr{L}$, but at least one of them is strictly better off. Now we show that the only core allocation is the competitive outcome, with debt limit, d, is given by the symmetric allocation, (y, ℓ), such that $\ell = \eta(y) \equiv v'(y)y$ and $y = \min\{y^*, \eta^{-1}(d)\}$.

First notice that, by standard arguments, the competitive outcome is in the core. For necessity, we restrict ourselves to symmetric allocations. For a justification of such assumption, see Mas-Colell *et al.* (1995). Note that to be in the core, $u(y) \geq \ell \geq v(y)$. First we show that $\ell = v'(y)y$. Suppose, by contradiction, $\ell \neq v'(y)y$. Assume that $\ell < v'(y)y$. The other direction has a similar proof. Let ε be so small that

$$[v'(y) - \varepsilon]y > \ell. \tag{C.48}$$

Consider a coalition with m buyers and n sellers such that with $\delta = m/n < 1$, we have

$$\frac{v(y) - v(\delta y)}{(1-\delta)y} > v'(y) - \varepsilon. \tag{C.49}$$

Consider the following allocation: each buyer consumes y and issues an IOU with face value ℓ, and each seller produces δy and receives an IOU with face value $\delta\ell$. Note that such allocation is feasible:

$$my = n\delta y \text{ and } m\ell = n\delta\ell.$$

Now, each buyer enjoys the same utility as before, but each seller has a higher utility: combining (C.48) and (C.49),

$$v(y) - v(\delta y) > [v'(y) - \varepsilon](1-\delta)y > (1-\delta)\ell,$$

and hence

$$\delta\ell - v(\delta y) > \ell - v(y).$$

This proves $\ell = v'(y)y = \eta(y)$. Finally, if $y < \min\{y^*, \eta^{-1}(d)\}$, then a buyer and a seller can form a coalition to increase surplus.

S4. Recursive formulation of the mechanism design problem

Here we show that we can solve the problem (C.16)-(C.18) recursively. First we show that recursive formulation with promised utility as a state variable is equivalent to the original sequence problem.

Lemma 6. *A sequence $\{y_t\}_{t=0}^{\infty}$ satisfies (C.17) and (C.18) if and only if there is a sequence $\{\omega_t\}_{t=0}^{\infty}$ such that, for all $t = 0, 1, 2, ...,$*

$$\omega_t \leq \alpha\left[u(y_t) - \eta(y_t)\right] + \beta\omega_{t+1}, \tag{C.50}$$

$$\eta(y_t) \leq \beta\omega_{t+1}/\lambda, \tag{C.51}$$

$$y_t \in [0, y^*], \tag{C.52}$$

$$\omega_t \in [0, \bar{\omega}]. \tag{C.53}$$

Proof. Suppose that $\{y_t\}_{t=0}^{\infty}$ satisfies (C.17) and (C.18). Then, define, for each $t = 0, 1, 2, ...,$

$$\omega_t = \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})]. \tag{C.54}$$

The right side of (C.17) is equal to $\beta\omega_{t+1}/\lambda$ for each t. Hence, $\{\omega_t, y_t\}_{t=0}^{\infty}$ satisfies (C.51). By definition of $\hat{y}$,

$$u(y_t) - \eta(y_t) \leq u(\hat{y}) - \eta(\hat{y}) \text{ for all } t \in \mathbb{N}_0.$$

It follows from (C.18) that $\{\omega_t\}_{t=0}^{\infty}$ satisfies (C.53). Finally, by (C.54),

$$\omega_t = \alpha[u(y_t) - \eta(y_t)] + \beta \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s+1}) - \eta(y_{t+s+1})] = \alpha[u(y_t) - \eta(y_t)] + \beta\omega_{t+1}$$

for all $t \in \mathbb{N}_0$. Hence, $\{\omega_t, y_t\}_{t=0}^{\infty}$ satisfies (C.50).

Conversely, suppose that $\{\omega_t, y_t\}_{t=0}^{\infty}$ satisfies (C.50)-(C.53). Then, $\{y_t\}_{t=0}^{\infty}$ satisfies (C.18) by (C.52). To show (C.17), define, for each $t \in \mathbb{N}_0$,

$$\omega_t' = \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})]. \tag{C.55}$$

By (C.51), it suffices to show that $\omega_t \leq \omega_t'$ for all $t \geq 0$. Let t be given. We show by induction on T that

$$\omega_t \leq \sum_{s=0}^{T} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+1}\omega_{T+1}. \tag{C.56}$$

When $T = 0$, this follows from (C.50). Suppose that it holds for T. Then,

$$\begin{aligned}
\omega_t &\leq \sum_{s=0}^{T} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+1}\omega_{T+1} \\
&= \sum_{s=0}^{T} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+1}\left\{\alpha\left[u(y_{T+1}) - \eta(y_{T+1})\right] + \beta\omega_{T+2}\right\} \\
&= \sum_{s=0}^{T+1} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+2}\omega_{T+2}.
\end{aligned}$$

This proves (C.56). Now, because, by (C.53), $\omega_{T+1} \leq \bar{\omega}$ for all T, it follows from the limit by taking T to infinity in (C.56) that $\omega_t \leq \omega_t'$. □

Because of Lemma 6, we may replace the constraints (C.17) and (C.18) by (C.50)-(C.53). Note that the initial condition for the promised utility, ω_0, is also a choice variable.

Define the planner's value function, $V(\omega)$, as follows:

$$V(\omega) = \max_{\{y_t\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \alpha\left[u(y_t) - v(y_t)\right]$$

subject to (C.50)-(C.53) with $\omega_0 = \omega$. From the Principle of Optimality V satisfies the following Bellman equations,

$$V(\omega) = \max_{y,\omega'} \left\{\alpha\left[u(y) - v(y)\right] + \beta V(\omega')\right\} \tag{C.57}$$

$$\text{s.t.} \quad -\eta(y) + \beta\frac{\omega'}{\lambda} \geq 0 \tag{C.58}$$

$$\beta\omega' \geq \left\{\omega - \alpha\left[u(y) - \eta(y)\right]\right\} \tag{C.59}$$

$$y \in [0, y^*],\ \omega' \in [0, \bar{\omega}]\,. \tag{C.60}$$

The proposition below shows that the above Bellman equation is well-defined and that V is uniquely determined. As a result, the maximization problem (C.16)-(C.18) is reduced to

$$\max_{\omega_0 \in [0,\bar{\omega}]} V(\omega_0).$$

Proposition 15. *Suppose that $y^* > y^{\max} > \hat{y}$.*

(1) The value function V is the unique solution to (C.57)-(C.60), and is continuous and weakly decreasing in ω.

(2) The function V is concave in ω if η is convex.

Proof. (1) First we show that, for any $\omega \in [0, \bar{\omega}]$, the set of elements $(y, \omega') \in [0, y^*] \times [0, \overline{\omega}]$ satisfying (C.58)-(C.60) is nonempty and hence the maximization problem is well-defined. For all $\omega \in [0, \bar{\omega}]$, define $y_\omega \leq \hat{y} \leq y^*$ as the unique solution to

$$\omega = \frac{\alpha}{1-\beta}[u(y_\omega) - \eta(y_\omega)]. \tag{C.61}$$

As $u(0) - \eta(0) = 0$ and $\frac{\alpha}{1-\beta}[u(\hat{y}) - \eta(\hat{y})] = \overline{\omega}$, such $y_\omega \in [0, \hat{y}]$ exists by the Intermediate Value Theorem. We claim that (y_ω, ω') satisfies (C.58)-(C.60) for any $\omega' \in [\omega, \bar{\omega}]$. First (C.60) holds by construction. Moreover, rearranging (C.61), we have

$$\beta\omega = \omega - \alpha[u(y_\omega) - \eta(y_\omega)]$$

which implies (C.59) for any $\omega' \geq \omega$. Finally, by (C.61) and the fact that $y \leq \hat{y} \leq y^{\max}$,

$$\eta(y_\omega) \leq \beta\frac{\omega}{\lambda} \leq \beta\frac{\omega'}{\lambda}$$

for any $\omega' \geq \omega$.

We now show that the Bellman equation (C.58)-(C.60) has a unique solution. Let $\mathscr{C}[0, \bar{\omega}]$ be the complete metric space of continuous functions over $[0, \bar{\omega}]$ equipped with the sup norm. Define $T : \mathscr{C}[0, \bar{\omega}] \to \mathscr{C}[0, \bar{\omega}]$ by

$$T(W)(\omega) = \max_{y, \omega'} \left\{ \alpha \left[u(y) - v(y) \right] + \beta W(\omega') \right\},$$

subject to (C.58)-(C.60). Note that $T(W) \in \mathscr{C}[0, \bar{\omega}]$ by the Theorem of Maximum. The mapping T satisfies the Blackwell sufficient condition (Lucas *et al.*, 1989, Theorem 3.3), and hence T is a contraction mapping, which admits a unique fixed point by the Banach Fixed-Point Theorem. Hence, V is the unique solution to the Bellman equation and is continuous.

Notice that by decreasing ω we increase the set of (y, ω') that satisfies (C.58)-(C.60), but without affecting the objective function. Hence, V is weakly decreasing.

(2) Assume now that η is convex. To show that V is concave, we show that T preserves concavity. Let $\omega_0, \omega_1 \in [0, \bar{\omega}]$ be given. Let (y_0, ω_0) and (y_1, ω_1) solves (C.58)-(C.60) for ω_0 and ω_1, respectively. let $\epsilon \in (0, 1)$ be given. Then,

$$\begin{aligned}
& T(W)(\epsilon\omega_0 + (1-\epsilon)\omega_1) \\
\geq\ & \alpha \left[u(\epsilon y_0 + (1-\epsilon) y_1) - v(\epsilon y_0 + (1-\epsilon) y_1) \right] + \beta W(\epsilon\omega_0' + (1-\epsilon)\omega_1') \\
\geq\ & \alpha\epsilon[u(y_0) - v(y_0)] + \alpha(1-\epsilon)[u(y_0) - v(y_0)] + \beta[\epsilon W(\omega_0') + (1-\epsilon) W(\omega_0')] \\
=\ & \epsilon T(W)(\omega_0) + (1-\epsilon) T(W)(\omega_1).
\end{aligned}$$

The first inequality follows from the fact that $(\epsilon y_0 + (1-\epsilon)y_1, \epsilon\omega_0' + (1-\epsilon)\omega_1')$ also satisfies (C.58)-(C.60) for $\omega = \epsilon\omega_0 + (1-\epsilon)\omega_1$ because η is convex. The second inequality follows from the concavity of $u - v$ and the assumed concavity of W. □

S5. Optima under arbitrary trading mechanism

We characterize the optimal credit equilibrium allocation taking the mechanism to set the terms of the loan contract, η, as given. Although Propositions 14 are obtained under competitive pricing, they hold for any arbitrary trading mechanism, η. For example, if η is determined by proportional bargaining, then $\hat{y} = y^*$, and Parts 1-2 of Proposition 14 imply that the best PBE corresponds to the highest steady state, $y_t = y^{\max}$ for all t. Proposition 14 also applies to generalized Nash bargaining: if $y^{\max} \leq \hat{y} \leq y^*$ or $y^* \leq y^{\max}$, then the best PBE is the highest steady state (in the proof of the proposition we only use the fact that $\hat{y}$ is the unique maximizer). However, under Nash bargaining, the loan contract η may not be convex in general, and hence Proposition 14 may not apply. Nevertheless, we showed that (3.34)-(3.37) defines a contraction mapping so that we can easily solve for the best PBE allocation numerically.

In Figure C.1 we adopt the same functional forms and parameter values as the ones in the bottom left panel of Figure 3.10. The top left panel plots y_t while the right panel plots V_t^b. It can be seen that the allocation that maximizes social welfare is non-stationary: $y_0 > y_1 = y_t$ for all $t \geq 1$, in accordance with Part 3(b) of Proposition 14. The logic for why the solution is non-stationary is similar to the one described in the case of price taking. Given that the buyer's surplus is hump-

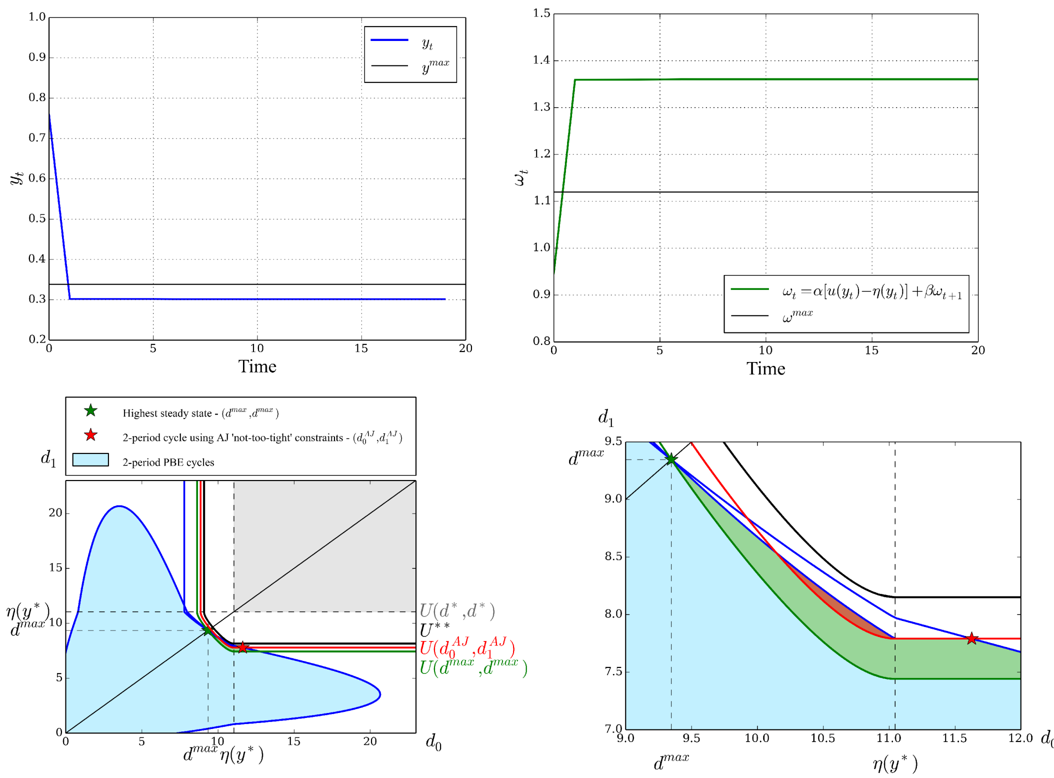

Figure C.1: Top panels: Best PBE under Nash bargaining; Bottom panels: 2-period cycles and their welfare properties under Nash bargaining.

shaped, one can implement a high level of output in the initial period by promising a high utility to buyers in the future, which is achieved by lowering future output. In the bottom panels of Figure C.1 we represent the set of 2-period cycles under the same parametrization. There are a continuum of cycles that dominate the periodic equilibria obtained under "not-too-tight" solvency constraints (the red area) and the highest steady state (the green area).